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BENDING AND SHEAR STRENGTH OF LONGITUDINALLY STIFFENED PLATE GIRDER

by

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ABSTRACT

The general objective of this report is to determine the effect of longitudinal stiffeners on the static behavior of plate girder panels and to determine the contribution of longitudinal stiffeners to the ultimate strength of plate girders.

The results of thirteen tests on nine full size girders are described and discussed. The ultimate strength theories developed by Basler and Thürlimann for transversely stiffened plate girders are also reviewed and discussed. Using these theories and the test results as a starting point, methods of predicting the strength of longitudinally stiffened plate girders are developed.

Bending tests on longitudinally stiffened plate girders indicate that by controlling lateral web deflections, a longitudinal stiffener can maintain a linear stress distribution in the girder section. It is suggested that a properly positioned and proportioned longitudinal stiffener can maintain this linear stress distribution until the ultimate moment is reached, thus eliminating the need for a reduction in ultimate bending stress. Stiffener positioning and proportioning requirements are formulated to ensure that the bending stress distribution remains linear. For test girders with stiffeners which fulfill these requirements, the observed ultimate loads agree very closely with those predicted by the theory.
Shear tests on longitudinally stiffened plate girders reveal that a longitudinal stiffener can control lateral web deflections to the extent that separate tension fields can be developed in the subpanels formed by the stiffener. Accordingly, a shear strength theory is formulated by assuming that the shear strengths of the subpanels can be developed independently of adjacent subpanels and that the ultimate shear force of the stiffened panel is the sum of the ultimate shear forces of the subpanels. Using this theory, the shear strength is computed for a number of panel sizes and stiffener positions. Comparison of these predictions with test results indicates that the theory provides a reliable, though somewhat conservative estimate of the shear strength of longitudinally stiffened plate girders.
Chapter 1

INTRODUCTION

1.1 Background

The essential difference between an I-shaped plate girder and a beam is the slenderness of the web. Because of this difference, the methods commonly used in the design of beams are not applicable to plate girders. The proportions of a plate girder have been determined from a stability consideration, that is, the theoretical web buckling stress has been the criterion for failure or limit of usefulness. Various arrangements of stiffeners have been used to reinforce the web and thus raise the buckling stress. The elastic or linear buckling of plates with various combinations of boundary conditions, loading and stiffener arrangements has been investigated extensively\(^1,2\), so that the theoretical web buckling stress can be determined without much difficulty.

Classical plate buckling theory is based on the assumptions that the plate is initially undeformed and that lateral deflections which develop under load are small relative to the plate thickness. In applying this theory to plate girder design, several difficulties are encountered. Initial web distortions are the rule rather than an exception in real plate girders and the magnitude of the lateral web deflections commonly approaches or even exceeds the web thickness\(^3\). Because of the
presence of initial web distortions, no clearly distinguishable web buckling load can be observed in a test of a real plate girder. It has been concluded from an extensive series of girder tests that lateral web deflections "increase continuously, without any considerable changes in the rate of increase, from zero load up to the instant when the proportional limit or yield point is exceeded in comparatively large portions of the plate."\(^4\) Furthermore, because of stress redistribution in the web and the supporting action of the flanges and stiffeners which frame the web, the maximum load which a girder can sustain is usually considerably higher than the theoretical web buckling load.\(^5\) In many cases where plate girder design is based on web buckling theory, the existence of post-buckling strength is tacitly recognized by the use of lower factors of safety against web buckling.\(^6\)

The post-buckling strength of plates can be investigated using the so-called "large deflection theory", which includes the effect of lateral deflections of the order of the plate thickness and membrane strains which are a function of these deflections. To be applicable to real problems, the influence of initial plate imperfections should be taken into account. The results of such an investigation could not necessarily be used directly to predict the ultimate strength of a plate girder since the boundary members around a web panel play an important role in developing the strength of a girder. Thus the strength of the whole assemblage consisting of the web plate and the boundary members needs to be investigated.
An extensive test program was conducted at Lehigh University to determine the post-buckling strength of welded, transversely stiffened plate girders during the period 1957 to 1960. Covering a wide range of geometric parameters, the full-size test girders were subjected to pure bending, high shear and combined bending and shear. It was concluded from ten bending tests that "the theoretical buckling load has no bearing on the static performance of a girder and that there is no justification in trying to use it as a basis for design." In all ten tests, girder failure was due to a failure of the compression flange. Five shear tests on "girders with high slenderness ratios revealed that such girders exhibit a considerable post-buckling strength...This post-buckling strength is due to a very pronounced tension field action requiring the presence of transverse stiffeners."

Methods of predicting the static strength of transversely stiffened plate girders, which are based on the behavior exhibited by the above-mentioned test girders, were presented in 1961. These methods have since been adopted as the basis for the design of plate girders for buildings in the United States. However, bridge girder specifications in the United States are still based on web buckling theory, and include provisions for the use of longitudinal stiffeners to increase the web buckling strength in bending. In 1963, a research program was initiated at Lehigh University to determine the strength of longitudinally stiffened plate girders. This report presents much of the results of this research.
1.2 Previous Work

Many investigators have studied the buckling strength of longitudinally stiffened plates; however, no analytical methods have been developed to determine the carrying capacity of longitudinally stiffened plates or longitudinally stiffened girders. Although a number of tests on longitudinally stiffened girders have been conducted, the methods of conducting the tests and the conclusions which were reached from the tests have unfortunately been too often influenced by the belief of the investigators that web buckling is the phenomenon which controls the strength of a girder.

A brief account of one bending test on an 18 ft. longitudinally stiffened steel box girder was given by Madsen in 1941. The test was not taken to failure but it was concluded that "longitudinal stiffeners are very effective in preventing buckling of the web."

In 1944, Gaber published the results of tests on 10 steel girders with spans of 3 and 6 meters. In spite of accurate workmanship, the webs of all 10 girders had initial deflections and, as load was applied, the deflections increased continuously from zero load to failure with no elastic stability phenomena being observed. With a uniformly distributed load applied to the center portion of the top flange of each girder, it was found that it was practically impossible to eliminate the influence of lateral web deflections on the load carrying capacity by means of longitudinal stiffeners of a reasonable size.
Massonnet conducted six ultimate load tests on two welded plate girders. The depth of the girders was one meter and the spans varied from 5.5 to 10 meters. Longitudinal stiffener position and size, loading condition and transverse stiffener spacing were all varied during the test series. It was concluded that the minimum stiffener rigidity prescribed by buckling theory is inadequate at ultimate load. A more recent report presents the results of two more girder tests where tubular flanges and stiffeners were used.

Perhaps the most extensive test program has been conducted by Rockey. Over 140 buckling tests on webs of bolted aluminum girders with one-sided longitudinal and transverse stiffeners have been reported. Based on the tests, design rules for spacing and proportioning transverse stiffeners and for proportioning longitudinal stiffeners were suggested.

Two full size and two model longitudinally stiffened steel plate girders were tested by Longbottom and Heyman to check the provisions of a proposed new British specification. With the test sections subjected to pure bending, all of the specimens failed by lateral buckling of the compression flange. The report does not indicate the extent, if any, that the longitudinal stiffeners contributed to the bending strength. This topic is further explored in Chapter 3.

Although the tests mentioned above established that the strength of longitudinally stiffened plate girders is greater than the web buckling strength, the effect of longitudinal stiffeners on girder behavior and the contribution of a longitudinal stiffener to the ultimate strength of
a girder have not been clearly defined. The experimental phase of the research on longitudinally stiffened plate girders at Lehigh University has been directed toward solving these problems. 19,20 This test program will be described in Chapter 2.
1.3 Definitions and Notation

A plate girder of the type considered in this report is shown in Fig. 1. A panel is defined as that portion of the web which is bounded by the flanges and two adjacent transverse stiffeners. The notation used to describe the panel size and the size of the web are indicated in Fig. 2. The web thickness is denoted by the symbol "t", the panel width or distance between transverse stiffeners is designated by "a" while the panel depth or distance between flanges is designated by "b". Two non-dimensional parameters will be used to describe the geometry of the web and of a web panel: the aspect ratio $\alpha = a/b$ and the web slenderness ratio $\beta = b/t$.

A longitudinal stiffener divides a panel into two subpanels, subpanel "0" and subpanel "1", as shown in Fig. 2. The location of the stiffener is defined by the distance from the compression flange to the stiffener, designated as $b_1$. If a second longitudinal stiffener is introduced between the first stiffener and the tension flange, its distance from the compression flange would be designated as $b_2$. It is more convenient to describe the location of longitudinal stiffeners in the non-dimensional form $\eta_1 = b_1/b$, $\eta_2 = b_2/b$, etc.

Other symbols used in the report will be defined when they are introduced in the development.
2.1 Introduction

It has been pointed out in Sect. 1.2 that, although quite a few tests on longitudinally stiffened plate girders have been reported in the literature, most of the tests were intended to determine the influence of longitudinal stiffeners on web buckling strength. The objectives of the experimental investigation described in this chapter were to determine the effect of longitudinal stiffeners on the post-buckling behavior of plate girders and to determine the contribution of longitudinal stiffeners to the static strength of plate girders. The test program and the results of the tests are presented in detail in Refs. 19 and 20.

A girder section can be subjected to pure bending, high shear or combined bending and shear, as indicated in Fig. 1. The loading conditions of pure bending and high shear were investigated using the test setups shown in Figs. 3 and 4.

Five test specimens were subjected to pure bending. The test specimens were bolted to reuseable end sections (Fig. 3), which were designed conservatively so that failure would occur in the test sections. The same flange, web and transverse stiffener sizes were used in all five specimens (Fig. 5). Since the longitudinal stiffener position was also kept constant, the only test variables were the longi-
tudinal stiffener size and transverse stiffener spacing. Specimen LBI had no longitudinal stiffener and served as a control specimen. The principal specimen parameters and the stiffener sizes are summarized in Table 1.

Eight shear tests were conducted on four girders using the set-up of Fig. 4. The flanges were over-designed so that shear failures could be obtained. Flange and web sizes were the same for the four girders (Fig. 6), and thus the location and size of the stiffeners were the only variables. It was intended that some of the panels in each girder would be stronger than the other panels so that after the weaker ones had been tested they could be reinforced, permitting a second test on the girder. This reinforcement consisted of one-sided diagonal stiffeners welded to the web and flanges and oriented along the tension diagonal of each failed panel as shown in the photograph in Fig. 7. This procedure was used successfully for three girders, permitting two tests on Girder LS1 and on Girder LS4 and three tests on Girder LS3. The test parameters and stiffener sizes for the eight shear tests are listed in Table 1.

The specimens were fabricated from structural carbon steel. All of the longitudinal components (flanges, webs and longitudinal stiffeners) were ordered at least two feet longer than required for the specimens, and before these components were welded together, the extra two foot lengths were cut off and shipped to the laboratory. These coupon plates were measured to determine the actual, as-delivered dimensions and standard tensile coupons were cut from them to determine the physical properties. Using the measured dimensions and yield points, various reference loads were calculated as described below.
The critical load $P_{cr}$ is the load at which the theoretical web buckling stress is reached. It is calculated from the formula

$$ \sigma_{cr} \tau_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \frac{1}{\beta^2} $$

where $\sigma_{cr}$ and $\tau_{cr}$ are the critical normal and shearing stresses respectively. The buckling coefficients $k$ for the various stiffened panels were obtained from Ref. 21. For the bending tests, the critical moment is $M_{cr} = \sigma_{cr} S$, where $S$ is the section modulus obtained by dividing the moment of inertia of the section by the distance from the neutral axis to the extreme fiber of the compression flange. For the test set-up used for the bending tests, the critical load is given by $P_{cr} = M_{cr}/120$ in. The critical shear force is $V_{cr} = \tau_{cr} A_w$, where $A_w$ is the area of the web, and the critical load for the shear girders is $P_{cr} = 2V_{cr}$.

The yield load is defined as the load at which yielding is first reached according to beam theory. For the bending tests, the yield moment is $M_y = \sigma_{yf} S$, where $\sigma_{yf}$ is the yield point of the compression flange, and the yield load is $P_y = M_y/120$ in. The shear force required to reach the yield stress in shear is $V_y = \tau_y I/Q$, where $\tau_y = \sigma_{yw}/\sqrt{3}$, $\sigma_{yw}$ is the yield point of the web, $I$ is the moment of inertia of the section and $Q$ is the static moment about the neutral axis. The yield load for the shear girders is given by $P_y = 2V_y$. 
The final reference load is the theoretical ultimate load for the test panels if no longitudinal stiffener were used and is denoted by $P_o$. Methods for calculating $M_o$, the theoretical ultimate moment for an unstiffened section, and $V_o$, the theoretical ultimate shear force for an unstiffened panel, are reviewed in Chapters 3 and 4, respectively. The theoretical ultimate load is given by $P_o = M_o/120$ in. for the bending tests and $P_o = 2V_o$ for the shear tests.

The reference loads were used to select test load increments and to compare with the experimentally obtained ultimate loads $P_{ex}$. Thus the ratio $P_{ex}/P_{cr}$ will indicate the magnitude of the post-buckling strength and $P_{ex}/P_o$ will provide a quantitative measure of how much the longitudinal stiffeners contributed to the static strength of the test specimens. The values of the reference loads for the thirteen tests are listed in Table 2.
2.2 Bending Tests

For the bending tests, load was applied with two hydraulic jacks and measured with a gage which indicated the load \( P \) (Fig. 3). Intermittent lateral support of the compression flange was supplied by pipes which were pinned to the test specimen and end fixtures at one end and to a lateral support beam at the other end. These supports were located at the transverse stiffeners of the test specimens, at the bolted joints and at the loading points.

The testing history and general loading behavior of a specimen can be traced on a load-versus-center line deflection curve. The \( P \) vs. \( v_L \) curves for specimens LB1 and LB4 are shown in Figs. 8 and 9. The corresponding curves for the other three bending tests were similar. The numbered circles indicate positions on the curves where loading was stopped and where measurements were taken. The reference load \( P_{cr} \) and the experimentally obtained ultimate load \( P_u^{ex} \) are also indicated on the figures. In each test, a specimen was considered to have reached its ultimate load when a substantial increase in the center line deflection was observed with no accompanying increase in the applied loads. In two tests (LB2 and LB4), the specimens were strained considerably beyond first attainment of ultimate load before failure occurred. (These failures were defined by a sudden drop in load accompanied by visible damage to the specimens.)

The reference loads and experimentally obtained ultimate loads for the five tests are listed in Table 3. Although buckling theory indicates that specimens LB2 to LB5, because of their longitudinal stiffeners,
should be over five times as strong as LB1, which did not have a longitudinal stiffener, $P_{u}^{ex}$ for all five tests was of about the same magnitude. The ratio $P_{u}^{ex}/P_{cr}$ clearly demonstrates the inadequacy of buckling theory in predicting bending strength. Since the ratio $P_{u}^{ex}/P_{y}$ is about 0.9 for each test, the fact that beam theory cannot be used to predict the bending strength is also established. The experimental ultimate loads are very close to the theoretical ultimate loads for the unstiffened panels ($P_{o}$), in fact the maximum difference between the two loads is only 2%. This indicates that the longitudinal stiffeners used in specimens LB2-LB5 did not contribute significantly to the bending strength.

In addition to the ultimate loads, test results were obtained in the form of lateral web deflections and stress distributions. Measured web deflections for specimens LB1 and LB3 are shown in Figs. 10 and 11, respectively. Data similar to that in Fig. 11 was obtained from tests LB2, LB4 and LB5. At the left of this figure, the web deflection patterns for four different loads are plotted. The two sections where these deflections were measured are indicated on the sketch of the test panel between the two plots. At the right, the lateral deflection of the longitudinal stiffener at the same two sections can be traced from zero load to ultimate load with the aid of the load-versus-lateral deflection ($P$ vs. $w$) curves. Similar curves are plotted for LB1 in Fig. 10. By comparing Figs. 10 and 11, the effectiveness of the longitudinal stiffener in controlling the growth of web deflections in LB3 can be evaluated. Although the initial deflected shapes were
different, the most important difference between the two figures is the slope of the $P$ vs. $w$ curves. The maximum increase in stiffener deflection between 0 kips and 120 kips was 14% for LB3, while the deflection at the same location on the web of LB1 increased 140% between the same two loads.

By Load No. 19, the stiffener of LB3 was severely buckled. The magnitude of the stiffener deformations after test LB3 was completed is evident in the photograph in Fig. 12. In tests LB2, LB4 and LB5, the longitudinal stiffeners also failed prior to reaching the ultimate load, with the result that a fairly rapid increase in stiffener deflection similar to that shown in Fig. 11 took place.

The distribution of stress in the section at the center of each test specimen was computed from strains which were measured at the various load numbers. Some of the resulting data for tests LB1 and LB3 are plotted in Figs. 13 and 14. Because of the large lateral web deflections in the upper portion of the web of LB1 (see Fig. 10), the stress in a substantial portion of the web between the neutral axis and the compression flange was very small, while that in the compression flange was greater than that predicted by beam theory (Fig. 13). This redistribution of stress from the web to the compression flange was prevented by the longitudinal stiffener of specimen LB3 until the stiffener had failed (Fig. 14). In tests LB2, LB4 and LB5, stress redistribution was controlled in a similar manner until the longitudinal stiffeners failed.
The stress distributions in Figs. 13 and 14 indicate that the yield stress was reached in the compression flange at the ultimate load in tests LB1 and LB3. Actually, in all five tests the ultimate load was reached as a result of general yielding of the compression flange. The extent of this yielding is evident in Figs. 12 and 15, where the yield patterns on the bottom surface of the compression flange and on the adjacent portions of the web are shown after tests LB1 and LB3 were completed. In the two cases where the tests were continued beyond the ultimate load until failure occurred (tests LB2 and LB4), the mode of failure was vertical buckling of the compression flange into the web. Specimen LB4 after failure is shown in the photograph in Fig. 16.

