Plastic Design in High Strength Steel

EXPERIMENTS ON HIGH STRENGTH STEEL MEMBERS

by

Peter F. Adams
Maxwell G. Lay
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Fritz Engineering Laboratory Report No. 297.8
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This report describes tests performed on members of ASTM A441 steel. The testing program included basic material property tests, beam tests under uniform moment and moment gradient as well as beam-column tests. The results of the tests are presented and discussed.
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I INTRODUCTION

Since World War II plastic design procedures have gained steady acceptance on the North American continent. Current design codes\(^{(1)}\)(\(^{(2)}\)) allow the use of these procedures for structural carbon steels only. This restriction is justified as the bulk of the research in this area to date has been performed on steels of this family.\(^{(3)}\)

With the development of high strength steels\(^{(4)}\) and their adoption under elastic design procedures, increasing interest has been taken in determining their suitability for plastic design. In extending plastic design to high strength steel many problems will involve only a consideration of the altered yield stress level. In other cases, where the problem is not one of strength alone or where the solution previously developed for the A7 type steel is semi-empirical, a more detailed investigation will be required.

The evaluation of plastic design procedures as applied to high strength steels has been hampered by a lack of information on the behavior of high strength steel members in the inelastic range.\(^{(5)}\) With the exceptions of the few investigations listed in Ref. 5, the only relevant data available on the behavior of high strength steel members consists of a series of tests on bolted joints\(^{(6)}\) and columns.\(^{(16)}\)

A project, which has as its aim the extension of plastic design procedures to include steels of up to 50 ksi yield stress, has been in progress at Lehigh University since 1962. Within the framework of this project, work is being carried on to determine material properties
and basic member behavior and to extend this knowledge to predict the behavior of frames. The research performed has, of necessity, led to a re-examination of past research in the light of the different material properties. Several investigations outlining these developments have been already presented. (7)(8)(9)

The experimental program required to complement the theoretical investigations mentioned above is also in progress. Tests were performed to evaluate the material properties and to gain further insight into the lateral-torsional and local buckling characteristics of high strength steel members. The steel chosen for the test program was ASTM A441, a high strength steel with a nominal yield strength of 50 ksi for thicknesses less than 3/4". This is a weldable steel, covered under ASTM Specifications, (10) which is presently acceptable under elastic design procedures. (1) Table I contains a summary of the chemical composition of the steel used in the Lehigh test program as well as values for the yield point and ultimate tensile strength for various material thicknesses. This table was drawn up from information obtained from the mill test reports.

The complete test program to date is summarized in Table 2 and it includes:

(1) 30 tension coupons
(2) 2 residual stress measurements
(3) 2 concentric stub column tests
(4) 2 eccentric stub column tests
(5) 6 beam tests-uniform moment
(6) 5 beam tests—moment gradient
(7) 2 beam column tests.

The sections used were as-rolled beams ordered from the manufacturer under normal purchasing conditions. No attempt was made to obtain special steels. The sections chosen represent a fairly wide range of width to thickness ratios.

The purpose of this report is to describe the tests listed above and to discuss the results. Future reports will combine these results with theoretical solutions in an attempt to obtain rational plastic design procedures for high strength steels.
2. MATERIAL PROPERTIES FROM TENSILE COUPONS

The main tool used in the determination of the material properties was the standard ASTM tension test. The specimens used were 1 1/2" wide and had an 8" gage length. Table 3 summarizes these tests. The tests were performed on a screw type machine and an automatically-recorded load-elongation curve was obtained. As all members of any one cross section were rolled from the same heat, the results within each section should be free from variations due to changes in the steel composition.

The measured value of the modulus of elasticity $E$ has been purposely omitted from Table 3. The modulus of elasticity is dependent on the binding forces between atoms and is almost insensitive to the presence of alloying elements.\(^\text{(23)}\) For this reason, and because it is impossible to obtain consistent values of $E$ from the standard tension test as presently performed, $E$ has been taken from a series of very careful coupon tests on low-alloy steels as 29,500 ksi.\(^\text{(11)}\)

In Table 3 $\sigma_{ul}$ represents the upper yield point, $\sigma_{ys}$ the maximum stress observed, $\sigma_{ys}/E$ the static yield point, $\epsilon_{yst}$ is a calculated value equal to $\sigma_{ys}/E$ (E=29,500 ksi), $\epsilon_{yst}$ is the strain at the onset of strain hardening. This latter property exhibits a considerable variation. However, $\epsilon_{yst}$ depends upon the distribution of inhomogeneities in the specimen as well as the amount of prestraining that has taken place in the coupon material before testing. When $\epsilon_{yst}$ is examined for a particular group of specimens from the same location
in the member, the variation is reasonable.

The values obtained for the strain hardening modulus, $E_{st}$, varied considerably. Studies of member behavior have shown that the strain hardening modulus is a critical property for predicting the post elastic behavior of beams and beam-columns. Thus, a reliable value of $E_{st}$ is essential. The determination of $E_{st}$ requires knowledge of the slope of the load-deformation curve in the strain hardening range. Attempts to measure this slope from dynamic tests have proven unsuccessful.

In order to obtain a consistent value for $E_{st}$, a special series of tension tests were performed. These tests have been described in a previous report, but a brief summary will be included for completeness.

Fig. 1 shows the automatically obtained load-strain relationship for Test HT-46, one of the special series of tests. To obtain $E_{st}$ from this test the difference in loads and strains between points 6 and 7 on the graph must be obtained. The strain difference is obtained with sufficient accuracy from measurement on the graph. However, the difference in loads is so small that it cannot be obtained in the same manner.

To obtain the value of the load at points 6 and 7 (Fig. 1) it is necessary to stop the testing machine and read the load value from the dial. This load is not stable, but decreases with time. This may be seen on Fig. 1. Fig. 2 presents the measured load drop-time at zero strain rate relationship for points 6 and 7 of Fig. 1. The load did not stabilize completely until a period of roughly 24 hours had elapsed. However, the
load drop-time relationship of the two stages coincided almost exactly. Thus, provided the load was recorded after a consistent time interval at zero strain rate, the load difference between two points could be obtained. This procedure was utilized to determine $\varepsilon_{\text{ST}}$ for seven specimens. The results of the tests are recorded in Table 4. It can be seen that the values of $\varepsilon_{\text{ST}}$ are consistent to within about 2% with the exception of Tests HT-44 and HT-49.

The relaxation of load with time at zero strain rate also occurs in the plastic range. Fig. 3 gives the load drop-time at zero strain rate relationship for points 3, 4, and 5 of Test HT-46 (Fig. 1). The curve obtained is similar to that obtained in the strain hardening range. Here, however, the problem is not one of determining a difference in load, but of determining a value of the load (stress) corresponding to the predicted duration of loading on the structure. Thus in Table 4 two values have been listed for the static yield stress, the first is that obtained after 30 minutes at zero strain rate, the second is the (estimated) fully stabilized yield stress.

For reference purposes the measured material properties from Tables 3 and 4 have been averaged and plotted as the idealized stress-strain curve shown in Fig. 4. It should be remembered that this curve represents average values only for the specimens tested. The specimens represent two different heats and four different rolled shapes.
3. RESIDUAL STRESS MEASUREMENTS

Two residual stress measurements, Tests HT-9 and HT-19 were performed and the results are given in Fig. 5. The residual stresses were determined by sectioning the plates into 1 in. wide nominal slices.\textsuperscript{(3)}

The distribution of residual stress in wide-flange shapes of both A7 steel and A242 steel has been extensively investigated.\textsuperscript{(15)(16)} The investigation of A242 steel was performed (1959) before the ASTM specified A441 steel. The two steels are essentially alike, the major difference being the specification of minimum percentages of copper and vanadium for A441 steel.\textsuperscript{(10)(17)} Thus, it is expected that the residual stress distribution in the two steels would be similar.

From previous investigations\textsuperscript{(15)(16)} the maximum compressive residual stress, $\sigma_{RC}$, was found to occur at the flange tips. For the A7 steel specimens tested $\sigma_{RC} = 12.8$ ksi or $0.39 \sigma_Y$. For the A-242 specimens, $\sigma_{RC} = 12.1$ ksi or $0.22 \sigma_Y$. The values above are average values for the specimens reported. As can be observed from Fig. 5, the average residual stress at the flange tips was $7.3$ ksi or $0.14 \sigma_Y$ for the A441 specimens. It must be remembered that this figure was obtained from only two investigations and no general conclusions should be made.

Earlier investigators\textsuperscript{(16)} have concluded that the distribution of residual stress does not depend on the yield stress level. It would appear from the results on A7 and A242 steels that this is so, but as yet an insufficient number of tests have been performed for this same conclusion to be applied to A441 steel.
4. CONCENTRIC STUB COLUMN TESTS

The non-dimensionalized load-deformation relationships obtained from two concentric stub column tests, Tests HT-8 and HT-20, are presented as Figs. 6 and 7. Since the concentric stub column test produces a uniform stress distribution over the cross section the stress at the onset of local buckling may be determined rather precisely.

The deformations which form the basis for the curves of Figs. 6 and 7 are the average contraction over the column length. Local buckling is initiated at some point of material weakness or initial curvature in the flange. Once local buckling has commenced the deformations tend to concentrate in the area of the buckle. This behavior is shown emphatically by the photograph of Fig. 8 which shows the stub columns at load #41 (Fig. 7). Thus, while it is the averaged deformations that form the basis of the load-deformation curves, the post local buckling deformations are concentrated almost entirely in the area of the buckle.

The post local buckling behavior may be followed through the sequence of photographs shown in Fig. 9. Fig. 9(a) shows the stub column of Test HT-20 before local buckling occurred. This corresponds to load No. 33 in Fig. 7. Fig. 9(b) shows the column after definite local buckling had been observed and unloading had commenced; (Load No. 39, Fig. 7) and Fig. 9(c) shows the condition of the stub column at the conclusion of the test. (Load No. 41, Fig. 7.) Fig. 10 shows the change in distance between flanges along one side of the stub column. The numbers against the curves refer, again, to the load numbers of Fig. 7. This plot, together with the photographs of Fig. 9, serves to emphasize the local nature of the post
local buckling deformations.