The results of the bending tests described in this section can be summarized as follows:

1. There is no rational correlation between the theoretical web buckling load and the bending strength of longitudinally stiffened plate girders.
2. Beam theory cannot be used to predict the bending strength of longitudinally stiffened plate girders.
3. The longitudinal stiffeners which were used in these tests had no significant effect upon the bending strength of the specimens.
4. The longitudinal stiffeners were very effective in controlling web deflections up to the loads at which the stiffeners buckled.
5. The longitudinal stiffeners had a significant effect upon the stress redistribution in the girders, causing the stress distribution to remain approximately linear until the stiffeners buckled.

6. In all of the tests, the ultimate load was reached as a result of general yielding of the compression flange.

7. Vertical buckling of the compression flange was observed in two tests; in both cases this occurred when the specimen was strained considerably beyond the first attainment of the ultimate load.
2.3 **Shear Tests**

The four shear girders were tested in a hydraulic testing machine with a concentrated load applied at midspan of each girder by the movable crosshead of the machine. As mentioned in Sect. 2.1, the flanges of the girders were overdesigned so that shear failures would occur. The compression flange in each test was supported laterally at the quarter points by pipes which were pinned to the girder at one end and to a lateral support beam at the other end.

Similar to the bending tests, the load-versus-center line deflection curves provided a convenient record of the testing history and general behavior of the girders. These P vs. $v$ curves are shown in Figs. 17 to 20 for Girders LS1 to LS4. Referring to Fig. 17, for example, the ultimate load in test LS1-T1 was reached at load No. 17. (The ultimate load is defined as the highest static load which a girder can sustain.) After unloading to load No. 18, the panels which had reached their ultimate load were reinforced with diagonal stiffeners as described in Sect. 2.1. This repair is indicated on the figure with a weld symbol. After the repair, test LS1-T2 was conducted, starting with load No. 19. A similar procedure was used for Girders LS3 and LS4 while Girder LS2 was subjected to only one test. Testing of all four girders was continued beyond the ultimate load of the last test until unloading of the specimen was observed (for example, the portion of the curve between load Nos. 36 and 37 in Fig. 17).
The reference loads \( P_{cr}, P_y \) and \( P_o \), the experimental ultimate load \( P_{ex} \) and the load ratios \( P_{ex}/P_{cr}, P_{ex}/P_y \) and \( P_{ex}/P_o \) are listed in Table 4 for the eight shear tests. Similar to the bending tests, the values of the ratios \( P_{ex}/P_{cr} \) and \( P_{ex}/P_y \) indicate that neither buckling theory nor beam theory can be used successfully to predict the ultimate shear strength. However, unlike the bending tests, the longitudinal stiffeners in each of the shear tests made a substantial contribution to the ultimate strength. As indicated by the values of the ratio \( P_{ex}/P_o \) in the last column of Table 4, the longitudinally stiffened panels developed from 6 to 38% more strength than the same panels would have developed according to the shear strength theory for transversely stiffened girders. (The panels of test LS1-T1 had no longitudinal stiffeners; the correlation between \( P_{ex} \) and \( P_o \) in this case is quite good.)

The longitudinal stiffeners were very effective in controlling web deflections in the shear tests. This can be seen from the plots of measured web deflections in Figs. 21 and 22. Fig. 21a shows the web deflections at the center of a panel of test LS1-T1, which had no longitudinal stiffener and served as a control test. Similar plots for tests LS1-T2, LS4-T1 and LS4-T2, with longitudinal stiffeners at \( \eta_1 = 0.33, 0.2 \) and 0.5, respectively, are shown in Figs. 21b, 22a and 22b. For each of these figures, the deflected shape of the web is shown for loads of 0 kips, 240 kips and the ultimate load for the particular test. A visual comparison of the web deflections of test LS1-T1 with those of the other three tests shows that the stiffeners were very effective in controlling the growth of the deflections. Similar plots for the tests
on girders LS2 and LS3 indicate that the longitudinal stiffeners in these tests were equally effective in controlling lateral web deflections.

The longitudinally stiffened panels developed higher shear strengths than predicted by shear strength theory for transversely stiffened girders because of the control of web deflections by the stiffeners. Thus, each individual subpanel developed its own tension field. This fact is well documented by photographs of the four girders after testing had been completed. In Fig. 7, which shows Girder LS4 after testing, the development of tension field action in each of the six subpanels in the left half of the girder is clearly seen from the yield line patterns. Similar photographs of Girders LS1, LS2 and LS3 after testing (Figs. 23, 24 and 25) indicate that separate tension fields developed in all of the subpanels of these girders also.

The results of the shear tests described in this section can be summarized as follows:

1. Neither web buckling theory nor beam theory can be used to predict the shear strength of longitudinally stiffened plate girders.
2. The longitudinal stiffeners used in the tests resulted in a significant increase in shear strength.
3. The longitudinal stiffeners were very effective in controlling web deflections.
4. Because of the control of web deflections by the longitudinal stiffeners, all subpanels developed their
own tension fields.

5. The shear strength of the longitudinally stiffened panels was attained only after the development of these tension fields.
2.4 Summary

Thirteen ultimate strength tests were conducted on nine longitudinally stiffened girder specimens. Five bending tests and eight shear tests indicate that neither web buckling theory nor beam theory can be used to predict the static strength of longitudinally stiffened plate girders. The test results show that longitudinal stiffeners can be very effective in controlling web deflections and, as a result, the type of stress redistribution which occurs in the post-buckling range can be significantly affected. For the case of pure bending, an effective longitudinal stiffener can reduce stress redistribution from the compressed portion of the web to the compression flange. For the case of high shear, longitudinal stiffeners can force the formation of individual, separate tension fields in all of the girder subpanels.

Using the test results described in this chapter and the strength theories for transversely stiffened plate girders\textsuperscript{7,8} as a starting point, the static strength of longitudinally stiffened plate girders is explored analytically in the following two chapters.
3.1 Introduction

The behavior of a transversely stiffened plate girder subjected to pure bending can be described using the test data on measured web deflections and bending stresses shown in Fig. 26. Test specimen LBl, from which the data was obtained, has already been described in Sect. 2.1 and some of the data in Fig. 26 has been plotted in Figs. 10 and 13.

Plotted in Fig. 26a are lateral web deflections measured at four different loads. The initial deflected configuration of the web is indicated for a load of zero kips and it can be seen that the maximum initial deflection was about one and a half times the thickness of the web. This situation is quite typical of welded girders with high web slenderness ratios. The figure indicates that the web deflections increased at a rather uniform rate in the upper half of the web, which was subjected to compressive bending stresses, while the deflections in the lower half of the web were somewhat reduced as load was increased due to the tensile stresses present in that region. Once again it can be stated that this behavior is typical of welded plate girders with high slenderness ratios.

The behavior of plate girders subjected to bending is further illustrated by the curves of bending stress distribution in Fig. 26b. The data shown in the figure was obtained by multiplying the strains...
measured with electrical resistance strain gages by the modulus of elasticity. For the web, each plotted point represents the average of two values obtained from gages mounted on opposite sides of the web, therefore the curves indicate the web membrane stresses. The linear stress distributions predicted by conventional beam theory, that is, $\sigma = M/S$, are also shown in the figure by light lines. The measured tensile stresses in the lower portion of the web correspond very closely to those predicted by beam theory, however, due to the increasing lateral web deflections in the compression zone, a redistribution of compressive stresses from the web to the compression flange occurs. The stresses in a significant portion of the web between the neutral axis and the compression flange are essentially zero while the compression flange and a portion of the web adjacent to it carry a stress which exceeds that predicted by beam theory. This stress redistribution has been observed in a number of other test girders and may be considered typical of transversely stiffened, slender web plate girders.

Because of the stress redistribution described above, Basler and Thürlimann reasoned that the bending strength of a plate girder is governed by the strength of the compression flange acting with a portion of the web as a column. It was assumed that the bending strength would be reached as a result of yielding or instability of the "compression flange column". Three types of instability were considered; lateral buckling, torsional buckling and vertical buckling, the directions of which are indicated by the arrows in Fig. 27. A flange stress reduction formula was also derived to compensate for the
increase in compression flange stress above the beam theory stress. In the following section, the bending strength theory, which was proposed for transversely stiffened plate girders, will be reviewed and discussed. The effect of a longitudinal stiffener on web deflections and stress distribution in the web will then be examined in Sect. 2.3, leading to the development of a method of predicting the bending strength of longitudinally stiffened plate girders. Theoretical predictions will be compared with the results of tests on eight longitudinally stiffened girders.
3.2 Review of Bending Strength Theory for Transversely Stiffened Plate Girders

The theory which is reviewed and discussed in this section was presented by Basler and Thürlimann in Ref. 7.

Compression Flange Stability

The first of three compression flange column buckling modes to be reviewed is the lateral buckling mode (see Fig. 27). In deriving lateral buckling formulas for plate girders, it was shown that the contribution of St. Venant torsion to the resistance against lateral buckling is small and that warping torsion is the dominant factor. By neglecting St. Venant torsion, a simple and slightly conservative estimate of the lateral buckling stress was obtained.

\[
\left(\frac{\sigma_{cr}}{\sigma_y}\right) = \frac{1}{\kappa_{_L}} \frac{1}{\kappa_{_L}}
\]

(3.1)

where

\[
\kappa_{_L} = \frac{\kappa}{r} \sqrt{\frac{\varepsilon_y}{\pi}}
\]

\(\kappa\) is the effective unsupported length, \(r\) is the radius of gyration of the compression flange column and \(\varepsilon_y\) is the yield strain. In the derivation of Eq. 3.1, it was shown that one-sixth of the area of the web \(A_w\) acts with the compression flange, therefore the radius of gyration is given by \(r = \sqrt{\frac{I_f}{A_f + A_w/6}}\), where \(I_f\) and \(A_f\) are the moment of inertia and the area of the compression flange, respectively. Eq. 3.1 is only applicable in the elastic range, \(\lambda_{_L} > \sqrt{2}\). It was suggested by Basler and Thürlimann that CRC Basic Column Formula be
used in the inelastic range, with the compressive residual stress taken to be \( \sigma_y/2, (\lambda_t = \sqrt{2}) \).  

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = 1 - \frac{\lambda_t^2}{4} \quad \text{for} \quad 0 \leq \lambda_t \leq \sqrt{2} \quad (3.2)
\]

Eqs. 3.1 and 3.2, which give the lateral buckling stress of the compression flange column of a plate girder subjected to pure bending, are plotted in Fig. 28.

The compression flange buckling mode referred to by Basler and Thürlimann as torsional buckling is commonly called local buckling in beam and column analysis. 23 By considering the compression flange as a long plate hinged at the flange-web junction and subjected to pure edge compression at its ends (Fig. 29), the torsional buckling curve shown in Fig. 30 was obtained. In the inelastic range, it was assumed that the magnitude of the compressive residual stress is \( \sigma_y/2, (\lambda_t = \sqrt{2}) \), that strain-hardening commences at \( \lambda_t = 0.45 \) and that the transition curve is tangent to the curves at these two points. The equations of the torsional buckling curve are then

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = 1 - 0.53 (\lambda_t - 0.45)^{1.36} \quad \text{for} \quad 0.45 \leq \lambda_t \leq \sqrt{2} \quad (3.3)
\]

and

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = \frac{1}{\lambda_t^2} \quad \text{for} \quad \lambda_t \geq \sqrt{2}, \quad (3.4)
\]
where $\lambda_t = 1.61 c / \sqrt{\varepsilon_y / d}$, $c$ is one half the width of the compression flange and $d$ is the thickness of the compression flange.

Vertical movement of the compression flange is resisted by the bending rigidity of the flange plate and by the restraint offered to the flange by the web. A sudden movement of the flange into the web is referred to as vertical buckling of the compression flange. The appearance of a test girder after vertical buckling has occurred is shown in Fig. 31, taken from Ref. 5. Basler and Thürlimann neglected the flange rigidity and, by equating the transverse flange force components which result from curvature due to bending to the Euler buckling load of a transverse strip of the web, derived an expression for the limiting web slenderness ratio ($\beta = b/t$) below which vertical buckling would not be expected to occur prior to compression flange yielding. This limiting slenderness ratio, given in Eq. 3.5, varies with the yield point of the flange material and the magnitude of the tensile residual stresses at the flange-web junction ($\sigma_r$).

$$\beta_{\text{max}} = \frac{0.48E}{\sigma_y(\sigma_y + \sigma_r)} \tag{3.5}$$

**Ultimate Bending Moment**

The ultimate bending moment, defined as the highest static moment which a girder section can resist, is in some way related to the web slenderness ratio $\beta$. In girders with high web slenderness ratios, large lateral deflections will develop in the compression zone of the web, resulting in a redistribution of stress from the web to the
compression flange (Sect. 3.1). The stresses in the compression flange and a portion of the web adjacent to the compression flange can then exceed the values predicted by the beam theory formula \( \sigma = \frac{M_y}{I} \), where \( y' \) is the distance from the neutral axis to the fiber for which the stress \( \sigma \) is being calculated and \( I \) is the moment of inertia of the entire girder section. As the web slenderness ratio is increased, this stress redistribution becomes more pronounced, or, stated in a different way, a smaller portion of the web is effective in resisting bending stresses with the compression flange. For the limiting case of \( \beta = \infty \), only the compression flange would be available to resist compressive bending stresses. For stocky webs, that is, for webs with low slenderness ratios, no stress redistribution will occur. At some lower limit of \( \beta \), the stresses in the web between the neutral axis and the compression flange will reach the yield point \( \sigma_y \), that is, the full plastic moment \( M_p \) of the section will be developed.

In the following development, the ultimate bending moment \( M_u \), will be non-dimensionalized by the yield moment \( M_y \), which is defined as the moment required to initiate yielding in the compression flange, \( M_y = \sigma_y/S \). The general relationship between the ultimate bending moment and the web slenderness ratio can be summarized as follows:

For \( \beta = \beta_A \), \( \frac{M_u}{M_y} = \frac{M_p}{M_y} \);

For \( \beta_A \leq \beta \leq \beta_0 \), \( \frac{M_p}{M_y} \geq \frac{M_u}{M_y} \geq 1 \);
For $\beta = \beta_A$, \[ \frac{M_u}{M_y} = 1; \]

For $\beta > \beta_A$, \[ \frac{M_u}{M_y} < 1. \]

$\beta$ is defined as the highest slenderness ratio for which $M_p$ can be developed and $\beta_A$ is the highest slenderness ratio for which a linear stress distribution can be developed according to beam theory.

Basler and Thürlimann further defined $\beta_A$ as the slenderness ratio at which, according to plate buckling theory, web buckling would occur when the applied moment reached $M_y$. This slenderness ratio was expressed as $\beta_A = 3.7 \sqrt{\frac{y}{y}}$, which gives $\beta_A = 170$ for structural carbon steel with $\sigma_y = 33$ ksi. Based on Fig. 8 of Ref. 24, $\beta_A$ was taken to be 53 for structural carbon steel. It was also proposed that, at the maximum slenderness ratio permitted by Eq. 3.5, a girder section consisting of the portion on the tension side of the neutral axis plus the compression flange acting with an effective width of the web equal to 30 times the web thickness should be considered available to resist bending moment. From Eq. 3.5, assuming $\sigma_y = 33$ ksi and $\sigma_r = 16.5$ ksi, the corresponding value of slenderness ratio is $\beta_B = 360$.

The ratio $\frac{M_u}{M_y}$ is plotted against the web slenderness ratio in Fig. 32. As explained above, for $\sigma_y = 33$ ksi, $M_u = M_p$ at $\beta_A = 53$, $M_u = M_y$ at $\beta_A = 170$ and $M_u = M_y$ of the reduced section at $\beta_B = 360$. The corresponding bending stress distributions are indicated in circles in the figure. The numerical values of $\frac{M_u}{M_y}$ at $\beta_A$ and $\beta_B$ depend on the...
area ratio $\rho = A_w/A_f$. Since a curve passing through the plotted points in Fig. 32 is essentially a straight line, Basler and Thürlimann assumed that the following linear $M_u/M_y$ vs. $\beta$ relationship would apply,

$$\frac{M_u}{M_y} = 1 - 0.0005 \rho (\beta - 5.7/\sqrt{\varepsilon_y})$$  \hspace{1cm} (3.6)

Eq. 3.6 is plotted in Fig. 33 for various values of $\rho$.

The influence of lateral or torsional buckling of the compression flange on the ultimate bending moment is incorporated in Eq. 3.6 simply by replacing $M_y$ by the respective critical moment $M_{cr}$. Since the stresses are related to moments by the expression $\sigma = M/S$, the equation for ultimate bending stress becomes

$$\sigma_u = \sigma_{cr} \left[ 1 - 0.0005 \rho (\beta - 5.7 \sqrt{E/\sigma_{cr}}) \right]$$  \hspace{1cm} (3.7)

A reduction in the ultimate bending stress $\sigma_u$ is required only when $\beta > 5.7 \sqrt{E/\sigma_{cr}}$; the reduction as a percentage of the critical stress $\sigma_{cr}$ is $0.05 \rho (\beta - 5.7 \sqrt{E/\sigma_{cr}})$.