In both tests first signs of localized yielding occurred at relatively low loads (See Figs. 6 and 7). However, this occurred at points of stress concentration and it was not until much higher loads \(\frac{P}{P_y} = 0.78\) for Test HT-8 and \(\frac{P}{P_y} = 0.82\) for Test HT-20) that general yielding was observed. From the residual stress measurements (see Fig. 5) it would be expected that yielding would commence at \(\frac{P}{P_y} = 0.82\) for Test HT-8 and at \(\frac{P}{P_y} = 0.75\) for Test HT-20. Thus visually observed values check reasonably well with the measured residual stresses.
5. ECCENTRIC STUB COLUMN TESTS

Two eccentric stub column tests, Test HT-2 and Test HT-18, were performed as part of the test program. The moment-curvature relationship for Test HT-2, which had a load eccentricity of 0.500 d, (d is the depth of the stub column section) is shown in Fig. 11. That for Test HT-18, which had a load eccentricity of 0.218 d, is given as Fig. 12.

These tests have been described in detail in an earlier report, (18) and the moment-curvature relationships are included here only for completeness. However, the overall significance of the stub column tests can be realized through an examination of Fig. 13 which compares the results obtained from the four stub column tests with the theoretical interaction curve. (19) The ratio of the areas of flanges to web chosen correspond to an average for the 8WF31 and 14WF78 sections. Also shown are the results of the tests on beams under uniform moment, which are discussed in the following section. In this figure w represents the web thickness and d, the total depth of the beam.
6. **BEAM TESTS**

A beam designed according to plastic theory must deliver the full plastic moment, \( M_p \), and maintain this moment while the hinge rotates inelastically an amount sufficient to ensure the development of a mechanism. This ability to deform inelastically is largely a function of the adequacy of the bracing system.

Beam tests on A7 steel which are pertinent to this problem have previously been reported.\(^{(20)}\) The series of tests on high strength steels are similar in most respects to these.

The objectives of the present tests on high strength steel are:

(a) To study the behavior of high strength as-rolled steel beams and to determine whether or not this behavior differs significantly from that observed for A7 beams.

(b) To examine carefully and record beam behavior, and to obtain the data necessary for a parallel theoretical study.\(^{(7)}\)

(c) To investigate the rotation capacity of high strength steel beams and to determine the optimum unbraced length.

To achieve these objectives two series of beam tests were performed. The first series used beams having the critical span subjected to uniform moment. Under this condition, yielding reduces the stiffness of the critical span and thus lateral deflections due to initial imperfections of the beam are greatly magnified. The combined lateral bending and axial strains produce a critical situation in the compression flange, leading to local buckling and eventually unloading. The second series of tests were performed on simply supported beams under a central concentrated load which produced a moment that varied from zero to a
maximum. In this case, yielding is localized at the area of maximum moment. Thus the beams retained much of their original stiffness and the strains required to produce local buckling of the compression flange had to be produced by in-plane bending only. The two situations are discussed in the following two sections. One intermediate type of test, Test HT-38, was also performed. In this test the adjacent spans were adjusted so that the moment at one interior support was 0.8 times that at the other. This test will be discussed separately.

The 10WF25 and 8B13 sections were chosen to represent respectively a typical and a weak beam section and also to facilitate comparison with previous tests. (20,21,22)
(a) BEAMS UNDER UNIFORM MOMENT

Test Program:

Table 5 summarizes the test program which consisted of six beams each having a center span under uniform moment. Test HT-38 is also included in Table 5, but will be discussed in a separate section. The maximum moments and the rotation capacities obtained for each test are given in Table 5. The full plastic moment, \( M_p \), was calculated by assuming the cross section to be composed of three rectangles; using the appropriate static yield stress level as obtained from tensile tests and finally calculating the moment of the forces (yield stress times corresponding area) about the centroid. The dimensions used in this calculation were taken from the AISC Manual\(^1\) as preliminary measurements indicated that actual dimensions were close to those listed.

The rotation capacities were obtained from the non-dimensionalized experimental moment-rotation curves and the capacities were taken at 0.95 \( M/M_p \)\(^7\) on the unloading branch of the \( M-\Theta \) curve.

Test Arrangement

The experimental set-up was similar to that described in detail in an earlier report.\(^20\) The beams were subjected to static concentrated loads at the ends and they were supported vertically at their third points to provide a center span under constant moment, as shown in the sketch of Table 5. Lateral bracing was provided at each load and reaction point.

After each increment of load (or deformation) vertical rotations
and deflections, lateral deflections and strains were measured.

To extend the range of strains that could be measured, a Whittemore Mechanical Gage was used in pre-punched holes for strain measurement beyond the range of the SR4 gages. Excellent correlation between the two methods of strain measurement was observed.

**Discussion of Results**

The test results can best be presented by tracing the behavior of a typical specimen (Test HT-29). The other tests in this series exhibited behavior similar to Test HT-29. A summary of the series will complete the discussion.