**Discussion**

In analyzing the stability of the compression flange column, it is unconservative to treat torsional buckling independently of lateral buckling, as Basler and Thürlimann pointed out. However, the conservative assumption that the restraint of the web on the flange can be neglected in the torsional buckling analysis compensates to some extent for the unconservative separate treatment of the two
buckling modes. The assumption that the contribution of St. Venant torsion to lateral buckling strength can be neglected is also on the conservative side.

The lateral and torsional buckling stress formulas (Eqs. 3.1-3.4) can only be checked experimentally in conjunction with the flange stress reduction formula (Eq. 3.7) since most of the available test results have web slenderness ratios which are high enough to require a reduction in the ultimate bending stress. The pertinent parameters, theoretical predictions and test results for nine bending tests on five transversely stiffened, structural carbon steel plate girders are summarized in Table 5. A detailed description of the tests is given in Ref. 5 while the computation of the theoretical predictions is given in Ref. 7. In only one test (G1-T1) is torsional buckling the predicted failure mode, and in this case the buckling stress calculated from Eq. 3.3 is low enough that there is no need for a reduction in flange stress according to Eq. 3.7. For this test the predicted torsional buckling stress was conservative, $P_{u}^{th} = 1.11$. In another test (G2-T2), the predicted failure mode could be either lateral or torsional buckling, according to Eqs. 3.2 and 3.3. For all of the other tests, lateral buckling is the controlling factor in the prediction of ultimate loads. (Girders G3 and G5 had tubular compression flanges, thus torsional buckling would not be considered as a possible failure mode.) The correlation between experimental and theoretical ultimate loads is generally quite good, with the theory being somewhat conservative in most cases. Based on these results, it
can be tentatively concluded that the buckling stress formulas (Eqs. 3.1-3.4) along with the flange stress reduction formula (Eq. 3.7) provide an effective means of estimating the bending strength of transversely stiffened plate girders. The applicability of these formulas will be further checked in Sect. 3.3 for longitudinally stiffened plate girders.

It is somewhat more difficult to verify the vertical buckling theory represented by Eq. 3.5, because the equation does not provide a means of predicting the stress at which vertical buckling should occur. However, the limiting web slenderness ratio $\kappa_{\text{max}}$ given by the equation agrees fairly well with the test results in Table 5. Of the specimens which had a compression flange consisting of a rectangular plate, one had a slenderness ratio larger than the limiting value according to Eq. 3.5 (Girder G4, $\kappa = 388$). The vertical buckling mode of failure was observed in this girder (see Fig. 31). In tests on two other girders having slenderness ratios less than the limiting value according to Eq. 3.5 (Girders G1 and G2, $\kappa = 185$), vertical buckling of the compression flange did not occur.

An examination of the results of the five bending tests described in Chapter 2 will be helpful in further discussing the vertical buckling problem. Since one of the original objectives of this test series was to investigate the effect of a longitudinal stiffener in increasing the resistance of the web to vertical buckling, the compression flange was designed so that neither local nor lateral buckling would occur prior to compression flange yielding. With web slenderness ratios between
444 and 447 (see Table 1), vertical buckling would surely be expected according to Eq. 3.5, especially in specimen LB1 which had no longitudinal stiffener. The results of the tests pertinent to this discussion of vertical buckling can be summarized as follows:

1) In each test, the ultimate load was reached as a result of general yielding of the compression flange.

2) The axial strain measured in the extreme fibers of the compression flange at ultimate load exceeded the yield strain in each test. Visual observations indicated that the compression flange was completely yielded at ultimate loads (see Figs. 12 and 15).

3) Vertical buckling of the compression flange was observed in two tests (LB2 and LB4). In both cases testing was continued well beyond ultimate load before vertical buckling occurred (see Fig. 9, for example). A photograph of specimen LB4 after failure is shown in Fig. 16.

The test results described above seem to contradict the vertical buckling theory represented by Eq. 3.5. However, this is not necessarily the case. Since it was assumed that the bending rigidity of the compression flange plate could be neglected in deriving Eq. 3.5, the equation can only predict a value of the slenderness ratio for which the resistance of the web to vertical buckling becomes negligible. While it is true that the additional resistance to vertical buckling provided by the bending rigidity of the compression flange will
normally be very small, some bending rigidity will exist until the
flange is completely yielded due to the bending moment acting on the
girder. The photographs in Figs. 16 and 31 also indicate that the
compression flange must be completely yielded for vertical buckling
to occur; compression flange deformations of the magnitude shown in
these figures could not occur in a steel plate without plastic hinges
being developed.

Two conditions must be fulfilled before vertical buckling of the
compression flange can occur: (1) the web must be slender enough to
permit large lateral web deflections to develop so that the resistance
to vertical buckling becomes negligible; (2) the compression flange
must be completely yielded so that its bending rigidity also becomes
negligible. According to the bending strength theory represented by
Eq. 3.7, however, the ultimate bending moment which a girder section
can sustain will be reached before the above condition (2) is ful-
filled. It appears, therefore, that vertical buckling of the compres-
sion flange can only occur after the ultimate bending moment has been
attained.

In view of the above conclusion, the ultimate bending strength
theory previously reviewed must be reexamined. One of the points used
to determine the bending strength reduction curve was derived using
the vertical buckling analysis (Fig. 32, $\beta_B = 360$). However, since
the other two points at $\beta_A = 53$ and $\beta_o = 170$ in Fig. 32 were deter-
mined independently of the vertical buckling analysis, and since the
reduction curve is a straight line through these two points, the reduction formula of Eq. 3.7 could still be used, and it could also be extended beyond \( \phi = 360 \). This is partially verified by the test on specimen LB1, described in Chapter 2. The ultimate load predicted for this specimen, which had a slenderness ratio of 444, was 156.4 kips. A flange stress reduction of 10.7\%, according to Eq. 3.7, was used in calculating this theoretical ultimate load. The experimentally obtained ultimate load was 156.5 kips, almost exactly the value predicted. The applicability of Eq. 3.7 to girders with slenderness ratios above 360 will be further substantiated with the results of four other tests in Sect. 3.3.

Summary

The behavior of a transversely stiffened plate girder subjected to pure bending can be described schematically with the \( \frac{M}{M_y} \) versus \( \frac{v}{h} \) curve of Fig. 34. If the lateral or torsional buckling stress \( \sigma_{cr} \) is less than the yield point, the moment-deflection curve will be similar to curve A in the figure. However, if \( \sigma_{cr} \geq \sigma_y \), behavior similar to curve B would result, where the ultimate moment is reached due to compression flange yielding. In this case, failure will occur after the ultimate moment has been attained due to lateral, torsional or vertical buckling of the compression flange (curves C or D, for example).

Based on the review of the bending strength theory for transversely stiffened plate girders in this section, it is concluded that the formulas for lateral and torsional buckling stresses (Eqs. 3.1-3.4) provide a good approximation of actual girder behavior. The restriction
on maximum web slenderness ratio (Eq. 3.5), based on a vertical buckling analysis, appears to be an unnecessary one since vertical buckling can only be expected to occur after the ultimate moment has attained. Therefore, it is suggested that the flange stress reduction formula (Eq. 3.7) can be applied to plate girders with slenderness ratios greater than 360. This formula has been checked experimentally only for slenderness ratios up to 450, however.
3.3 Influence of Longitudinal Stiffeners on Bending Strength

Compression Flange Stability

Compression flange instability in the form of lateral or torsional buckling may be the factor which limits the bending strength of a plate girder. The torsional buckling of the compression flange is a function only of the compression flange dimensions and material properties according to Eqs. 3.3 and 3.4. The effect of web restraint, which was neglected in deriving these equations is negligible. Therefore, there is no possibility that a longitudinal stiffener will significantly increase the stress at which torsional buckling of the compression flange will occur. Lateral buckling of the compression flange is a function of the effective length, the material properties and the radius of gyration of the compression flange column according to Eqs. 3.1 and 3.2. Assuming that a longitudinal stiffener is located close enough to the compression flange to act with the flange in resisting lateral buckling, the effect of the stiffener on the lateral buckling stress of the compression flange column will now be explored.

Consider the three cases of Fig. 35, where the thickness of the web is assumed to be zero. The radius of gyration for each of the three sections is listed below.

\[
\text{Case I } \quad r_1 = \frac{c}{\sqrt{3}}
\]

\[
\text{Case II } \quad r_2 = \frac{c}{\sqrt{3}} \left( 1 + \frac{A_s}{A_f} \right) \left( \frac{c_s}{c} \right)^2 \frac{A_s}{A_f}
\]
Case III \[ r_3 = \frac{c}{\sqrt{3}} + \frac{(c_s/c)^2 A_s}{A_f} + \frac{A_s}{A_f} \]

For a longitudinal stiffener to have a beneficial effect on the lateral buckling stress of the compression flange column, the radius of gyration of the column with a stiffener must be larger than that of the column without a stiffener. An examination of the cases listed above shows that for \( c_s < c \), \( r_1 \) is greater than \( r_2 \) and \( r_3 \); for \( c_s = c \), \( r_1 = r_2 = r_3 \); and for \( c_s > c \), \( r_2 \) and \( r_3 \) are greater than \( r_1 \). Thus, only when the width of a longitudinal stiffener exceeds the half-width of the compression flange can the stiffener increase the lateral buckling stress of the compression flange column. An increase in the size of the compression flange itself would obviously be a more economical way to increase the lateral buckling strength. Thus, it can be concluded that lateral and torsional buckling strength are not affected by a longitudinal stiffener.

**Control of Web Deflections and Stress Redistribution**

One possible way in which a longitudinal stiffener could influence the bending strength of a plate girder can be investigated with the aid of the web deflection and stress distribution data of Fig. 36. Test specimen LB3, from which the data was obtained, has already been described in Sect. 2.1 and some of the data in Fig. 36 has been plotted in Figs. 11 and 14.
The measured lateral web deflections for four different loads are plotted in Fig. 36a. In comparing these web deflections with those plotted in Fig. 26a, a number of differences are apparent. The web of specimen LB3 was initially deformed in a single wave pattern while specimen LBl (Fig. 26a) had an initial deflected configuration with two waves. The maximum initial deflection in the web of LB3 was almost double the web thickness compared with a maximum initial deflection of about one and a half times the thickness for LBl. The magnitude and pattern of initial web deflections are to a large extent random, but are influenced to some extent by the amount of heat input during welding and by the welding sequence. The most significant difference between the two girders is the extent of the increase in web deflections due to the applied loads. The maximum increase in deflection at a load of 120 kips for LBl was 275% while the maximum increase for LB3 at the same load was only about 45%. This resulted from the fact that in specimen LB3 web deflection growth under load was controlled by the longitudinal stiffener.

Further information on the influence of a longitudinal stiffener on bending strength can be obtained from a comparison of the stress distributions in specimen LB3 (Fig. 36b) and LBl (Fig. 26b). Although the large initial web deflections of LB3 caused the web membrane stresses to deviate somewhat from beam theory (indicated by light lines in the figure), a redistribution of stress from the web to the compression flange of the type shown in Fig. 26b for specimen LBl did not occur in specimen LB3. Beam theory could be used to predict the
compression flange stresses in LB3 very accurately for the loads shown in Fig. 36a. Since the two girders were identical in every respect except for the presence of a longitudinal stiffener in specimen LB3, the stiffener must be credited with preventing an extensive stress redistribution.

**Ultimate Bending Strength**

In the above discussion it has been shown that when a longitudinal stiffener is effective in controlling lateral web deflections, stress redistribution from the web to the compression flange is also controlled or prevented. A linear stress distribution in the girder section results and beam theory can then be used to predict the compression flange stresses. If this type of behavior can be maintained until the ultimate bending moment is reached, the longitudinal stiffener will have a significant and beneficial effect on the bending strength. Since no stress redistribution will occur, a reduction in the ultimate bending stress is not required, and the simple expression

$$\sigma_u = \sigma_{cr} \tag{3.8}$$

can be used to compute the ultimate bending stress. (In this equation $\sigma_{cr}$ is the buckling stress for lateral or torsional buckling from Eqs. 3.1-3.4, whichever is lower.) A longitudinal stiffener should be properly positioned and adequately proportioned so that the ultimate bending stress can be computed according to Eq. 3.8.
**Longitudinal Stiffener Requirements**

In order for a longitudinal stiffener to control web deflections and prevent stress redistribution from the web to the compression flange, it obviously must be located somewhere between the neutral axis and the compression flange. Although plate girder bending strength is not directly related to web buckling strength (Sect. 2.2), the control of lateral web deflections by means of a longitudinal stiffener is similar to the problem of increasing web buckling strength by forcing a nodal line in the deflection pattern of the web. Thus, it is assumed that the optimum stiffener position from a web buckling viewpoint is also the most effective position for controlling web deflections. An analysis of the stability of a longitudinally stiffened web panel subjected to pure bending has shown that the optimum stiffener position is between \( \eta_1 = 0.2 \) and \( \eta_1 = 0.22 \), depending on the degree of restraint offered to the web by the flanges. The web deflections of specimen LB3 (Fig. 36), as well as those of the other three specimens described in Sect. 2.2, confirm that the one-fifth depth position \( \eta_1 = 0.2 \) is effective in controlling web deflections to the extent that the stress distribution in the web remains essentially linear. The longitudinal stiffener position \( \eta_1 = 1/5 \) will be adopted in the following discussion. It should be noted that if extremely high web slenderness ratios are used (say \( \beta \gg 450 \)), a single longitudinal stiffener will not be adequate to control web deflections in the entire region between the neutral axis and the compression flange. The problem of positioning and proportioning multiple longitudinal stiffeners is beyond the scope of this report, however.
In addition to the location requirement, an effective longitudinal stiffener must be proportioned so that it will control web deflections and stress redistribution for loads up to the ultimate load. The ratio of the width of the stiffener plate $c_s$ to its thickness $d_s$ must be kept low enough to avoid premature local buckling of the stiffener. An example of such premature local buckling in a test girder is shown in Fig. 12. When the local buckling shown in the figure occurred in specimen LB3, the lateral web deflections became large enough to result in a significant stress redistribution from the web to the compression flange. (See the $P$ vs. $w$ curves in Fig. 11 and the stress distribution plotted for load No. 20 in Fig. 14). If it is conservatively required that the stiffener stress reach the yield point before local buckling occurs and if the restraint offered to the stiffener by the web is neglected, the limiting width-thickness ratio for the stiffener plate is

$$
\frac{c_s}{d_s} \leq 2400/\sqrt{\sigma_y} \tag{3.9}
$$

where $\sigma_y$ is the yield point of the stiffener material. For structural carbon steel with $\sigma_y = 36$ ksi, $(c_s/d_s)_{\text{max}} = 13$.

With regard to longitudinal stiffener rigidity, two requirements are suggested. The first is that the stiffener possess the minimum rigidity required to form a nodal line in the deflected web up to the theoretical web buckling load (elastic). Based on a buckling analysis of a plate with a longitudinal stiffener at $\eta_l = 1/5$ and subjected to pure bending, at least two formulas for minimum stiffener rigidity are
The formula proposed by Massonnet is the simpler one and will be adopted in this work. In the formula, given below, the required stiffener rigidity ratio \( \gamma_L^* \) is expressed as a function of the aspect ratio \( \alpha \) and the stiffener area ratio \( \delta_L = \frac{A_s}{A_w} \)

\[
\gamma_L^* = 3.87 + 5.1\alpha + (8.82 + 77.6 \delta_L)\alpha^2
\]  

(3.10)

where \( \gamma_L^* \) is defined as \( \frac{I_L}{I_w} \) and \( I_w \), the moment of inertia of the web, is defined by \( I_w = \frac{bt^3}{12(1-\nu^2)} \).

Above the theoretical web buckling load, the stiffener should possess sufficient rigidity to control web deflections up to the ultimate load. A convenient method of ensuring that this is the case is by considering the stability of the stiffener acting with a portion of the web as a column in a manner analogous to the compression flange column discussed in Sect. 3.2. Such a column is shown in Fig. 37a, along with the compression flange column and the linear stress distribution assumed for the girder section. Neglecting the thickness of the compression flange, the requirement for the lateral buckling stress of the longitudinal stiffener column is

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_L \geq 0.6 \left( \frac{\sigma_{cr}}{\sigma_y} \right)_F
\]  

(3.11)

for \( \eta_1 = 1/5 \), where \( \left( \frac{\sigma_{cr}}{\sigma_y} \right)_L \) is the lateral buckling stress of the compression flange column according to Eqs. 3.1 and 3.2. Eq. 3.11 ensures that the longitudinal stiffener column will not fail prior to the compression flange column. In Fig. 37b, this requirement is shown
graphically. The stiffener column slenderness parameter \( \lambda_{ls} \) is given by

\[
\lambda_{ls} = \frac{a}{r_L} \sqrt{\frac{E}{f_y}}
\]

(Separation of the two longitudinal stiffener functions is based on the assumption that the web slenderness ratio is high enough that the stiffened web plate buckles elastically before the longitudinal stiffener column buckles.)

The section to be used in computing \( I_L \) and \( r_L \) is yet to be defined. It is customary to calculate the moment of inertia of unsymmetrical (one-sided) stiffeners with respect to an axis through the web-stiffener interface. In a number of tests where strain measurements were made to determine the width of the web plate which participates in the transverse bending of a longitudinal stiffener, it was determined that the mean effective width is \( 20t \). The moment of inertia \( I_s \) calculated using the conventional method is greatly exaggerated compared with the value obtained using an effective width of the web with the stiffener. The stiffener section properties \( I_L, A_{sl}, \text{ and } r_L \) to be used in Eqs. 3.10-3.12 should be computed for a section consisting of the stiffener plate and \( 20t \) of the web.