The non-dimensionalized moment-rotation relationship for test HT-29 is shown in Fig. 14. The rotations plotted are the average of those measured at the two reaction points. \( \Theta_p \) is the rotation at which the full plastic moment would be attained, under ideal elastic behavior.

Up to load #4 on Fig. 14, the \( M-\Theta \) relationship was linear. Visual confirmation of the initiation of yielding was provided by the flaking of the whitewash coating from the specimen during the application of load #5. This is noted as "first yield" on Fig. 14.

As loading continued, flange yielding progressed inward from the compression flange tips to the center. Yielding also penetrated the tension flange, but at slightly higher loads due to the more favorable residual stress distribution. At load #8 the full plastic moment was attained and yielding had started to penetrate into the web.

On further loading the beam continued to rotate with slight change in load. At load #12 indications of local buckling were present and at load #14 a definite local buckle could be observed in the compression
flange. The buckle was in the form of a symmetrical wave with its maximum ordinate at the mid span of the beam.

The test was terminated at $\Theta/\Theta_p = 6.65$, at this point the $M/M_p$ value had dropped to 0.97 and large deflections caused the loading jacks to become unstable. The rotation capacity for Test HT-29 was 5.7. (See Table 5) For all tests in this series the gradual unloading after the formation of a local buckle was typical.

The non-dimensionalized moment-vertical deflection curve is given as Fig. 15. In this Figure $U_0$ represents the vertical deflection at the center of the beam and $U_{op}$ represents this deflection at $M/M_p = 1.0$, assuming ideal elastic behavior.

The lateral deflections along the length of the beam are shown in Fig. 16. The numbers against each plot correspond to the load numbers as given on the $M-\Theta$ curve, Fig. 14.

As expected, the tension flange movements were small throughout the test and only the deflected shape at load #14 is given. Figure 17(a) serves to further emphasize this point. The compression flange movements were also small up to load #8. After the attainment of $M_p$ the lateral deflections of the compression flange increased rapidly causing relative lateral displacements of the two flanges. Figures 17(a) and (b) illustrate this behavior. The maximum lateral deflection occurred at the mid-span of the compression flange and was 1.26 inches at termination of the test.

Figs. 18(a) to 18(d) show the progression of yielding in the critical span. The photographs refer to Test HT-37, however, similar yield patterns were observed for all tests. Fig. 18(a) shows the
underside of the compression flange at mid-span. This photograph was taken one load increment before the full plastic moment was attained and before the observation of significant lateral deflection. The only yield lines present reveal a uniform compressive strain in the flange.

Fig. 18(b) was taken at a rotation of approximately three times $\theta_p$. In this photograph the tension yield lines are clearly visible in the top flange and the top part of the web. Compression yield lines can be seen toward the bottom part of the web. On the bottom flange it can be clearly seen that the near side is in tension at mid-span and is moving toward the eye. The major portion of the yield lines now present are due to the lateral deformations.

Fig. 18(c) was taken after the beam had started to unload. By this time the flange exhibited a definite local buckle which is not clearly visible in this photograph; however, it may be seen in Fig. 18(d) which was taken from the opposite side of the beam and shows a better view of the pattern of yield lines. The compressive yielding in the central area, the lateral inflection point areas, and finally the tensile yielding near the supports are all visible.

Consideration of the deflections of the beam together with an examination of the deflected shape of the compression flange in the critical span revealed that its behavior is essentially that of a restrained column. The yield line pattern shown in Figs. 18(a) to 18(d) and again in Fig. 19 confirmed this behavior. Fig. 19 was also taken from Test HT-37 after the completion of the test. As loading progressed the strain across the compression flange was at first uniform, then after the
full plastic moment was attained and significant lateral deflections observed the bending action predominated and the mid-span portion of the flange deflected laterally. However, the entire span was restrained by the stiff side spans which remained elastic. The compression flange of the critical span was thus almost completely fixed at the ends. The yield line pattern in Fig. 19 reveals slight compression yielding over the whole span, then superimposed on this is the lateral bending pattern of tension yield lines at mid-span and more compression yield lines in the restraining areas near the supports. Measurements on the yield line pattern gave an effective length of between 0.5 and 0.6. Complete fixity at the ends would require an effective length of 0.5. Part of the discrepancy may be accounted for by the movement of the supports during testing. It may be seen from Fig. 16 that in the progress of Test HT-29 lateral movements of 0.09 inches occurred at the braces.

It is difficult to determine for Test HT-29 whether these movements were due to deformations of the braces themselves or slippage in the bolted connections. For later tests the connections were spot welded and bolted. Even, after this, for instance for Test HT-41, considerable (0.25") inelastic deformation occurred.