It should be noted that if a longitudinal stiffener fails to fulfill the proportioning requirements given by Eqs. 3.9-3.11, there is no justification in assuming that a stress redistribution from the web to the compression flange will be prevented and that the ultimate bending stress can be computed according to Eq. 3.8. In the case of
an inadequately proportioned stiffener, the ultimate bending stress should be calculated from Eq. 3.7, which is primarily intended for transversely stiffened plate girders. This will be verified experimentally in the discussion of "Correlation with Test Results".

Transverse Stiffener Requirements

A longitudinal stiffener, in performing its role of controlling web deflections, will subject the transverse stiffeners to concentrated forces at the intersection of the two stiffeners. If the longitudinal stiffener were removed from the web, the deflected shape of the web would approximate a sine curve between transverse stiffeners. Therefore, it will be assumed that the longitudinal stiffener is subjected to a sinusoidal load by the web, as shown in Fig. 38a. The reactions at the ends of the stiffener are \( R = \frac{P_0 a}{\pi} \) and the moment at midspan is \( M_L = \frac{P_0 a^2}{2\pi} \). If it is conservatively required that the bending stress in the longitudinal stiffener reach the yield point, the corresponding value of \( P_0 = \sigma_y S_L \frac{2}{a^2} \), where \( S_L \) is the section modulus of the longitudinal stiffener.

A transverse stiffener, at its intersection with the longitudinal stiffener, will be subjected to a concentrated force \( 2R \) from the two adjacent longitudinal stiffener spans (Fig. 38b). Using \( \eta_l = 1/5 \) for the position of the longitudinal stiffener, the maximum moment in the transverse stiffener can be determined as \( M_T = 8 \pi \frac{\sigma_y S_L}{25\alpha} \). The maximum bending stress in the transverse stiffener is permitted to reach \( \sigma_y \), resulting in the expression \( S_T = \frac{8\pi}{25} \frac{S_L}{\alpha} \), where \( S_T \) is the
required section modulus of the transverse stiffener. Since the fraction $8\pi/25$ is very close to unity, the simple formula

$$S_T \geq \frac{S_L}{\alpha}$$

(3.13)

is obtained for the required section modulus of the transverse stiffener.

Due to the conservative assumptions made in the above derivation, Eq. 3.13 will result in a conservative design. However, the resulting transverse stiffeners will normally not have to be larger than they would be if designed by other available criteria. If a longitudinal stiffener exceeds its minimum rigidity requirements, the value of $S_T$ required by Eq. 3.13 can be reduced by multiplying it by the ratio of the required longitudinal stiffener rigidity to the rigidity actually supplied.

Correlation With Test Results

Two series of tests are available to substantiate the expressions which have been developed in this section for computing the bending strength of longitudinally stiffened plate girders and for proportioning the stiffeners of these girders. The first series has been described in Sect. 2.2. The principal specimen parameters, the stiffener properties and the correlation of the test results with theory are summarized in Table 6. For each of the four specimens, the longitudinal stiffener width-thickness ratio exceeded the maximum value of 13 permitted by Eq. 3.9. In the case of Specimen LS4, neither of the longitudinal stiffener rigidity requirements given by Eqs. 3.10
and 3.11 were fulfilled either. Therefore, the ultimate bending stress has been computed using Eq. 3.7. The resulting flange stress reduction varied from 10.5 to 10.7% for the four specimens. The theoretical ultimate loads \( P_{u}^{th} \), the experimentally obtained loads \( P_{u}^{ex} \) and the \( P_{u}^{ex}/P_{u}^{th} \) ratios are listed at the bottom of Table 6. The excellent correlation between theory and test results confirms the applicability of Eq. 3.7 to girders with inadequately proportioned longitudinal stiffeners and provides further evidence that this equation can be successfully used for slenderness ratios up to 450, as tentatively concluded in Sect. 3.2.

The second series of tests, performed by Longbottom and Heyman, is described in detail in Ref. 18. The test setup and dimensions of the component plates in the test section of the four specimens are shown in Fig. 39, while the girder parameters, stiffener properties and test results are summarized in Table 6. All of the girders in this series had longitudinal stiffeners of sufficient proportions to fulfill the requirements of Eqs. 3.9-3.11. In addition, girders E and 4 both had a second longitudinal stiffener located at mid depth \( (\eta_2 = 1/2) \) which was not considered in calculating the values in Table 6. Since the longitudinal stiffener requirements were fulfilled, the reduction in ultimate bending stress listed in parentheses in the table for each of the four girders was not used in calculating \( P_{u}^{th} \); it is listed so

\*The test specimens were designed before the longitudinal stiffener requirements presented in this Chapter had been developed; therefore, the proportions of the longitudinal stiffeners were based on other considerations.\(^{19}\)
that the magnitude of the increase in bending strength due to the longitudinal stiffener can be noted. The correlation of the experimental results with theory is very good, and indicates that when a properly proportioned longitudinal stiffener is used, Eq. 3.8 provides a reliable estimate of the bending strength. Since all of the girders failed by lateral buckling of the compression flange, further confirmation of the formula for lateral buckling stress (Eq. 3.2) was also obtained.

Summary

A method of predicting the bending strength of longitudinally stiffened plate girders has been presented in this section, along with requirements for proportioning both longitudinal and transverse stiffeners. The results of bending tests on eight longitudinally stiffened girders have been summarized, showing that if the longitudinal stiffeners do not fulfill the requirements, a reduction in the ultimate bending stress must be used. It has also been shown that if properly proportioned longitudinal stiffeners are provided, a significant increase in bending strength can result.
Chapter 4

SHEAR STRENGTH

4.1 Introduction

The type of shear panel which will be discussed in this chapter is shown in Fig. 40. The panel consists of a rectangular portion of the web bounded by the flanges and transverse stiffeners. It will be assumed that the moment present on any section in the panel is small so that the shear strength of the panel can be studied independently.

It has been well established that plate buckling theory is not adequate to predict the shear strength of a transversely stiffened plate girder panel. Test results indicate that the ultimate shear force which a panel can sustain is considerably higher than the critical shear force calculated according to buckling theory. For one series of tests the ratio of the ultimate shear force to the critical shear force varies from about 2 to 4.

An element subjected to pure shearing stresses $\tau$ is shown at the left of Fig. 41a. These stresses correspond to the principal stresses shown at the right of the figure, where the tensile principal stress $\sigma_1$ is numerically equal to both the compressive principal stress $\sigma_2$ and the shear stress $\tau$. The state of stress shown in Fig. 41a is the type usually assumed in beam theory; in the following discussion it will be referred to as "beam action shear". As the shear force on a plate girder panel is increased, it will reach a stage where the compressive
stress \( \sigma_2 \) can no longer increase because the web deflects laterally. For an ideal panel which is initially perfectly plane, this stage occurs when the shear force reaches the critical value predicted by plate buckling theory. The stress in the direction of the tension diagonal continues to increase as the applied shear force increases beyond the critical shear force. A field of tensile stresses \( \sigma_t \) of the type shown on the element in Fig. 41b develops and is the source of the post-buckling shear strength of the panel. This state of stress is termed "tension field action shear".

The ultimate shear strength of a panel is the sum of the beam action and tension field action shear forces and will be reached when the combination of the two states of stress shown in Fig. 41 fulfills the yield condition. Tension field action can develop only if the panel framing members, that is, the flanges and transverse stiffeners, which serve to anchor the tension field stresses, have sufficient strength and rigidity. After the static ultimate shear force is reached, the panel yields and large shear deformations result. Final failure occurs when these shear deformations become so pronounced that one of the flanges bends into the web. This process can be traced on the load-deflection curve for test girder LS2 in Fig. 18, where a substantial yield plateau was obtained before unloading and failure occurred. The extent of the shear deformations in this girder is evident in Fig. 24.
In 1961, Basler presented a shear strength theory for plate girders which incorporated both beam action and tension field action. A number of tests of full size girders have demonstrated that this theory can be used successfully to approximate the shear strength of transversely stiffened plate girders. In the following section, Basler's shear strength theory will be reviewed and discussed. This theory will be used as a basis for developing a method of predicting the shear strength of longitudinally stiffened plate girders in Sect. 4.3. The applicability of the method will be checked with the results of seven shear tests on longitudinally stiffened plate girders.
4.2 Review of Shear Strength Theory For Transversely Stiffened Plate Girders

The shear strength theory which is reviewed and discussed in this section was developed by Basler in Ref. 8.

Tension Field Action and the Ultimate Shear Force

One of the basic assumptions made in developing the shear strength theory was that externally applied shear forces are resisted by beam action, or pure shear, (Fig. 4la) up to the theoretical web buckling stress, and that any additional applied shear forces are resisted by tension field action (Fig. 4lb). It was further assumed that the ultimate shear force $V_u$ is equal to the sum of the beam action contribution $V$ and the tension field action contribution $V_o$:

$$ V_u = V + V_o \quad (4.1) $$

Defined as the shear force carried by the web at the theoretical web buckling stress, the beam action contribution is given by

$$ V = \tau_{cr} A_w, \quad (4.2) $$

where the critical shear stress is

$$ \tau_{cr} = k \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (4.3) $$

The buckling coefficient $k$ for a simply supported plate subjected to pure shear is a function of the aspect ratio. According to Ref. 30, the $k$ - values obtained from a buckling analysis can be closely approximated by the following relationships (plate with simply supported boundaries)

$$ k = 4.00 + 5.34/\alpha^2, \quad \text{for } \alpha \leq 1, \quad (4.4) $$

and

$$ k = 5.34 + 4.00/\alpha^2, \quad \text{for } \alpha \geq 1. $$
Since some restraint will always be exerted on the web by the flanges and the web in adjacent panels, it is conservative to use the buckling coefficient for a simply supported plate.

The type of tension field used by Basler to determine the tension field action contribution to the ultimate shear force is shown in Fig. 42a. Assuming that the flanges of a conventional welded plate girder do not have sufficient bending rigidity in the plane of the web to resist vertical stresses at the flange-to-web junction, the transverse stiffeners were relied upon to resist the vertical components of the tension field stress $\sigma_t$. The horizontal components were assumed to be anchored by the adjacent panels. To determine the slope $\phi$ of the field, it is noted that the strip is capable of carrying a shear force equal to $\sigma_t \cdot s \cdot t \cdot \sin \phi$, where $s$, the width of the strip, is equal to $b \cdot \cos \phi - a \cdot \sin \phi$. Differentiating this shear force with respect to the angle $\phi$ and setting the resulting expression equal to zero, the optimum value of $\phi$ for the assumed field is found to be

$$\phi = \tan^{-1} \left( \sqrt{1 + \frac{\phi^2}{\alpha^2}} - \alpha \right). \quad (4.5)$$

The tension field contribution was evaluated using the free-body diagram in Fig. 42b. The total tension field force acting at the angle $\phi$ with the horizontal is $\sigma_t \cdot t \cdot a \cdot \sin \phi$. The change in flange force $\Delta F_f$ is determined by summation of forces in the horizontal direction and is equal to $\sigma_t \cdot t \cdot a \cdot \sin \phi \cdot \cos \phi$. Finally, by summation of moments about point 0, the total shear force $V_\sigma$ is $b \cdot \Delta F_f / 2$. When the value of $\Delta F_f$ given above is substituted in this expression along with the value of $\phi$ from Eq. 4.5, Basler's equation for the contribution of tension field action to the ultimate shear force is obtained.
Substituting Eqs. 4.2 and 4.6 into Eq. 4.1, the ultimate shear force is found to be

\[ V_u = \tau_{cr} A_w + \frac{\sigma_t A_w}{2\sqrt{1 + \sigma^2}} \]  

(4.7)

where \( A_w = b \cdot t \), the area of the web. It is convenient to non-dimension-alyze Eq. 4.7 by dividing by the plastic shear force \( V_p = \tau_y A_w \), where the shear yield stress is taken to be \( \tau_y = \sigma_y / \sqrt{3} \) according to Mises' yield condition for plane stress. Carrying out this operation,

\[ \frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{3}{2} \cdot \frac{\sigma_t}{\sigma_y} \cdot \frac{1}{\sqrt{1 + \sigma^2}} \]  

(4.8)

It was assumed by Basler that the ultimate shear strength would be reached when the combination of beam action and tension field stresses in the web fulfilled the yield condition. The approximate form of Mises' yield condition was used,

\[ \frac{\sigma_t}{\sigma_y} = 1 - \frac{\tau_{cr}}{\tau_y} \]  

(4.9)

The final form of the shear strength formula is obtained when Eq. 4.9 is inserted in Eq. 4.8,

\[ \frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{3}{2} \cdot \frac{1 - \tau_{cr}/\tau_y}{\sqrt{1 + \sigma^2}} \]  

(4.10)
Two limitations were imposed on the shear strength formula to account for the influence of strain hardening in panels with low slenderness ratios. The first limitation was that the theoretical buckling stress \( \tau_{cr} \) computed from Eq. 4.3 is only considered to be an ideal value \( \tau_{cri} \) when it exceeds the proportional limit (which is taken to be 0.8 \( \tau_y \)). Based on a series of shear tests on girders with web slenderness ratios between 50 and 70, \(^{31}\) Basler proposed an empirical formula for \( \tau_{cr} \) when \( \tau_{cri} \) given by Eq. 4.3 exceeds 0.8 \( \tau_y \):

\[
\tau_{cr} = \sqrt{0.8 \tau_y (\tau_{cri})} 
\]

Thus, the following set of equations are used to determine \( \tau_{cr} / \tau_y \):

\[
\tau_{cr} = \tau_{cri} \quad \text{for} \quad \tau_{cri} \leq 0.8 \tau_y \quad (4.11a)
\]

\[
\tau_{cr} = \sqrt{0.8 \tau_y (\tau_{cri})} \quad \text{for} \quad \tau_{cri} > 0.8 \tau_y \quad (4.11b)
\]

where

\[
\tau_{cri} = k \frac{\pi^2 E}{12(1 - \nu^2)} \left( \frac{1}{h} \right)^2 \quad (4.11c)
\]

The second limitation is that if the critical shear stress \( \tau_{cr} \) exceeds the shear yield stress \( \tau_y \), it is assumed that no tension field action will be developed. Therefore, the ultimate shear strength is given by

\[
\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \cdot \frac{1 - \tau_{cr}/\tau_y}{\sqrt{1 + \alpha}} \quad \text{for} \quad \frac{\tau_{cr}}{\tau_y} \leq 1, \quad (4.12a)
\]

and

\[
\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} \quad \text{for} \quad \frac{\tau_{cr}}{\tau_y} > 1. \quad (4.12b)
\]
Eqs. 4.12 are shown graphically in Fig. 43. Since the ratio $V_u/V_p$ is a function of $\alpha$, $\beta$ and the yield strain $\varepsilon_y$, one variable has to be kept constant on a two-dimensional plot. For Fig. 43, $\sigma_y = 36$ ksi. ($\varepsilon_y = 0.0012$) and ultimate strength curves are shown for five values of the aspect ratio $\alpha$ covering the practical range from $\alpha = 1/2$ to $\alpha = 3$. There are two points of discontinuity on each of these curves: the first, between $V_u/V_p = 0.85$ and $V_u/V_p = 0.95$, indicates the transition from Eq. 4.11a to 4.11b ($\tau_{cr}/\tau_y = 0.8$) and the second, at $V_u/V_p = 1.0$, indicates the transition from Eq. 4.12a to 4.12b.

Also plotted on Fig. 43 are a number of test points to show the correlation between Basler's theory and test results. The principal specimen parameters, the theoretical ultimate shear force and the experimentally measured ultimate shear force $V_{u \text{ ex}}$ for 22 tests are listed in Table 7. The first 8 tests in the table are the girder tests which were used to establish the empirical formula of Eq. 4.11b. The values of the correlation ratio $V_{u \text{ ex}}/V_{u \text{ th}}$ in the last column of the table range from 0.95 to 1.07 for these tests, indicating that Eq. 4.11b provides a good estimate of the shear strength in the strain hardening range for girders with low slenderness ratios.

The 10 shear tests described in Parts 3 and 4 of Ref. 5 were structural carbon steel plate girder tests. Covering a wide range of aspect ratios and slenderness ratios, these tests provide a means of checking the accuracy of Eq. 4.11a in predicting ultimate shear strength. The $V_{u \text{ ex}}/V_{u \text{ th}}$ ratios for these tests indicate that correlation between
theory and tests can generally be expected to be within 10% and that
Eq. 4.11b tends to be conservative for high aspect ratios (\( \alpha = 3 \)).
In addition to the graphical representation of theory and test results
in Fig. 43, a simpler presentation can be made for the seven tests on
girders with a slenderness ratio of about 260. In Fig. 44, \( \frac{V_u}{V_p} \) is
plotted against the aspect ratio \( \alpha \) for constant values of \( \beta = 260 \) and
\( \sigma_y = 36 \) ksi. This figure further illustrates the accuracy of Eq. 4.11a
for the practical range \( 0.5 \leq \alpha \leq 3 \).

The last four tests listed in Table 7 were structural alloy steel
girder tests which were conducted to determine the applicability of
Eq. 4.11a to girders with an extreme value of \( \varepsilon_y \)\(^{29} \). The results of
these tests agree reasonably well with the theory except for Test Hl-
Tl, which has a \( \frac{V_{ex}}{V_{th}} \) ratio of 1.33. Since \( \alpha = 3 \) for this test,
the tendency for Eq. 4.11a to be conservative for high aspect ratios
seems to be more pronounced for constructional alloy steel girders than
it is for structural carbon steel girders.