In order to obtain a complete picture of the strain distribution in the critical span, strains were measured by SR4 gages supplemented by Whittemore gage readings where necessary. The locations of the SR4 gages for Test HT-37 are shown in Fig. 23(b). The gages shown are those attached to the underside of the bottom flange. In addition, nine gages were placed at the mid-span of the beam. These gages are not shown in Fig. 23(b) but were located so as to give the complete strain distribution at mid-span. The Whittemore gage locations were punched to
coincide with the SR4 gage locations at mid-span. The gaging for all tests was of a similar pattern and only the longitudinal gage spacing varied depending on the span of the beam. Fig. 20 shows the strain distribution across the web at mid-span for selected stages of loading. For low loads, before the attainment of the plastic moment (#5) the neutral axis is located at mid-depth of the beam. By the time $M_p$ is reached (#8) the neutral axis had dropped below mid-depth. The neutral axis gradually rose as rotation continued (#11) and finally dropped once again (#14) at the point of unloading. It should be noted that the above behavior was not typical of all tests in the series. The neutral axis location seemed to fluctuate randomly as inelastic rotation took place. This behavior may possibly be due to yield lines passing randomly through the various strain gages. No consistent variation could be observed between tests.

The strain distribution across the tension flange is shown in Fig. 21. For low loads (#5) a uniform tensile strain exists. At load #8, corresponding to the attainment of the full plastic moment, the lateral deflection contribution comes into the picture. As rotation increases (#11 and #14) this deflection contribution remains approximately constant and the flange elongates with increasing inelastic rotation.

In contrast to this behavior, the strain distribution in the compression flange is given in Fig. 22. Again the strain distribution at low loads (#5) was almost uniform. Bending action entered into the picture at load #8, corresponding to the load at which significant lateral deflections were observed. As the inelastic rotation proceeded the bending action continued to dominate the strain distribution. The curve
representing the strain distribution across the flange rotated about a point located approximately 0.75 inches from the unloading edge of the flange. The distribution for loads #8 and #11 appears somewhat irregular but this is due to yield lines passing randomly through the gage length. At load #14, the point at which unloading commenced, the distribution was again linear.

It is interesting to note the magnitudes of the observed strains at the various stages of loading. At load #8, the point at which the full plastic moment was attained the maximum strain in the compression flange was 0.006, much less than that associated with the onset of strain hardening 0.019. For load #14, however, the point at which definite local buckling was observed, the loading side of the compression flange was almost completely strain-hardened, that is, fully yielded.

In Fig. 23(a) the strains at various points along the compression flange are plotted for loads corresponding to the attainment of the full plastic moment \( M_p \) and definite local buckling (L.B.). Test HT-37 was again used to illustrate this figure because the gage locations were such that the results serve to emphasize the restrained column action of the compression flange. All tests behaved in a similar fashion. The difference between the two strain plots represents the change in lateral curvature during the inelastic rotation under the full plastic moment.

Locations C and D were close to the mid-span of the beam and showed increasing compressive strains on the loading edge of the
flange. Fig. 18 shows the condition of this area of the flange. $M_p$ corresponds to load #6 for this test and L.B. occurs at a load between #17 and #23. Locations B and E show only minor changes in strain distribution as these locations were close to the points of inflection. Locations A and F show the reversed curvature expected as these points were close to the lateral braces and in the restraining portion of the critical span.
(b) SUMMARY OF UNIFORM MOMENT TESTS

Fig. 24 shows the non-dimensionalized moment-rotation curves for all tests in this series. With the exception of Test HT-36, which unloaded almost immediately on reaching the full plastic moment, all tests sustained considerable inelastic rotation before unloading. The rotation capacities for the tests are listed in Table 5.

It is apparent from Fig. 24 that the rotation capacity increased as the unbraced slenderness ratio, L/ry, decreased. For the tests performed, with the exception of HT-36, unloading occurred only after severe local buckling of the compression flange had been observed. Local buckling was observed visually, therefore it was impossible to determine exactly the rotation at which it occurred. As can be seen in Fig. 24, in some cases a considerable amount of post-local buckling capacity was observed. One other feature of this series that should be noted is the very gradual decrease in moment capacity after unloading commenced.

The strain distributions across the compression flange, for the load at which definite local buckling was observed, are plotted in Fig. 25. For all tests the distribution was remarkably consistent and the loading side of the flange was completely strain hardened at the time local buckling occurred. It should be emphasized that except for Test HT-36, local buckling did not coincide with unloading but that unloading followed only after severe local buckling. For Test HT-36 (L/ry=45) local buckling occurred at a rotation significantly higher than unloading. The
unloading point has been selected as the point where \( \frac{M}{M_p} = 0.95 \) to be consistent with the definition of rotation capacity used and to remove any subjective interpretations of the results.

Fig. 26 shows a plot of the lateral movement of the compression flange, \( \mu_c \), minus that of the tension flange, \( \mu_t \), at mid-span of the beam. This is plotted for all tests. The theoretical solution is shown for \( \mu_c \) as obtained from Reference 7. In all cases \( \mu_t \) is very small. The agreement with theoretical predictions is reasonable. The bars for each test represent limits taken at the points of local buckling and unloading.

Fig. 27 shows a plot of the experimentally determined rotation capacity against the unbraced slenderness ratio for all tests under uniform moment. The results plot smoothly and it appears that an \( L/\sqrt{\gamma} \) value of approximately 25 would result in an optimum rotation capacity. In Fig. 27, \( R \) is the rotation capacity as defined on the diagram.
(c) **BEAMS UNDER MOMENT GRADIENT**

Beams under uniform moment and beams under a moment which varies from zero at one end of the span to a maximum at the other, represent the two extreme cases when considering lateral and local buckling. For beams under uniform moment, complete yielding of the compression flange in the critical span led to a flexible, elastically restrained column in which the large lateral deflections produced strains leading to local buckling of the flange, and eventually to unloading. For simply supported beams under a central concentrated load the portion of the compression flange which yields is relatively small. Thus, the equivalent column is composed of a large elastic portion and a small yielded portion which occurs under the load point. This column is much stiffer than that resulting from the uniform moment case. The lateral deflections remain relatively small until the strains in the compression flange reach the values necessary for local buckling. At this point the cross section is no longer symmetrical and lateral deflections increase rapidly followed by unloading. Thus, lateral buckling is not the critical phenomenon.

Table 6 summarizes the test program for beams under moment gradient. The four simply supported beams were tested in the 300 kip hydraulic testing machine. Lateral braces were provided at the load and reaction points. The loading procedure was similar to that used for the tests under uniform moment.

The vertical deflection was measured under the load point and lateral deflections were measured at one point in each side span. The
support rotations were measured at each end of the beam. Electrical-resistance strain gages supplemented by Whittemore gages were used to determine strains at various points on the beam. In addition, strain gages were used in the center of each side span at midheight to determine principal strains.

Test HT-42 (see Table 6) will not be reported, as a premature stiffener failure invalidated the results. The stiffeners under the load point for this test met the requirements of Section 2.5 of Ref. 1, however, the stiffener was completely yielded at the attainment of the plastic moment and the expected increase in moment due to strain hardening did not take place. Test HT-43 is a repeat of Test HT-42 but the stiffeners were designed in accordance with section 1.10.5 of Ref. 1 which relates to bearing stiffeners for plate girders. Test HT-43 behaved in a satisfactory manner. Moment-rotation curves for the three tests in this series are shown in Fig. 28.

Test HT-43

The dimensions for test HT-43 were chosen to represent the lowest L/ry ratio that was practical to test. In this case the moment gradient applied to the beam was so steep that only a very small portion, directly under the load point, yielded. Thus the usual type of local buckle did not have room to form. At load #15 (Fig. 28) the first indications of flange wrinkling became evident. As loading progressed the wrinkles became more pronounced and unloading commenced at load number 19 as a result of the general "softness" and extreme deformations of the center yielded portion. Strains were measured on the compressive
flange at a distance of 6" on either side of the load point and showed an almost uniform compressive strain over the flange of less than 0.002 throughout the complete test. (c.f. Fig. 22 for uniform moment). The maximum lateral deflection measured was 0.11" at the center of the side spans at load number 19.

Test HT-28

The M-Θ curve for HT-28 is shown in Fig. 28. In this test the first sign of local buckling was observed at load #19 and unloading commenced at load #20. The non-dimensionalized M-Θ curve for this test are shown in Fig. 29. There was none of the localized deformation of the type observed for test HT-43, due to the spreading of the yielded zone.

Test HT-52

The M-Θ curve for test HT-52 is shown in Fig. 28. It is interesting to note that in spite of the large difference in length between tests HT-28 and HT-52, local buckling occurred at approximately the same increment of rotation after the full plastic moment was attained. This behavior explains the importance of local buckling in beams under moment gradient.

The lateral deflection of the compression flange at unloading (load #15) was 0.34". This was measured at points 9-1/2 on either side of the point of load application.

Possibly because of the restraining influence of the loading head, all three test beams deformed laterally in two waves symmetrical about the loading point, rather than one 'S' wave as would normally be expected.
Test HT-38

Test HT-38 was performed to investigate the case of a beam for which the center span was subjected to a moment which varied linearly from a maximum at one interior support to 0.8 $M_{\text{MAX}}$ at the other. This situation is shown in the insert of Table 5 and the M-θ curve for this test is given as Fig. 30.

The M-θ curve rose slightly above the full plastic moment due to the effect of strain hardening. Severe local buckling at load #17 (Fig. 30) was followed closely by the commencement of unloading.

The rotation capacity of Test HT-38 was found to be 4.7 as compared with 1.5 for Test HT-36 which had the same L/ry but was under uniform moment.

It was expected that beams tested as simply supported would exhibit an even greater advantage over those under uniform moment. However, a comparison of Tests HT-29 and HT-28 does not bear this out, only a slight advantage being observed for the simply supported beam (Test HT-28). It should be noted in comparing these tests, however, that all simply supported beam tests were performed on 8 B 13 sections while all other tests used 10 WF 25 sections. The ratio of width to thickness of the flange of the 10 WF 25, b/t, is 13.40 while the b/t for the 8 B 13 is 15.75. The "critical" b/t ratio for A441 steel is about 14.
7. BEAM-COLUMN TESTS

The moment-rotation curves for Tests HT-39 and HT-40 are given as Figs. 31 and 32. These tests have been completely reported in a previous report (24) and the above curves are included only for completeness. The two beam-columns had a nominal L/ry of 80 and P/P_y of 0.4. Due to unavoidable material variations the actual properties were slightly different and are given in Fig. 31 and 32. Test HT-39 utilized three lateral braces while Test HT-40 was completely unbraced.
8. SUMMARY

This report has described the results of an extensive series of tests on members of A441 steel. As the members were rolled from steel of only two different heats, the results cannot be interpreted as providing average values, however, it is thought that the results represent typical member behavior.