**Transverse Stiffener and End Panel Requirements**

Transverse stiffeners perform two functions in developing the shear
strength of a plate girder panel; initially to preserve the shape of
the girders's cross section, and to transfer the vertical components
of the tension field stresses from one end of the stiffener to the
other. The first requirement is a stiffness requirement. The stiff-
ener must be rigid enough to force a nodal line in the lateral web
deflection pattern at the stiffener location. The stiffener stiffness
requirements in design specifications which are based on buckling strength are intended to ensure that the required rigidity is provided.

Since the stiffener acts as a column in carrying the vertical components of the tension field stresses (see Fig. 42a), the second requirement is an area requirement. The axial force on the stiffener $F_s$ can be determined from a summation of forces in the vertical direction on the free-body diagram in Fig. 42b, $F_s = \sigma_t \cdot t \cdot a \cdot \sin^2 \xi$. Substituting the value of $\xi$ from Eq. 4.5,

$$F_s = \sigma_t \cdot t \cdot a \cdot \left[ \frac{1}{2} - \frac{\alpha}{2\sqrt{1 + \alpha^2}} \right]$$  \hspace{1cm} (4.13)

Dividing Eq. 4.13 by $\sigma_y$ and using $\frac{\sigma_t}{\sigma_y}$ from Eq. 4.9 leads to an expression for the required stiffener area $A_{sT}$, which in non-dimensional form is

$$\delta_T^* = \frac{A_{sT}}{A_w} = \frac{1 - \tau_{cr}/\tau_y}{2} \left[ \alpha - \frac{\alpha}{\sqrt{1 + \alpha^2}} \right], \hspace{1cm} D.$$  \hspace{1cm} (4.14)

Basler introduced the factor $D$ in Eq. 4.14 to reflect the efficiency of stiffeners furnished in pairs as opposed to one-sided stiffeners. A one-sided stiffener will be subjected to moment as well as axial force since it will be loaded eccentrically. By allowing the one-sided stiffener to become fully yielded under the combined moment and axial force, and using the case of a pair of stiffeners as a reference ($D = 1.0$), it was determined that $D = 2.4$ for a one-sided stiffener.
The horizontal components of the tension field stresses are anchored by the adjacent panels (see Fig. 42a). In the case of an end panel, however, no adjacent panel is available to perform this function. Two methods can be used to provide anchorage for the horizontal components. The first is to use an end plate which forms a strong end post over the end support, as shown in Fig. 45. The required area \( A_e \) for such an end plate can be estimated from

\[
A_e = \frac{(V - V_T)b}{8e \sigma_y},
\]

where \( e \) is the width of the endpost (Fig. 45) and \( V \) is given by Eq. 4.2.

The alternative to providing an end post to anchor the horizontal components of the tension field stresses in an end panel is to limit the size of the end panel so that tension field action does not develop. This can be accomplished by letting \( V = V_T \) and determining the corresponding maximum panel width \( a \) from Eqs. 4.2-4.4.

Discussion

According to the assumptions made in deriving the shear strength theory, the behavior of a plate girder panel subjected to shear can be described schematically by the plot of shear force versus panel deformation \( \Delta \) shown in Fig. 46a. The beam action shear force \( V_T \) increases linearly to its maximum value while the tension field action shear force remains zero up to this point \( (\Delta_T) \). Beyond \( \Delta_T \), \( V_{\sigma} \) increases linearly from zero to its maximum value while \( V_T \) remains
constant. The ultimate shear force \( V_u \) is assumed to be the sum of the \( V_t \) and \( V_\sigma \) contributions. Beyond \( \Delta \), the theory does not account for any unloading or eventual reduction below \( V_u \); it is assumed that as \( \Delta \) increases beyond \( \Delta_u \), \( V_t \) and \( V_\sigma \) remain constant and therefore \( V_u = V_t + V_\sigma \) also remains constant.

Actual girder behavior is probably better described by the schematic \( V \) vs. \( \Delta \) plot of Fig. 46b. Due to the presence of initial web deflections and residual stresses, tension field action contributes to the ultimate shear force right from the beginning. As the maximum value of \( V_t \) is approached, however, the participation of tension field action increases. The ultimate shear force is approached when yielding is initiated in the web and as this yielding spreads, \( V_t \) and \( V_\sigma \) finally reach their maximum values. Beyond the first attainment of \( V_u \), there is no reason to believe that the magnitude of either \( V_t \) or \( V_\sigma \) changes significantly until the panel deformations become so severe that one of the panel boundary members fails, causing a reduction in \( V_\sigma \) and resulting in the unloading curve shown in the figure. Four of the tests described in Sect. 2.3 were continued until unloading and failure occurred (Tests LS1-T2, LS2-T1, LS3-T3 and LS4-T2). In each of these tests the compression flange was the boundary member which failed due to severe panel deformations. These panel deformations and flange failures are evident in Figs. 7, 23, 24 and 25. The load versus centerline deflection curves for these tests (Figs. 17-20) are similar to the \( V_u \) curve in Fig. 46b.
The accuracy of the shear strength theory is not affected by the differences between Figs. 46a and 46b since only the magnitude of $V_u$ is predicted by the theory. The assumed geometry of the tension field, however, does have an effect on the accuracy of the theory.

Gaylord has shown that the tension field contribution used by Basler is actually that of a complete tension field acting at the angle $\phi$ given by Eq. 4.5. This result was obtained because the forces $F_w$ and $1/2 V_\sigma$ shown acting on the web in the free-body diagram of Fig. 42a are equivalent to tension field stresses acting on the web. This can be seen from Fig. 47, which is identical to Fig. 42 except that a full tension field is shown acting at the angle $\phi$. Summation of moments about point 0 in Fig. 47b yields the same result as that obtained by Basler from Fig. 42b. Thus two tension field models were used by Basler: a partial field (Fig. 48a) to determine the angle $\phi$ and a full field acting at the angle $\phi$ (Fig. 48b) to determine the magnitude of $V_\sigma$.

The contribution of the partial tension field of Fig. 48a to the ultimate shear force can be evaluated using $s = b \cdot \cos \bar{\phi} - a \cdot \sin \bar{\phi}$ and the value of $\bar{\phi}$ from Eq. 4.5:

$$\left( \frac{\sigma_u}{\sigma_p} \right) = \frac{\sqrt{3}}{2} \frac{\sigma_t}{\sigma_y} \left( \sqrt{1 + \frac{\sigma_t^2}{\sigma_y^2}} - \alpha \right). \quad (4.15)$$

The ratio of $(V_\sigma)$ to $(V_\sigma)_B$, which is the tension field action contribution derived by Basler and given by Eq. 4.6, is

$$\frac{(V_\sigma)}{(V_\sigma)_B} = 1 + \alpha^2 - \alpha \sqrt{1 + \alpha^2}. \quad (4.16)$$
This ratio is plotted against the aspect ratio in Fig. 49. It can be seen from the figure that for high aspect ratios, \( \frac{(V_p)}{\sigma_p} / \frac{(V_B)}{\sigma_B} \) approaches 0.5. Thus an ultimate shear strength formula using \( \frac{(V_p)}{\sigma_p} \) from Eq. 4.15 would be even more conservative than Eq. 4.10 and would not give a satisfactory prediction of shear strength.

The most efficient tension field is one which acts at \( \xi = 45^\circ \) from the horizontal (Fig. 48c) since this field would yield the maximum value of the tension field contribution:

\[
\left( \frac{V}{V_p} \right)_{45^\circ} = \sqrt{2} \frac{\sigma_T}{\sigma_y} \tag{4.17}
\]

The ratio of \( \left( \frac{V}{\sigma} \right)_{45^\circ} \) to \( \left( \frac{V}{\sigma_B} \right) \) is

\[
\left( \frac{V}{\sigma} \right)_{45^\circ} / \left( \frac{V}{\sigma_B} \right) = \sqrt{1 + \alpha^2}. \tag{4.18}
\]

The ratio is also plotted in Fig. 49, indicating that an ultimate shear strength formula using \( \left( \frac{V}{\sigma} \right)_{45^\circ} \) from Eq. 4.17 would considerably overestimate the shear strength of a panel for any practical value of \( \sigma \).

It is probable that the distribution of tension field stresses in a real girder panel is actually more like that shown in Fig. 50. Distributions similar to this have been measured in test girders. The higher flexibility of the flanges relative to the webs of adjacent panels in providing anchorage for the tension field would account for this type of distribution. It is logical, therefore, to investigate a partial width field acting along the panel diagonal as in Fig. 48d since this type of field provides a reasonable approximation of the distr-
bution shown in Fig. 50. Basler showed in the Appendix to his paper that a diagonal field having a width equal to \( b/2 \) gives a tension field contribution \( \left( \frac{V}{\sigma_D} \right)_D \) which is identical to that given by Eq. 4.6,

\[
\left( \frac{V}{\sigma_D} \right)_D / \left( \frac{V}{\sigma_B} \right)_B = 1
\]

The diagonal tension field with width \( b/2 \) is the most desirable model of these pictured in Fig. 48 since it is reasonable approximation of actual panel behavior and also leads to the shear strength formula (Eq. 4.6) which gives the good correlation with test results shown in Fig. 44. One possible difficulty with this model is that for very close transverse stiffener spacing \( (\alpha < 0.58) \), an unconservative estimate of shear strength results since the assumed field would provide a tension field contribution greater than that of a complete field acting at the same angle. Judging by the values of the correlation ratio \( \frac{V_{ex}}{V_{th}} \) in Table 7 for tests G6-T3 and H2-T2, this is not a serious limitation at \( \alpha = 0.5 \). Since practical girders will not normally have transverse stiffeners spaced closer than \( 1/2 \) of the web depth \( (\alpha < 0.5) \), the applicability of the diagonal tension field model to real girders is valid.

In addition to the tension field geometry, two other assumptions made in deriving the shear strength theory can affect its accuracy. The first of these is that the buckling coefficient \( k \) for a simply supported plate should be used. The effect of this assumption can be seen in Fig. 51, which shows the variation of \( k \) with the aspect ratio \( \alpha \) for the two extreme cases of simply supported edges on all sides and
of fixed edges along the flanges. The difference between these cases amounts to approximately 5% for $\alpha = 0.5$ and approximately 66% for $\alpha = 3.0$. Since the actual degree of restraint on the web by the flanges is not known and it is desirable to be conservative in predicting shear strength, the assumption of simply supported edges is justified. However, it should be noted that the influence of this conservative assumption becomes greater with higher aspect ratios and accounts partly for the tendency of the shear strength theory to be conservative for high aspect ratios.

The other assumption which affects the accuracy of the shear strength theory is that the yield condition for the combined tension field action normal stresses and the beam action shear stresses can be approximated by Eq. 4.9. In Ref. 8, Basler showed that this approximate yield condition results in a conservative estimate of the shear strength which is at most 10% lower than that obtained using the exact form of Mises' yield condition. The fact that Eq. 4.9 leads to more conservative values of $V_u$ as $\alpha$ increases also accounts in part for the tendency of the shear strength theory to be conservative for high aspect ratios.

**Summary**

The behavior of a plate girder panel subjected to shear can be described schematically by the shear force versus panel deformation curve of Fig. 46b. The ultimate shear force can be considered to be the sum of the beam action shear force and the tension field action shear force. Based on the review of the shear strength theory for
transversely stiffened girders in this section, it is concluded that the diagonal tension field model (Fig. 48d) is a reasonable approximation of actual girder behavior. The resulting equation for ultimate shear force (Eq. 4.8) provides an effective means of estimating the shear strength of transversely stiffened plate girders.
4.3 Shear Strength of Longitudinally Stiffened Plate Girders

Tension Field Action and the Ultimate Force

Shear tests on longitudinally stiffened plate girders have shown conclusively that each individual subpanel can develop its own tension field independently of adjacent subpanels (Sect. 2.3). Photographs of test girders (Figs. 7, 23, 24 and 25) provide visual evidence of this fact. In the following development of a method of predicting the shear strength of longitudinally stiffened girder panels, the fundamental assumption that each subpanel will develop its own shear strength is based on this experimental evidence. The effectiveness of the method in predicting shear strength will be checked with test results later.

A longitudinally stiffened panel with separate tension fields in subpanels "1" and "0" is shown in Fig. 52. The subpanel dimensions are \( a_1, b_1 \) and \( a_0, b_0 \), where \( b_0 = b - b_1 \). Using the notation \( \eta_1 = \frac{b_1}{b} \), the corresponding subpanel slenderness ratios are \( \beta_1 = \frac{b_1}{t} = \eta_1 \beta \) and \( \beta_0 = \frac{b_0}{t} = \beta(1 - \eta_1) \), while the subpanel aspect ratios are \( \alpha_1 = \frac{a_1}{b_1} = \frac{\alpha}{\eta_1} \) and \( \alpha_0 = \frac{a_0}{b_0} = \frac{\alpha}{(1 - \eta_1)} \). The subpanel shear strengths will be designated \( V_{u1} \) and \( V_{u0} \).

The ultimate shear force of the longitudinally stiffened panel is assumed to be the sum of the shear strengths of the subpanels,

\[
V_u = V_{u1} + V_{u0}. \tag{4.19}
\]

In non-dimensional form, using the notation \( p_1 = \tau y_1 t, \) \( p_0 = \tau y_0 t, \)
\[
\frac{V_{u1}}{V_{p1}} = (V_u/V_p)_1 \quad \text{and} \quad \frac{V_{u0}}{V_{p0}} = (V_u/V_p)_0, \quad \text{Eq. 4.19 becomes}
\]

\[
\frac{V_u}{V_p} = \left(\frac{V_u}{V_p}\right)_1 \eta_1 + \left(\frac{V_u}{V_p}\right)_0 (1 - \eta_1)
\]

(4.20)

The shear strength theory discussed in Sect. 4.2 will be used to evaluate the components \((V_u/V_p)_1\) and \((V_u/V_p)_0\). Thus, a diagonal tension field having a width \(b_1/2\) is assumed to develop in subpanel "1" and a diagonal field of width \(b_0/2\) is assumed to develop in subpanel "0" (Fig. 52). Formulas for the ultimate shear strengths of these subpanels can be written using Eq. 4.8.

\[
\left(\frac{V}{V_p}\right)_1 = \left(\frac{\tau_{cr}}{\tau_y}\right)_1 + \frac{\sqrt{3}}{2} \left(\frac{\sigma_t}{\sigma_y}\right)_1 \frac{1}{\sqrt{1 + (\sigma/\sigma_y)_1^2}}, \quad \text{for} \quad \left(\frac{\tau_{cr}}{\tau_y}\right)_1 \leq 1,
\]

(4.21)

\[
\left(\frac{V}{V_p}\right)_0 = \left(\frac{\tau_{cr}}{\tau_y}\right)_0 + \frac{\sqrt{3}}{2} \left(\frac{\sigma_t}{\sigma_y}\right)_0 \frac{1}{\sqrt{1 + \left[\sigma/(\sigma_y)_0\right]^2}}, \quad \text{for} \quad \left(\frac{\tau_{cr}}{\tau_y}\right)_0 \leq 1,
\]

where the limit on the \(\tau_{cr}/\tau_y\) ratios is based on the assumption that no tension field action will be developed if the critical shear stress exceeds the shear yield stress.

The use of approximate yield condition (Eq. 4.9) to evaluate the \(\sigma_t/\sigma_y\) ratios in Eq. 4.21 has been purposely avoided. It has been pointed out that this approximate form is more conservative for higher aspect ratios. The subpanels of a longitudinally stiffened panel will usually have quite high aspect ratios. For example, the panel aspect
ratios for the shear tests listed in Table 1 varied from 0.75 to 1.5, while the subpanel aspect ratios varied from 1.12 to 5.0. In each test subpanel "1" had an aspect ratio of 2.0 or more. Therefore, to avoid having the predicted shear strength according to Eq. 4.20 be excessively conservative, the use of Mises' yield condition in its exact form is desirable. According to Ref. 8, the corresponding \( \sigma_t / \sigma_y \) ratios are

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_1 = \sqrt{1 + \frac{1}{3} \left( \frac{\tau_{cr}}{\tau_y} \right)_1^2 \left[ \frac{3}{2} \sin(2\theta_1) \right]^2 - 3} - \frac{\sqrt{3}}{2} \left( \frac{\tau_{cr}}{\tau_y} \right)_1 \sin(2\theta_1)
\]

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_0 = \sqrt{1 + \frac{1}{3} \left( \frac{\tau_{cr}}{\tau_y} \right)_0^2 \left[ \frac{3}{2} \sin(2\theta_0) \right]^2 - 3} - \frac{\sqrt{3}}{2} \left( \frac{\tau_{cr}}{\tau_y} \right)_0 \sin(2\theta_0)
\]

These equations can be simplified by substituting the values of \( \sin(2\theta_1) \) and \( \sin(2\theta_0) \), where \( \theta_1 \) and \( \theta_0 \) are the angles of the two subpanel diagonals:

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_1 = \sqrt{1 - \left( \frac{\tau_{cr}}{\tau_y} \right)_1^2 + A_1^2 - A_1}, \quad A_1 = \left( \frac{\tau_{cr}}{\tau_y} \right)_1 \frac{\alpha_1 \sqrt{3}}{1 + \alpha_1^2}
\]

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_0 = \sqrt{1 - \left( \frac{\tau_{cr}}{\tau_y} \right)_0^2 + A_0^2 - A_0}, \quad A_0 = \left( \frac{\tau_{cr}}{\tau_y} \right)_0 \frac{\alpha_0 \sqrt{3}}{1 + \alpha_0^2}
\]

(4.22)

Provision was made in the shear strength theory for transversely stiffened girders for panels with low slenderness ratios to develop a shear strength greater than the plastic shear force \( V \) (Eq. 4.12b). In
a longitudinally stiffened panel, it is possible for one subpanel to have a low slenderness ratio while the other does not. For example, consider a girder having $g = 300$ and $\eta_1 = 0.2$; the subpanel slenderness ratios are $\delta_1 = 60$ and $\delta_0 = 240$. According to Fig. 43, subpanel "1" would be well into the strain-hardening range before it fails. However, there is no assurance that subpanel "0" could tolerate the associated shear deformations without having one of the subpanel boundary members fail. Therefore, to fulfill the requirement of compatibility of subpanel deformations, it is necessary to eliminate the possibility of a subpanel reaching the strain-hardening range:

$$
\begin{align*}
\frac{V_u}{V_p} & = 1, \text{ for } \left(\frac{\tau_{cr}}{\tau_y}\right)_1 > 1,
\frac{V_u}{V_p} & = 1, \text{ for } \left(\frac{\tau_{cr}}{\tau_y}\right)_0 > 1,
\end{align*}
$$

(4.23)

The values of the critical shear stresses of the subpanels can be found from Eqs. 4.11, using the subpanel $g$-ratios in Eq. 4.11c. Since the degree of fixity along the subpanel borders will vary considerably and would be difficult to evaluate even for a specific web, flange and longitudinal stiffener size, simply supported edges will be assumed to determine the subpanel buckling coefficients. From Eqs. 4.4, these $k$-values are given by
It should be noted that since the flanges will always restrain the web to some extent, the use of Eqs. 4.24 to determine the subpanel k-values should result in a conservative estimate of the subpanel shear strengths.