The tests performed to obtain material properties included standard tension coupon tests, a special series of coupon tests designed to determine the strain-hardening properties of the material, residual stress measurements and both concentric and eccentric stub column tests. Member tests were performed on beams under uniform moment and moment gradient and beam-columns.

These tests were performed to provide a base of experimental evidence on which plastic design of high strength steel could rest. In conjunction with the experimental program, parallel theoretical investigations are currently in progress. A series of future reports will describe these investigations. The two will then be combined to provide a rational basis for plastic design in high strength steels.
9. ACKNOWLEDGEMENTS

This study is part of a general investigation "Plastic Design in High Strength Steel" currently being carried out at Fritz Engineering Laboratory, Department of Civil Engineering. Professor W. J. Eney is Head of the Civil Engineering Department and Professor L. S. Beedle is Director of the Laboratory. The investigation is sponsored jointly by the Welding Research Council, and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, the American Iron and Steel Institute, Lehigh University Institute of Research, the Bureau of Ships and the Bureau of Yards and Docks. The Column Research Council acts in an advisory capacity.

The authors wish to express their appreciation to Messrs. R. A. Aglietti and B. P. Parikh for their willing assistance in the performance of the test program and to the Fritz Laboratory Staff, who aided in the performance of the various tests. Thanks are due also to Miss Nancy Turner who typed the report and Mr. H. Izquierdo who did the drawings.
10. NOMENCLATURE

A \quad \text{Area}

A_f \quad \text{Flange Area}

E \quad \text{Modulus of Elasticity}

E_{ST} \quad \text{Modulus of Elasticity in Strain-Hardening Range}

I \quad \text{Moment of Inertia}

L \quad \text{Length}

M \quad \text{Moment}

M_p \quad \text{Full Plastic Moment}

M_{pc} \quad \text{Plastic Moment Reduced by Axial Load Effect}

M_{MAX} \quad \text{Maximum Moment Acting On Member}

P \quad \text{Axial Force}

P_y \quad \sigma_y A

X,Y,Z \quad \text{Principal Section Axes}

b \quad \text{Breadth of Beam Flange}

d \quad \text{Depth of Beam}

r_y \quad \text{Radius of Gyration About Y Axis}

t \quad \text{Flange Thickness}

u \quad \text{Lateral Deflection}

u_c \quad \text{Lateral Deflection of Compression Flange}

u_t \quad \text{Lateral Deflection of Tension Flange}

w_{0} \quad \text{In Plane Deflection at Mid-Span}

w_{0p} \quad \text{In Plane Deflection at Mid-Span Corresponding to } M_p \text{ (Elastic Behavior)}

w \quad \text{Web Thickness}
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
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<td>( \beta )</td>
<td>Angle of Twist of Cross Section</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>Yield Stress</td>
</tr>
<tr>
<td>( \sigma_Y^S )</td>
<td>Yield Stress Corresponding to Zero Strain Rate</td>
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<td>( \sigma_m )</td>
<td>Upper Yield Point</td>
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<td>( \sigma_{ult.} )</td>
<td>Maximum Stress</td>
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<td>( \sigma_Y^S / E )</td>
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<tr>
<td>( \phi )</td>
<td>Curvature</td>
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Values in ksi

**TABLE 1(a) PHYSICAL PROPERTIES FROM MILL TEST REPORTS**

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**TABLE 1(b) CHEMICAL PROPERTIES FROM MILL TEST REPORTS**
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<th>HEAT</th>
<th>TENSION COUPONS</th>
<th>RESIDUAL STRESS MEASUREMENTS</th>
<th>STUB COLUMNS</th>
<th>BEAM TEST</th>
<th>BEAM COLUMNS</th>
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**TABLE 2**

**SUMMARY OF TEST PROGRAM**
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<th>$\sigma_{YS}$</th>
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**TABLE 3**

MATERIAL PROPERTIES FROM STANDARD COUPON TESTS
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**TABLE 3 - continued**
All coupons cut from flange of 10WF25 heat 143G540. Coupons std. except machined to 0.25" thick.