As an example, the non-dimensionalized ultimate shear forces of the subpanels given by Eqs. 4.21 and 4.23 are plotted in Fig. 53 for \( \alpha = 1.5, \eta_1 = 0.33 \) and \( \varepsilon_y = 0.0012 \). Also shown in the figure for comparison purposes is the \( \frac{V_u}{V_p} \) curve for the same panel if the longitudinal stiffener were not used. For this case, \( \left( \frac{V_u}{V_p} \right)_1 \) is greater than \( \left( \frac{V_u}{V_p} \right) \) unstiffened for the entire range of \( \beta \) ratios while \( \left( \frac{V_u}{V_p} \right)_0 \) is less than \( \left( \frac{V_u}{V_p} \right) \) unstiffened for \( \beta > 230 \). The points of discontinuity on each curve represent the transition from the elastic range to the inelastic range, that is, \( \tau_{cr} = 0.8 \tau_y \). According to Eq. 4.20, the ultimate shear force for the stiffened panel for a particular value of \( \beta \) is obtained by multiplying the corresponding values of \( \left( \frac{V_u}{V_p} \right)_1 \) and \( \left( \frac{V_u}{V_p} \right)_0 \) by \( \eta_1 \) and \( (1 - \eta_1) \), respectively, and multiplying the sum of these two products by the plastic shear force \( V_p \).

Figs. 54, 55 and 56 show the results of such a calculation for \( \eta_1 = 0.2, \eta_1 = 0.33 \) and \( \eta_1 = 0.5 \), respectively. In each figure, the ultimate shear strength curves are plotted for three values of aspect ratio;
\( \alpha = 0.75, \alpha = 1.0 \) and \( \alpha = 1.5 \). For \( \eta_1 = 0.2 \) (Fig. 54), the transition points for \( (V_u/V_p)_1 = 1 \) occur near \( \beta = 380 \) and those for \( (\tau_{cr})_0 = 0.8 \tau_y \) occur near \( \beta = 150 \). (The point where \( (\tau_{cr})_1 = 0.8 \tau_y \) occurs above \( \beta = 400 \) for \( \eta_1 = 0.2 \)). Three transition points are shown on the curves for \( \eta_1 = 0.33 \) in Fig. 55: \( (\tau_{cr})_1 = 0.8 \tau_y \) at \( \beta \approx 295 \), \( (V_u/V_p)_1 = 1 \) at \( \beta \approx 240 \) and \( (\tau_{cr})_0 = 0.8 \tau_y \) at \( \beta \approx 160 \). Since the two subpanels are the same size when \( \eta_1 = 0.5 \), only the transition points for \( (\tau_{cr})_1 = (\tau_{cr})_0 = 0.8 \tau_y \) are shown in Fig. 56. The ultimate shear force ratio \( V_u/V_p \) for a longitudinally stiffened panel can be determined by selecting the figure for the proper \( \eta_1 \) value and reading off the ordinate where the slenderness ratio \( \beta \) intersects the proper aspect ratio curve.

The optimum stiffener position \( \eta_1 \) varies with the slenderness ratio. For the lower range of \( \beta \)-ratios, \( \eta_1 = 0.5 \) gives the highest value of \( V_u/V_p \); for \( 220 \leq \beta \leq 280 \), \( \eta_1 = 0.33 \) is the optimum position and for \( \beta > 280 \), \( \eta_1 = 0.2 \) gives the highest \( V_u/V_p \) value. This situation is shown in Fig. 57, where the ultimate shear strength curves for longitudinally stiffened panels having \( \alpha = 0.75, 1.0 \) and 1.5 are shown. Only three stiffener positions (\( \eta_1 = 0.2, \eta_1 = 0.33 \) and \( \eta_1 = 0.5 \)) are considered in the figure and for each value of \( \beta \), the \( V_u/V_p \) curve shown is for the stiffener position which gives the highest value of \( V_u/V_p \).

As shown in Fig. 57 are the ultimate shear strength curves for unstiffened panels with \( \alpha = 0.75, 1.0 \) and 1.5, so the increase in shear strength due to the longitudinal stiffener can be seen graphically. A better indication of this, however, can be obtained from Fig. 58, where
the ratio of the shear strength of the stiffened panel to the shear strength of the unstiffened panel $\Delta$ is plotted against the slenderness ratio. Again, only three values of aspect ratio and three stiffener positions are considered. The efficiency ratio $\Delta$ is 1.0 until the shear strength of the unstiffened panel becomes less than $V_p$. This occurs for $90 < \beta < 220$, and below this range of $\beta$-values, no advantage is gained by using longitudinal stiffeners. A peak in the efficiency curves is reached at the highest $\beta$-value for which $(V_u/V_p)_{\text{stiffened}} = 1.0$. At about $\beta = 220$, an abrupt transition occurs as the optimum stiffener position changes from $\eta_1 = 0.5$ to $\eta_1 = 0.33$, and a similar transition occurs at about $\beta = 280$ when the optimum $\eta_1$-value changes from 0.33 to 0.2. Of the three aspect ratios considered, $\alpha = 1.5$ provides the greatest increase in shear strength due to a longitudinal stiffener with an increase of over 10% for the entire range $100 < \beta < 400$ and a maximum increase of 47% at $\beta = 155$. Similar efficiency curves could be prepared using Eqs. 4.20-4.24 to include more stiffener positions and a larger number of aspect ratios, so that for any $\alpha$-value the optimum $\eta_1$-value could be determined more accurately and for these $\alpha$ and $\eta_1$ values the increase in shear strength due to a longitudinal stiffener could also be determined.

Stiffener Requirements

A longitudinal stiffener must fulfill two requirements if the ultimate shear strength of the stiffened panel is to be attained. It must be rigid enough to force a nodal line in the deflected web so that separate tension fields will form in the subpanels, and it must also have sufficient area to transfer the horizontal components of the
tension fields from one side of a panel to the other (see Fig. 52).
The first requirement can be satisfied by providing the minimum
stiffener rigidity $\gamma^*_L$ obtained from a web buckling analysis. Unfortunately for $\gamma^*_L$ are not available for the various stiffener positions considered in this discussion. However, charts have been published in Ref. 21 to determine $\gamma^*_L$ for $\eta_L = 0.2, 0.25, 0.33, 0.4$ and $0.5$ and for $0.7 \leq \alpha \leq 3.8$. Curves plotted from data obtained from these charts are shown in Fig. 59 for $\eta_L = 0.2, 0.33$ and $0.5$ and the values of $\gamma^*_L$ for these same $\eta_L$ - values and $\alpha = 0.75, 1.0$ and $1.5$ are listed in Table 8.

The horizontal component of the tension field force (Fig. 52) is
\[ F_h = \sigma_t t b \cos \theta = \sigma_t t b \frac{\alpha}{\sqrt{1 + \alpha^2}} \]

The corresponding longitudinal stiffener area $A_{sL}$ is $F_h/\sigma_y$, and in non-dimensional form
\[ \delta_L = \frac{A_{sL}}{A_w} = \frac{\sigma_t}{\sigma_y} \frac{\alpha}{2/1 + \alpha^2} \]  

Eq. 4.25 can be simplified using the approximate expression for $\frac{\sigma_t}{\sigma_y}$ from Eq. 4.9,
\[ \delta_L = \frac{\alpha}{2/1 + \alpha^2} \left(1 - \frac{T_{cr}}{\tau_y}\right) \]  

Unless the longitudinal stiffener is at mid height ($\eta_L = 0.5$), the horizontal components of the tension fields in the two subpanels will be different in magnitude. These forces will be at least partially anchored by the tension fields in the adjacent subpanels. If it is
conservatively assumed that the horizontal components are all applied at the corners of the subpanels (see Joint B in Fig. 52), the longitudinal stiffener will be required to carry the difference between the two horizontal components, thus

\[ \delta_L^* = |(\delta_L)_0 - (\delta_L)_1| \]  

(4.27)

The transverse stiffeners must have sufficient area to transfer the vertical component of the tension field force from the top of a subpanel to the bottom (see Fig. 52). A stiffener must therefore have sufficient area to carry the larger of the vertical components of the two subpanel tension field forces. The magnitude of these forces can be determined from

\[ F_v = \sigma_t \cdot \frac{b}{2} \cdot \sin \theta = \sigma_t \cdot t \cdot \frac{b}{2} \cdot \frac{1}{\sqrt{1 + \frac{\alpha^2}{\tau_y}}} \]

The corresponding transverse stiffener area is \( A_{sT} = F_v / \sigma_y \), and in non-dimensional form,

\[ \delta_T^* = \frac{A_{sT}}{A_w} = \frac{1}{2\sqrt{1 + \frac{\gamma}{\tau_y}}} \left( 1 - \frac{\tau c r}{\gamma} \right) \]

where \( \sigma_t / \sigma_y \) was obtained from Eq. 4.9. As pointed out above, the required transverse stiffener area \( \delta_T^* \) is the larger of the two values \( (\delta_T)_1 \) and \( (\delta_T)_0 \) which are determined from Eq. 4.28.

Both the longitudinal and transverse stiffeners are compression members and should be proportioned according to Eq. 3.9 to avoid premature local buckling. Also the area requirements given by Eqs. 4.26 and 4.28 should be modified if the ratio of the effective
stiffener length to its radius of gyration (that is, the stiffener slenderness ratio \( \ell/r \)) is large enough to cause premature lateral buckling. In this case the area requirement would be multiplied by the ratio of \( \sigma_y \) to \( \sigma_{cr} \), where \( \sigma_{cr} \) is the lateral buckling stress determined from Eq. 3.1. As pointed out in Sect. 3.3, an effective width of 20 times the web thickness can be assumed to act with a one-sided stiffener in resisting axial force or lateral bending. Therefore the stiffener area and section properties used in establishing the stiffener proportions can be computed for a "T" section consisting of the one-sided stiffener and 20t of the web.

Correlation With Test Results

The shear strength theory developed in this section can be checked experimentally with the results of the shear tests on longitudinally stiffened plate girders described in Section 2.3. The principal specimen parameters, stiffener properties and ultimate loads for the seven tests are summarized in Table 9.

For each test, the vertical component of the tension field in subpanel "0" exceeded that of subpanel "1", so that the required transverse stiffener area ratios \( \delta_T^* \) listed in the table have been computed from Eq. 4.28 using \( \alpha_0 \) and \( \left( \tau_{cr} / \tau_y \right)_0 \). The transverse stiffener area requirement was satisfied for all tests except LS4-T1, where only 91% of \( \delta_T^* \) was supplied.

The longitudinal stiffener rigidity requirements (from Table 8) were exceeded for all seven tests; however, the longitudinal stiffener
area requirements were fulfilled in only four tests. For test LS3-T1, only 76% of $\delta_{L}^{*}$ was supplied, while for tests LS3-T2 and LS4-T1, 95% and 87% of $\delta_{L}^{*}$, respectively, were supplied. However, a longitudinal stiffener failure was observed in only one of these cases. In test LS3-T1, between load Nos. 12 and 13 (see Fig. 19), a sudden increase in lateral stiffener deflection was observed, resulting in a rapid drop in the applied load. In spite of this failure, the panel was still able to maintain a load higher than that predicted by the theory. The behavior of the longitudinal stiffeners in the three tests where $\delta_{L}^{*}$ is a good indication that the stiffener area requirement given by Eq. 4.27 is conservative.

In the last section of Table 9 the theoretical ultimate loads predicted by the theory, the experimentally obtained ultimate loads and the correlation ratio $P_{u}^{th}/P_{u}^{ex}$ are listed. For the seven tests this ratio varied from 1.00 to 1.18, with a mean value of 1.10. While it is difficult to establish a trend from seven tests, it could be postulated that the theory is about 10% conservative, with experimental scatter for the seven tests ranging from -10% to +8% from this point. There does not appear to be any tendency for the theory to be more conservative for some values of $\alpha$ and $\eta_{1}$ than for others.

The conservative nature of the shear strength theory could be attributed to two factors. The first of these is the assumption that the buckling coefficient for the subpanels should be that associated with a plate with simply supported edges. It has been pointed out previously that the flanges will exert some restraint on the web plate.
However, the extent of this restraint is difficult to establish so the assumption of simply supported edges was intentionally made to be conservative. The other factor which could contribute to the conservativeness of the theory is the type of tension field geometry which was assumed. The model which was used, that of a diagonal tension field with width $b/2$, was selected because it appeared to approximate the behavior of transversely stiffened plate girders rather closely. Observations of the tension fields which developed in the longitudinally stiffened girder tests indicated that the model was also a reasonable approximation for the range of $\alpha$ and $\eta_1$ ratios which was tested. For this reason, it can be concluded that the use of buckling coefficients for simply supported panels is the main reason for the conservative nature of the shear strength theory developed in this section.

**Summary**

A method of predicting the shear strength of longitudinally stiffened plate girders has been developed in this section. The method is based on the assumptions that the subpanels formed by a longitudinal stiffener can develop independent tension fields, that the shear strength of a subpanel can be estimated using a diagonal tension field with a width of one-half the subpanel depth and that the ultimate shear strength of a stiffened panel is equal to the sum of the shear strengths of the subpanels. Transverse and longitudinal stiffener requirements have also been established based on the assumed tension field geometry.
Based on the theory, the shear strengths of panels with aspect ratios equal to 0.75, 1.0 and 1.5 and with longitudinal stiffeners located 0.2, 0.33 and 0.5 times the web depth from the compression flange have been computed and presented graphically. The optimum stiffener position was found to vary with the web slenderness ratio and the increase in shear strength due to a longitudinal stiffener was found to vary with both aspect ratio and slenderness ratio. An increase in shear strength of over 40% can be obtained for some values of aspect ratio and slenderness ratio and a minimum increase of 10% can be attained for slenderness ratios from 100 to 400 if the proper aspect ratio and stiffener position are used.

The results of seven shear tests on longitudinally stiffened plate girders have been summarized and compared with the shear strength theory. The test results indicate that the stiffener proportioning requirements are conservative and that the ultimate shear strength predicted by the theory provides a conservative estimate of actual girder shear strength.
Chapter 5

SUMMARY

The strength of I-shaped plate girders under various loading conditions is significantly affected by lateral web deflections. For this reason, the methods commonly employed in evaluating the strength of beams are not applicable to plate girders. Classical plate buckling theory, which is based on the assumptions that the plate is initially undeformed and that lateral deflections which develop under load are small relative to the plate thickness, has also been found to be inadequate in predicting the strength of plate girders. Because of initial web deflections no clearly defined buckling load is observed in a real girder test, and because of stress redistribution in the web, a considerable post-buckling strength can be developed. The web boundary members play an important role in this stress redistribution, so that the post-buckling strength cannot necessarily be determined using large deflection plate theory where initial plate imperfections and web deflections of the order of the plate thickness are considered.

Methods of predicting the static strength of transversely stiffened plate girders have been developed by Basler and Thürlimann, based on the behavior of a large number of test girders. However, methods of predicting the static strength of longitudinally stiffened plate girders have not been available. For this reason, the studies...
described in this report were undertaken. Because the tests on longitudinally stiffened girders reported in the literature have often been directed toward determining the web buckling load rather than the ultimate load, experimental studies were included in the research program.