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<th>$\sigma_{YS_{(30 \text{ min.})}}$ ksi</th>
<th>$\sigma_{YS}$ ksi</th>
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**TABLE 4**

**MATERIAL PROPERTIES FROM SPECIAL COUPON TESTS**
### Table 5

**Beams Under Uniform Moment**

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<td>FOR HT-38 ONLY</td>
<td>1.035</td>
<td>35,45,45</td>
<td>4.7</td>
</tr>
<tr>
<td>TEST NUMBER</td>
<td>LOADING CONDITION (BEAMS SUPPORTED LATERALLY AT LOAD &amp; REACTION POINTS)</td>
<td>M/M_p(MAX) TEST RESULTS</td>
<td>L/ry</td>
<td>ROTATION CAPACITY</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------------------</td>
<td>-------------------------</td>
<td>------</td>
<td>------------------</td>
</tr>
<tr>
<td>HT-28</td>
<td></td>
<td>1.142</td>
<td>35</td>
<td>6.3</td>
</tr>
<tr>
<td>HT-42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HT-43</td>
<td></td>
<td>1.130</td>
<td>22.9</td>
<td>6.00</td>
</tr>
<tr>
<td>HT-52</td>
<td></td>
<td>1.137</td>
<td>72.3</td>
<td>4.2</td>
</tr>
</tbody>
</table>

**TABLE 6**

BEAMS UNDER MOMENT GRADIENT
Fig. 1
LOAD-STRAIN RELATIONSHIP
TEST HT-46
Fig. 2
LOAD DROP-TIME RELATIONSHIP
STRAIN HARDENING RANGE
TEST HT-46
Fig. 3 LOAD DROP-TIME RELATIONSHIP INELASTIC RANGE TEST HT-46
$E = 29,500$ ksi (See Ref. 11)

$\sigma_{YS} = 52.0$ ksi

$\sigma_{YS(30MIN)} = 54.0$ ksi

$E_{st} = 704$ ksi

Fig. 4

MATERIAL PROPERTIES
Fig. 5

RESIDUAL STRESS DISTRIBUTION
Fig. 6  LOAD-DEFLECTION RELATIONSHIP
TEST HT-8

\[ \delta_y = \frac{P_y L}{AE} \]
Fig. 7  LOAD-DEFLECTION RELATIONSHIP
TEST HT-20
Fig. 8 POST BUCKLING DEFORMATION
TEST HT-20
Fig. 9  STUB COLUMN DEFORMATIONS

(a) BEFORE LOCAL BUCKLING

(b) AFTER SEVERE LOCAL BUCKLING

(c) AFTER COMPLETION OF TEST
Fig. 10  

FLANGE DEFORMATIONS  
TEST HT-20
Fig. 11  MOMENT-CURVATURE RELATIONSHIP
TEST HT-2
Fig. 12  MOMENT-CURVATURE RELATIONSHIP
TEST HT-18

Severe Local Buckling

Web Fully Yielded; Local Buckling noted

Yielding to Specimen

Web Yields
Yielding in Compression Flange

Yielding at Web Between Specimen and Base Plate

M_p = 2640 Kip-in.

HT-18
14 WF 78
Fig. 13  MOMENT-THRUST INTERACTION DIAGRAM
First Yield

\[ \frac{M}{M_p} = 5 \frac{M_p L}{6EI} \]

Local Buckling

\[ \theta_p = \frac{5 M_p L}{6EI} \]

Fig. 14  MOMENT-ROTATION RELATIONSHIP
TEST HT-29
Fig. 15
MOMENT-VERTICAL DEFLECTION
RELATIONSHIP
TEST HT-29

\[ v_{op} = \frac{M_p L^2}{8EI} \]

First Yield
Definite Local Buckling
Fig. 16  LATERAL DEFLECTIONS
TEST HT-29
Fig. 17(a)  
LATERAL MOVEMENTS  
TEST HT-29

Fig. 17(b)  
TWIST OF CROSS SECTION  
TEST HT-29
Fig. 18  PROGRESSION OF YIELDING
TEST HT-37
Fig. 19  TEST HT-37  SHOWING LATERAL DEFLECTION
Fig. 20

STRAINS AT MID SPAN
TEST HT-29
Fig. 21
TENSION FLANGE
STRAIN DISTRIBUTION
TEST HT-29
Fig. 22

COMPRESSION FLANGE
STRAIN DISTRIBUTION
TEST HT-29
Fig. 23(a)  COMPRESSION FLANGE STRAINS
TEST HT-37

23(b)  LOCATION OF SR4 GAGES
TEST HT-37
Local Buckling

HT-31: $L/r_y = 30$

Local Buckling

HT-29: $L/r_y = 35$

Local Buckling

HT-37: $L/r_y = 37.5$

Local Buckling

HT-30: $L/r_y = 40$

Local Buckling

HT-36: $L/r_y = 45$

Fig. 24

MOMENT ROTATION RELATIONSHIPS
Fig. 25  STRAINS AT LOCAL BUCKLING
Fig. 26 RELATIVE FLANGE MOVEMENTS
Fig. 27
ROTATION CAPACITY
BEAMS UNDER UNIFORM MOMENT
Fig. 28  MOMENT-ROTATION RELATIONSHIPS  BEAMS UNDER MOMENT GRADIENT
Fig. 29
MOMENT-ROTATION
RELATIONSHIP TEST HT-28
Fig. 30
MOMENT-ROTATION
RELATIONSHIP TEST HT-38
First Yield, Twisting Observed

Fig. 31

MOMENT-ROTATION RELATIONSHIP TEST HT-39

BW 31
Braced
\( \gamma' = 50 \text{ ksi} \)
\( P = 0.425 P_y \)
\( L_{r*} = 81.1 \)
Fig. 32
MOMENT-ROTATION RELATIONSHIP TEST HT-40
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