The loading conditions of pure moment and high shear are investigated both experimentally and theoretically. For the two loading conditions, the ultimate strength formulas represent an attempt to approximate the observed behavior of the test girders, using the ultimate strength formulas for transversely stiffened plate girders as a starting point. The principal results of the investigation will be summarized according to loading condition in the following two sections.
5.1 **Bending Strength**

In a transversely stiffened plate girder subjected to pure bending, the compressed portion of the web develops substantial lateral web deflections, resulting in a redistribution of compressive stresses from the web to the compression flange (Sect. 3.1). The bending strength of a girder is therefore limited by the strength of the compression flange acting with a portion of the web as a column. It has been shown that the bending strength will be reached as a result of yielding or instability of the "compression flange column". Three types of instability were considered: lateral buckling, torsional buckling and vertical buckling. A flange stress reduction formula was also derived to compensate for the increase in compression flange stress due to stress redistribution.

A review of this theory (Sect. 3.2) indicates that the formulas for lateral and torsional buckling stresses, along with the flange stress reduction formula, provide an effective means of estimating the bending strength of transversely stiffened plate girders. However, it is concluded that a limit on the maximum web slenderness ratio based on a vertical buckling analysis is unnecessary since vertical buckling can occur only after the ultimate moment has been reached. The flange stress reduction formula can be extended beyond this "maximum web slenderness ratio", giving good correlation with test results up to a slenderness ratio of 450.
The bending tests on longitudinally stiffened plate girders indicated that a longitudinal stiffener can effectively control lateral web deflections so that little or no redistribution of stress from the web to the compression flange results (Sect. 3.3). It is suggested that a properly positioned and proportioned longitudinal stiffener can maintain a linear stress distribution in the girder section until the ultimate moment is reached, thus eliminating the need for a reduction in ultimate bending stress. A stiffener position one-fifth of the web depth from the compression flange is adopted and stiffener proportioning requirements are formulated to ensure that the bending stress distribution remains linear. The bending strength formulas and stiffener requirements for longitudinally stiffened girders are summarized in the first section of the Appendix.

The bending strength theory is compared with the results of eight bending tests on longitudinally stiffened girders. For four of these tests the longitudinal stiffener proportions were inadequate to control stress redistribution up to ultimate moment according to the theory, so a reduction in ultimate bending stress was used in predicting the ultimate loads. Excellent correlation between the predicted and experimentally measured ultimate loads was obtained (Table 6). The other four test specimens had longitudinal stiffeners of sufficient proportions according to the theory and no reduction in ultimate bending stress was required in predicting ultimate loads. The correlation for these four tests (Table 6) is very good and confirms the applicability of the theory to girders with properly proportioned longitudinal stiffeners.
Thus, a method of predicting the bending strength of longitudinally stiffened plate girders has been presented along with requirements for proportioning stiffeners. The method is based on the behavior of actual girders. It has also been shown that when properly proportioned longitudinal stiffeners are used, a significant increase in bending strength can result.
5.2 Shear Strength

In a transversely stiffened plate girder subjected to high shear, lateral web deflections increase until a stage is reached where the compressive principal stress associated with beam theory shear (Fig. 4la) can no longer increase. The stress in the direction of the tension diagonal can increase, however, and as the applied shear force is increased, a field of tensile stresses develops. This tension field action is the source of the post-buckling shear strength. When the combination of beam action shear stresses and tension field action normal stresses in the web fulfills the yield condition, the ultimate shear strength will be reached.

A method of estimating the shear strength of transversely stiffened girders has been developed in Ref. 8. The shear contribution of the type of tension field shown in Fig. 42a was evaluated and added to the beam action contribution to obtain a prediction equation for the ultimate shear force (Sect. 4.2). This equation provides a reasonable estimate of actual girder shear strength (Table 7), with the theory tending to be conservative for panels with high aspect ratios. Two factors appear to be responsible for this tendency: the approximate form of the yield condition which was used (Eq. 4.9) and the assumption that the buckling coefficient used in evaluating the beam action contribution should be that for a simply supported plate.
The shear tests on longitudinally stiffened plate girders revealed that a longitudinal stiffener can effectively control lateral web deflections to the extent that separate tension fields can be developed in the subpanels formed by the stiffener. Accordingly, a shear strength theory for longitudinally stiffened girders is formulated in Sect. 4.3, based on the assumption that the shear strengths of the subpanels can be developed independently of the adjacent subpanels and that the ultimate shear force of the stiffened panel is the sum of the ultimate shear forces of the subpanels. The tension field geometry assumed in this theory is that of a partial diagonal field with a width equal to one-half of the web depth, since this model provides a reasonable approximation of actual girder behavior. In order to avoid an unduly conservative theory, it is suggested that the exact form of the yield condition (Eq. 4.22) be used. Longitudinal and transverse stiffener requirements are formulated based on the assumed tension field geometry. The shear strength formulas and stiffener requirements are summarized in the second section of the Appendix.

Using this theory, the shear strength of panels with aspect ratios equal to 0.75, 1.0 and 1.5 and with longitudinal stiffeners located 0.2, 0.33 and 0.5 times the web depth from the compression flange are computed and presented graphically (Figs. 54-56). It is shown that the optimum stiffener position varies with the web slenderness ratio and that the increase in shear strength due to a longitudinal stiffener is a function of the aspect ratio and the slenderness ratio. A substan-
tial increase in shear strength can be realized for some combinations of aspect ratio, slenderness ratio and stiffener position.

The applicability of the theory is checked experimentally with the results of seven shear tests on longitudinally stiffened girders with varying aspect ratio and stiffener position. The test results indicate that the stiffener proportioning requirements are conservative and that the ultimate shear strength predicted by the theory provides a conservative but reasonably close estimate of actual girder shear strength.

Thus, a method of predicting the shear strength of longitudinally stiffened plate girders has been presented along with requirements for proportioning stiffeners. The method is based on assumptions which approximate the behavior of actual girders. It has also been shown that a longitudinal stiffener can make a substantial contribution to the shear strength of a plate girder.
5.3 **Suggestions for Future Research**

This report has been limited to a study of the static bending and shear strength of plate girders with a single longitudinal stiffener. A number of related topics remain to be investigated:

1) the effect of interaction between bending moments and shear forces on the carrying capacity of longitudinally stiffened plate girders;

2) the use of multiple longitudinal stiffeners on a girder web;

3) the effect of longitudinal stiffeners on the strength of non-rectangular girder panels (curved or straight haunches);

4) the fatigue strength of longitudinally stiffened plate girders.

The above list is not intended to be exhaustive but indicates some of the problems concerning longitudinally stiffened plate girders which should be studied. It is anticipated that some of these problems will be investigated at Lehigh University in the future.
APPENDIX

SUMMARY OF ULTIMATE STRENGTH FORMULAS

1. Bending Strength Formulas

Lateral Buckling of the Compression Flange:

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = 1 - \frac{\lambda_l^2}{4}, \text{ for } 0 \leq \lambda_l \leq \sqrt{2}
\]

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = \frac{\lambda_l^2}{4}, \text{ for } \lambda_l > \sqrt{2}
\]

where \( \lambda_l = \frac{f}{r} \sqrt{\frac{2}{\pi}} \)

Torsional Buckling of the Compression Flange:

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = 1 - 0.53 \left( \lambda_t - 0.45 \right)^{1.36}, \text{ for } 0.45 \leq \lambda_t \leq \sqrt{2}
\]

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = \frac{1}{\lambda_t^2}, \text{ for } \lambda_t > \sqrt{2}
\]

where \( \lambda_t = 1.61 \frac{c}{\sqrt{\varepsilon_y/d}} \)

Ultimate Bending Stress:

\[ \sigma_u = \sigma_{cr}, \text{ where } \sigma_{cr} = (\sigma_{cr})_l \text{ or } (\sigma_{cr})_t, \text{ whichever is lower} \]
Longitudinal Stiffener Requirements:

\[ \eta_1 = 0.5 \]
\[ c_s/d_s \leq 24000/\sqrt{\sigma_y} \]
\[ \gamma_L \geq \gamma_L^*, \quad \text{where} \quad \gamma_L^* = 3.87 + 5.1 \alpha + (8.82 + 77.6 \delta_L)\alpha^2 \]
\[ (\sigma_{cr}/\sigma_y)_{fs} \geq 0.6 \left( \sigma_{cr}/\sigma_y \right) \]

Transverse Stiffener Requirement:

\[ S_T \geq S_L/\alpha \]

2. Shear Strength Formulas

Buckling Coefficients:

\[ k_1 = 4.00 + 5.34/\alpha_1^2, \quad \text{for} \quad \alpha_1 \leq 1 \]
\[ k_1 = 5.34 + 4.00/\alpha_1^2, \quad \text{for} \quad \alpha_1 > 1 \]
\[ k_0 = 4.00 + 5.34/\alpha_0^2, \quad \text{for} \quad \alpha_0 \leq 1 \]
\[ k_0 = 5.34 + 4.00/\alpha_0^2, \quad \text{for} \quad \alpha_0 > 1 \]

Critical Shear Stresses:

\[ (\tau_{cr})_1 = (\tau_{cri})_1, \quad \text{for} \quad (\tau_{cri})_1 \leq 0.8 \tau_y \]
\[ (\tau_{cr})_1 = \sqrt{0.8(\tau_{cri})_1 \tau_y}, \quad \text{for} \quad (\tau_{cri})_1 > 0.8 \tau_y \]

where \[ (\tau_{cri})_1 = \frac{k_1}{\beta_1^2} \frac{\pi^2 E}{12(1 - \nu^2)} \]

\[ (\tau_{cr})_0 = (\tau_{cri})_0, \quad \text{for} \quad (\tau_{cri})_0 \leq 0.8 \tau_y \]
\[ (\tau_{cr})_0 = \sqrt{0.8(\tau_{cri})_0 \tau_y}, \quad \text{for} \quad (\tau_{cri})_0 > 0.8 \tau_y \]

where \[ (\tau_{cri})_0 = \frac{k_0}{\beta_0^2} \frac{\pi^2 E}{12(1 - \nu^2)} \]
Tension Field Stresses:

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_{/1} = 1 - \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1}^2 + A_1 \frac{2}{\sigma_y} - A_1, \quad \text{where} \quad A_1 = \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1} \frac{\alpha_1 \sqrt{3}}{1 + \alpha_1^2}.
\]

\[
\left( \frac{\sigma_t}{\sigma_y} \right)_{/0} = 1 - \left( \frac{\tau_{cr}}{\tau_y} \right)_{/0}^2 + A_0 \frac{2}{\sigma_y} - A_0, \quad \text{where} \quad A_0 = \left( \frac{\tau_{cr}}{\tau_y} \right)_{/0} \frac{\alpha_0 \sqrt{3}}{1 + \alpha_0^2}.
\]

Subpanel Ultimate Shear Forces:

\[
\left( \frac{V_u}{V_p} \right)_{/1} = \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1} + \frac{\sqrt{3}}{2} \left( \frac{\sigma_t}{\sigma_y} \right)_{/1} \frac{1}{\sqrt{1 + \alpha_1^2}}, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1} \leq 1
\]

\[
\left( \frac{V_u}{V_p} \right)_{/0} = 1, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1} > 1
\]

\[
\left( \frac{V_u}{V_p} \right)_{/0} = \left( \frac{\tau_{cr}}{\tau_y} \right)_{/0} + \frac{\sqrt{3}}{2} \left( \frac{\sigma_t}{\sigma_y} \right)_{/0} \frac{1}{\sqrt{1 + \alpha_0^2}}, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_{/0} \leq 1
\]

\[
\left( \frac{V_u}{V_p} \right)_{/0} = 1, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_{/0} > 1
\]

Ultimate Shear Strength of Stiffened Panel:

\[
\frac{V_u}{V_p} = \left( \frac{V_u}{V_p} \right)_{/1} \eta_1 + \left( \frac{V_u}{V_p} \right)_{/0} (1 - \eta_1)
\]

Longitudinal Stiffener Requirements:

\[
\gamma_L \geq \gamma_L^*, \quad \text{where} \quad \gamma_L^* \text{ is given by Fig. 59}
\]

\[
\delta_L \geq \delta_L^*, \quad \text{where} \quad \delta_L^* = \left| \left( \delta_L \right)_{/0} - \left( \delta_L \right)_{/1} \right|
\]

\[
\left( \delta_L \right)_{/1} = \frac{\alpha_1}{2\sqrt{1 + \alpha_1^2}} \left[ 1 - \left( \frac{\tau_{cr}}{\tau_y} \right)_{/1} \right]
\]
and \((δ_L)_0 = \frac{α_0}{2\sqrt{1 + α_0^2}} \left[1 - \left(\frac{τ_{cr}}{τ_y}\right)_0 \right]\)

**Transverse Stiffener Requirements:**

\(δ_T \geq δ_T^*\), where \(δ_T^*\) is the larger of the two values

\[(δ_T)_1 = \frac{1}{2\sqrt{1 + α_1^2}} \left[1 - \left(\frac{τ_{cr}}{τ_y}\right)_1 \right]\]

or \((δ_T)_0 = \frac{1}{2\sqrt{1 + α_0^2}} \left[1 - \left(\frac{τ_{cr}}{τ_y}\right)_0 \right]\)
NOMENCLATURE

1. Lower Case Letters
   a: panel width or distance between transverse stiffeners
   b: panel depth or distance between flanges; with subscript "l", distance from compression flange to longitudinal stiffener
   c: half of flange width; with subscript "s", width of a longitudinal stiffener
   d: flange thickness; with subscript "s", thickness of a longitudinal stiffener
   k: buckling coefficient
   t: effective length of a column
   r: radius of gyration
   s: tension field width
   t: web thickness
   v: girder deflection in the plane of the web
   w: lateral web deflection

   A: area of cross section
   E: modulus of elasticity, 30,000 ksi.
   F: force
   I: moment of inertia
   M: bending moment
P: test load on a girder
Q: static moment of area
S: section modulus
V: shear force

3. Greek Letters
\(\alpha\): panel aspect ratio, \(a/b\)
\(\beta\): web slenderness ratio, \(b/t\)
\(\gamma\): stiffener rigidity ratio, \(12(1 - \nu^2) I_s/bt^3\)
\(\delta\): stiffener area ratio, \(A_s/A_w\)
\(\varepsilon\): strain
\(\eta\): longitudinal stiffener position, \(\eta_l = b_l/b\)
\(\theta\): inclination of panel diagonal
\(\lambda\): column buckling parameter
\(\nu\): Poisson's ratio, 0.3
\(\rho\): ratio of web area to flange area, \(A_w/A_f\)
\(\sigma\): normal stress
\(\tau\): shear stress
\(\xi\): inclination of tension field

4. Subscripts
\(a\): above
\(cr\): critical
\(cri\): ideal critical
\(f\): flange
\(\xi\): lateral buckling
o: without longitudinal stiffener
p: plastic
s: stiffener
t: tension; torsional buckling
u: ultimate
w: web
y: yield
0: subpanel "0"
l: subpanel "l"
L: centerline
L: longitudinal stiffener
T: transverse stiffener
σ: as carried in tension
τ: as carried in shear

5. **Superscripts**
ex: experimental
th: theoretical
*: required
<table>
<thead>
<tr>
<th>Test</th>
<th>Loading</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\eta_1$</th>
<th>Longitudinal Stiffeners</th>
<th>Transverse Stiffeners</th>
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<tbody>
<tr>
<td>LB1</td>
<td>Pure Bending</td>
<td>1.0</td>
<td>444</td>
<td>---</td>
<td>none</td>
<td>3&quot;x1/4&quot;</td>
</tr>
<tr>
<td>LB2</td>
<td>Pure Bending</td>
<td>1.0</td>
<td>447</td>
<td>0.2</td>
<td>2&quot;x1/8&quot;</td>
<td>3&quot;x1/4&quot;</td>
</tr>
<tr>
<td>LB3</td>
<td>Pure Bending</td>
<td>1.0</td>
<td>447</td>
<td>0.2</td>
<td>2 1/2&quot;x1/8&quot;</td>
<td>3&quot;x1/4&quot;</td>
</tr>
<tr>
<td>LB4</td>
<td>Pure Bending</td>
<td>1.5</td>
<td>447</td>
<td>0.2</td>
<td>2&quot;x1/8&quot;</td>
<td>3&quot;x1/4&quot;</td>
</tr>
<tr>
<td>LB5</td>
<td>Pure Bending</td>
<td>0.75</td>
<td>447</td>
<td>0.2</td>
<td>2&quot;x1/8&quot;</td>
<td>3&quot;x1/4&quot;</td>
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<tr>
<td>LS1-T1</td>
<td>High Shear</td>
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<td>256</td>
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<td>none</td>
<td>3&quot;x3/4&quot;</td>
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<td>High Shear</td>
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<td>3&quot;x3/4&quot;</td>
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<td>LS2-T1</td>
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<td>0.33</td>
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<td>3&quot;x3/4&quot;</td>
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<td>276</td>
<td>0.33</td>
<td>2&quot;x1/2&quot;</td>
<td>5&quot;x3/8&quot;</td>
</tr>
<tr>
<td>LS3-T2</td>
<td>High Shear</td>
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<td>276</td>
<td>0.33</td>
<td>3 1/2&quot;x 1/2&quot;</td>
<td>5&quot;x3/8&quot;</td>
</tr>
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<td>276</td>
<td>0.33</td>
<td>3 1/2&quot;x1/2&quot;</td>
<td>5&quot;x1/2&quot;</td>
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<td>LS4-T1</td>
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<td>260</td>
<td>0.20</td>
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<td>3&quot;x1/2&quot;</td>
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<td>LS4-T2</td>
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<td>0.50</td>
<td>3 1/2&quot;x1/2&quot;</td>
<td>4 1/2&quot;x1/2&quot;</td>
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Table 1 Test Parameters and Stiffener Sizes
<table>
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<tr>
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<th>P_{cr} (kips)</th>
<th>P_y (kips)</th>
<th>P_o (kips)</th>
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<tbody>
<tr>
<td>LB1</td>
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<td>175.7</td>
<td>155.0</td>
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<td>LB2</td>
<td>81.3</td>
<td>172.2</td>
<td>153.6</td>
</tr>
<tr>
<td>LB3</td>
<td>81.4</td>
<td>169.1</td>
<td>149.9</td>
</tr>
<tr>
<td>LB4</td>
<td>81.1</td>
<td>163.8</td>
<td>143.6</td>
</tr>
<tr>
<td>LB5</td>
<td>81.7</td>
<td>166.5</td>
<td>148.3</td>
</tr>
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<td>523.6</td>
<td>351.5</td>
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<td>351.5</td>
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<td>LS2-T1</td>
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<td>408.7</td>
<td>276.9</td>
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<td>302.7</td>
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<td>357.7</td>
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<td>357.7</td>
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Table 2: Reference Loads for Girder Tests
### Table 3 Bending Test Results

<table>
<thead>
<tr>
<th>Test</th>
<th>P&lt;sub&gt;cr&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;y&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;o&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;u&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;cr&lt;/sub&gt;</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;y&lt;/sub&gt;</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;o&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB1</td>
<td>15.1</td>
<td>175.7</td>
<td>155.0</td>
<td>156.5</td>
<td>10.36</td>
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<td>81.3</td>
<td>172.2</td>
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<td>152.0</td>
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<td>169.1</td>
<td>149.9</td>
<td>150.0</td>
<td>1.84</td>
<td>0.89</td>
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<tr>
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<td>163.8</td>
<td>143.6</td>
<td>147.0</td>
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<td>150.8</td>
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### Table 4 Shear Test Results

<table>
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<tr>
<th>Test</th>
<th>P&lt;sub&gt;cr&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;y&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;o&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;u&lt;/sub&gt; (kips)</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;cr&lt;/sub&gt;</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;y&lt;/sub&gt;</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;^ex/P&lt;sub&gt;o&lt;/sub&gt;</th>
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<tr>
<td>LS1-T1</td>
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<td>523.6</td>
<td>351.5</td>
<td>363.5</td>
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<td>408.7</td>
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<td>G3-T2</td>
<td>G4-T1</td>
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<td>$\rho$</td>
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<tr>
<td>c/d</td>
<td>24</td>
<td>16</td>
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<td>16</td>
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<td>Pred. Mode</td>
<td>T*</td>
<td>L**</td>
<td>LorT</td>
<td>L</td>
<td>L</td>
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* T: Torsional Buckling
** L: Lateral Buckling

Table 5 Test Results from Ref. 7
<table>
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<th>SOURCE</th>
<th>REFERENCE 19</th>
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<td>LS2 LS3 LS4 LS5</td>
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<td>3.0 5.0 6.4 6.7</td>
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<tr>
<td>$(\sigma_{cr}/\sigma_y)_{xf}$</td>
<td>0.991 0.991 0.981 0.995</td>
<td>0.759 0.736 0.887 0.898</td>
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<td>$\gamma^*$</td>
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<td>13.4 13.4 12.3 12.3</td>
</tr>
<tr>
<td>$0.6(\sigma_{cr}/\sigma_y)_{xf}$</td>
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<tr>
<td>$(\sigma_{cr}/\sigma_y)_{xs}$</td>
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<tr>
<td>$s_L/\alpha$ (in.$^3$)</td>
<td>0.110 0.096 0.099 0.102</td>
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<td>$s_T$ (in.$^3$)</td>
<td>0.562 0.562 0.562 0.562</td>
<td>0.013 0.064 0.583 0.583</td>
</tr>
<tr>
<td>% Red.</td>
<td>10.7 10.6 10.5 10.5</td>
<td>(17.2) (24.2) (16.4) (26.1)</td>
</tr>
<tr>
<td>$P_{tu}^{th}$ (k)</td>
<td>153.6 149.9 143.6 148.3</td>
<td>28.3 38.6 189.7 238.8</td>
</tr>
<tr>
<td>$P_{tu}^{ex}$ (k)</td>
<td>152.0 150.0 147.0 150.8</td>
<td>28.2 36.3 194.2 237.4</td>
</tr>
<tr>
<td>$P_{tu}^{ex}/P_{tu}^{th}$</td>
<td>0.99 1.00 1.02 1.02</td>
<td>1.00 0.94 1.02 0.99</td>
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Table 6 Correlation of Bending Strength Theory With Test Results
<table>
<thead>
<tr>
<th>Source</th>
<th>Test</th>
<th>$a/b$</th>
<th>$b/t$</th>
<th>$A_w$ (in.$^2$)</th>
<th>$\sigma_y$ (ksi)</th>
<th>$V_u^{th}$ (kips)</th>
<th>$V_u^{ex}$ (kips)</th>
<th>$V_u^{ex}/V_u^{th}$</th>
</tr>
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<tbody>
<tr>
<td>Group A</td>
<td>WB-1</td>
<td>3</td>
<td>56.5</td>
<td>3.47</td>
<td>43.3</td>
<td>108</td>
<td>109</td>
<td>1.01</td>
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<tr>
<td>Ref. 31</td>
<td>WB-2</td>
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<td>54.9</td>
<td>3.57</td>
<td>47.8</td>
<td>120</td>
<td>128</td>
<td>1.07</td>
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<tr>
<td></td>
<td>WB-3</td>
<td>3</td>
<td>58.9</td>
<td>4.36</td>
<td>49.6</td>
<td>139</td>
<td>139</td>
<td>1.00</td>
</tr>
<tr>
<td>Group B</td>
<td>WB-6</td>
<td>3</td>
<td>70.0</td>
<td>4.40</td>
<td>33.1</td>
<td>97</td>
<td>96</td>
<td>0.99</td>
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<tr>
<td>Ref. 31</td>
<td>WB-7</td>
<td>3</td>
<td>60.6</td>
<td>3.88</td>
<td>33.7</td>
<td>100</td>
<td>95</td>
<td>0.95</td>
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<tr>
<td></td>
<td>WB-8</td>
<td>3</td>
<td>59.7</td>
<td>4.10</td>
<td>29.7</td>
<td>101</td>
<td>100</td>
<td>0.99</td>
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<tr>
<td></td>
<td>WB-9</td>
<td>3</td>
<td>50.0</td>
<td>3.12</td>
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<td>92</td>
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<td>WB-10</td>
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<td>30.3</td>
<td>94</td>
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<tr>
<td>Part 3</td>
<td>G6-T1</td>
<td>1.5</td>
<td>259</td>
<td>9.65</td>
<td>36.7</td>
<td>112</td>
<td>116</td>
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<td>Ref. 5</td>
<td>G6-T2</td>
<td>0.75</td>
<td>259</td>
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<td>157</td>
<td>150</td>
<td>0.95</td>
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<tr>
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<td>G6-T3</td>
<td>0.5</td>
<td>259</td>
<td>9.65</td>
<td>36.7</td>
<td>180</td>
<td>177</td>
<td>0.98</td>
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<tr>
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<td>G7-T1</td>
<td>1.0</td>
<td>255</td>
<td>9.80</td>
<td>36.7</td>
<td>142</td>
<td>140</td>
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<tr>
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<td>G7-T2</td>
<td>1.0</td>
<td>255</td>
<td>9.80</td>
<td>36.7</td>
<td>142</td>
<td>145</td>
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<td>Part 4</td>
<td>E1-T1</td>
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<td>131</td>
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<td>131</td>
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<td>38.2</td>
<td>76</td>
<td>85</td>
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<td>254</td>
<td>9.85</td>
<td>38.2</td>
<td>119</td>
<td>117</td>
<td>0.98</td>
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<td>382</td>
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<td>85</td>
<td>79</td>
<td>0.93</td>
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<tr>
<td>Ref. 29</td>
<td>H1-T1</td>
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<td>127</td>
<td>19.65</td>
<td>108.1</td>
<td>473</td>
<td>630</td>
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<td></td>
<td>H1-T2</td>
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<td>127</td>
<td>19.65</td>
<td>108.1</td>
<td>710</td>
<td>769</td>
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<td>H2-T1</td>
<td>1.0</td>
<td>128</td>
<td>19.50</td>
<td>110.2</td>
<td>875</td>
<td>917</td>
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<td>H2-T2</td>
<td>0.5</td>
<td>128</td>
<td>19.50</td>
<td>110.2</td>
<td>1143</td>
<td>1125</td>
<td>0.98</td>
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Table 7 Summary of Shear Tests on Welded, Transversely Stiffened Plate Girders (Ref. 8)
<table>
<thead>
<tr>
<th>α</th>
<th>γ* L for η₁ = 0.2</th>
<th>γ* L for η₁ = 0.33</th>
<th>γ* L for η₁ = 0.5</th>
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<tbody>
<tr>
<td>0.75</td>
<td>0.4</td>
<td>2.5</td>
<td>11</td>
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<td>3</td>
<td>8</td>
<td>35</td>
</tr>
<tr>
<td>1.5</td>
<td>17</td>
<td>32</td>
<td>140</td>
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Table 8 Longitudinal Stiffener Rigidity Requirements  
(Ref. 21)

<table>
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<tr>
<th>TEST</th>
<th>LS1-T2</th>
<th>LS2-T1</th>
<th>LS3-T1</th>
<th>LS3-T2</th>
<th>LS3-T3</th>
<th>LS4-T1</th>
<th>LS4-T2</th>
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</thead>
<tbody>
<tr>
<td>α</td>
<td>1.0</td>
<td>1.0</td>
<td>1.5</td>
<td>1.5</td>
<td>0.75</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>β</td>
<td>256</td>
<td>275</td>
<td>276</td>
<td>276</td>
<td>276</td>
<td>260</td>
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<td>η₁</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.20</td>
<td>0.50</td>
</tr>
<tr>
<td>δ* T</td>
<td>0.212</td>
<td>0.210</td>
<td>0.160</td>
<td>0.160</td>
<td>0.232</td>
<td>0.258</td>
<td>0.143</td>
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<tr>
<td>δ T</td>
<td>0.315</td>
<td>0.315</td>
<td>0.275</td>
<td>0.275</td>
<td>0.342</td>
<td>0.235</td>
<td>0.315</td>
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<td>γ* L</td>
<td>8</td>
<td>8</td>
<td>32</td>
<td>32</td>
<td>2.5</td>
<td>17</td>
<td>35</td>
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<td>γ L</td>
<td>264</td>
<td>164</td>
<td>49</td>
<td>116</td>
<td>116</td>
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<td>δ* L</td>
<td>0.238</td>
<td>0.238</td>
<td>0.275</td>
<td>0.275</td>
<td>0.201</td>
<td>0.302</td>
<td>0.000</td>
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<tr>
<td>δ L</td>
<td>0.502</td>
<td>0.288</td>
<td>0.208</td>
<td>0.262</td>
<td>0.262</td>
<td>0.262</td>
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<tr>
<td>P th(kips)</td>
<td>375.0</td>
<td>296.0</td>
<td>253.0</td>
<td>253.0</td>
<td>315.8</td>
<td>381.0</td>
<td>342.6</td>
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<tr>
<td>P ex(kips)</td>
<td>414.0</td>
<td>315.5</td>
<td>278.5</td>
<td>296.0</td>
<td>338.0</td>
<td>380.5</td>
<td>405.5</td>
</tr>
<tr>
<td>P ex/P th</td>
<td>1.10</td>
<td>1.06</td>
<td>1.10</td>
<td>1.17</td>
<td>1.07</td>
<td>1.00</td>
<td>1.18</td>
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</table>

Table 9 Correlation of Shear Strength Theory With Test Results
Fig. 1 Plate Girder Panel Loading Conditions

Fig. 2 Typical Longitudinally Stiffened Panel
Fig. 3 Setup for Bending Tests

Fig. 4 Setup for Shear Tests
Fig. 5 Cross Section of Bending Test Specimens

Fig. 6 Cross Section of Shear Test Specimens
Fig. 7 Girder LS4 After Testing

Fig. 8 Load-Vs-Centerline Deflection Curve
for Specimen LBI
Fig. 9 Load-Vs-Centerline Deflection Curve for Specimen LB4

Fig. 10 Web Deflections of Specimen LBI
Fig. 11 Web Deflections of Specimen LB3

Fig. 12 Yield Pattern and Longitudinal Stiffener
Buckles After Test LB3
Fig. 13 Stress Distribution in Specimen LB1

Fig. 14 Stress Distribution in Specimen LB3
Fig. 15 Yield Pattern in Compression Flange and Web After Test LB1

Fig. 16 Failure Due to Vertical Buckling in Specimen LB4
Fig. 17 Load-Vs-Centerline Deflection Curve for Girder LS1

Fig. 18 Load-Vs-Centerline Deflection Curve for Girder LS2
Fig. 19  Load-Vs-Centerline Deflection Curve
for Girder LS3

Fig. 20  Load-Vs-Centerline Deflection Curve
for Girder LS4
Fig. 21 Web Deflections at the Center of Two Panels of Girder LS1

Fig. 22 Web Deflections at the Center of Two Panels of Girder LS4
Fig. 23 Girder LS1 After Testing

Fig. 24 Girder LS2 After Testing
Fig. 25 Girder LS3 After Testing

Fig. 26 Test Measurements on Specimen LBI
Fig. 27 Buckling Modes of Compression

Flange Column

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = 1 - \frac{\lambda^2}{4}
\]

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = \frac{1}{\lambda^2}
\]

Fig. 28 Lateral Buckling Curve
(a) Assumed Model

(b) Loading

Fig. 29 Torsional Buckling of Compression Flange

\[ \frac{\sigma_{cr}}{\sigma_y} |_{\lambda_t} = 1 - 0.53(\lambda_t - 0.45)^{3.6} \]

\[ \frac{\sigma_{cr}}{\sigma_y} |_{\lambda_t} = \frac{1}{\lambda_t^2} \]

Fig. 30 Torsional Buckling Curve
Fig. 31 Vertical Buckling of Compression Flange
(Test G4-T2, Ref. 5)

Fig. 32 Ultimate Bending Moment as Influenced by Web Slenderness Ratio

\[ \frac{M_u}{M_y} \]

\[ \sigma_y = 33 \text{ ksi} \]
\[ \rho = 1.0 \]

\[ \beta \]
Fig. 33 Flange Stress Reduction Curves

Fig. 34 Schematic Moment-Vs-Centerline Deflection Curve
CASE I
\[ A = A_f = 2c d \]
\[ I = \frac{2}{3} c^3 d \]

CASE II
\[ A_t = 2c d \]
\[ A_s = 2c_s d_s \]
\[ A = 2c d + 2c_s d_s \]
\[ I = \frac{2}{3} (c^3 d + c_s^3 d_s) \]

CASE III
\[ A_t = 2c d \]
\[ A_s = c_s d_s \]
\[ A = 2c d + c_s d_s \]
\[ I = \frac{1}{3} (2c^3 d + c_s^3 d_s) \]

Fig. 35 Section Properties of Various Compression Flange Columns

Specimen LB3
\[ \alpha = 10, \beta = 447 \]
\[ P_{cr} = 81.4^k \]
\[ P_{cr} = 150.0^k \]

Scale for \( w \):
0 \ 0.1" \ 0.2"

Scale for \( \sigma \):
0 \ 5 \ 10ksi

(a) Lateral Web Deflections (b) Stress Distributions

Fig. 36 Test Measurements on Specimen LB3
Fig. 37 Lateral Buckling of Longitudinal Stiffener Column

Derivation of Transverse Stiffener Requirements
Fig. 39 Bending Test Specimens From Ref. 18
Fig. 40 Typical Shear Panel

(a) Beam Theory Shear Stress

(b) Tension Field Stress

Fig. 41 Stress States in Plate Girder Web
Fig. 42 Basler's Tension Field Model (Ref. 8)

Fig. 43 Shear Strength Curves for Transversely Stiffened Plate Girders (Ref. 8)
Fig. 44 Correlation Between Theory and Test Results, Transversely Stiffened Plate Girders

\[ \frac{V_u}{V_p} \]

\[ \sigma_y = 36 \text{ ksi} \]
\[ \beta = 360 \]

Fig. 45 End Post as Anchorage for Tension Field (Ref. 8)
Fig. 46 Schematic Shear Panel Behavior

Fig. 47 Complete Tension Field Acting at Angle $\phi$
Fig. 48 Geometry of Tension Field Models

Fig. 49 Comparison of Shear Strengths Using Various Tension Field Models
**Fig. 50** Probable Distribution of Tension Field Stresses

**Fig. 51** Influence of Boundary Conditions on Buckling Coefficient
Fig. 52 Tension Field Model for Longitudinally Stiffened Panel

Fig. 53 Subpanel Ultimate Shear Forces, $\alpha = 1.5$ and $\eta_1 = 0.33$
Fig. 54 Shear Strength Curves for $\eta_1 = 0.2$

Fig. 55 Shear Strength Curves for $\eta_1 = 0.33$
Fig. 56 Shear Strength Curves for $\eta_1 = 0.5$

Fig. 57 Shear Strength Curves Using Optimum Stiffener Position
Fig. 58 Increase in Shear Strength Due to the Use of a Longitudinal Stiffener

\[ \Delta = \frac{(V_u)_{\text{Stiffened}}}{(V_u)_{\text{Unstiffened}}} \]

\( \alpha = 1.5 \)
\( \alpha = 1 \)
\( \alpha = 0.75 \)

Fig. 59 Longitudinal Stiffener Rigidity Requirements
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