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STATIC STRENGTH OF LONGITUDINALLY STIFFENED PLATE GIRDERS

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The general objective of this report is to determine the effect of longitudinal stiffeners on the static behavior of plate girders based on observations of test girders, and to develop methods of estimating the static strength of longitudinally stiffened plate girders.

Bending tests on longitudinally stiffened plate girders indicate that by controlling lateral web deflections, a longitudinal stiffener can maintain a linear stress distribution until the ultimate moment is reached, thus eliminating the need for a reduction in ultimate bending stress. Stiffener positioning and proportioning requirements are formulated to ensure that the bending stress distribution remains linear. For test girders with stiffeners which fulfill these requirements, the observed ultimate loads agree very closely with those predicted by the theory.

Shear tests on longitudinally stiffened plate girders reveal that a longitudinal stiffener can control lateral web deflections to the extent that separate tension fields can be developed in the subpanels formed by the stiffener. Accordingly, a shear strength theory is formulated by assuming that the shear strengths of the subpanels can be developed independently of adjacent subpanels and that the ultimate shear force of the stiffened panel is the sum of the ultimate shear forces of the subpanels. Using this theory, the shear strength is computed for a number of panel sizes and stiffener
positions. Comparison of these predictions with test results indicates that the theory provides a reliable, though somewhat conservative estimate of the shear strength of longitudinally stiffened plate girders.
FOREWARD

Prior to 1961 the provisions for the design of steel plate girders in most specifications were based on the theoretical buckling strength of the web. Theoretical and experimental research on transversely stiffened plate girders at Lehigh University has shown that there is no consistent relationship between the ultimate strength and the theoretical buckling strength of a steel girder.\textsuperscript{1,2,3,4} Specifications for transversely stiffened plate girders for buildings based on this work are now being used in this country.\textsuperscript{5}

In 1963 a new plate girder research project was started at Lehigh University with the general objective of determining the contribution of longitudinal stiffeners to the static load-carrying capacity of plate girders. The analytical phase of this research consisted of separate investigations of the bending and shear strength of longitudinally stiffened plate girders. The purpose of this report is to present the results of these analytical studies. Parallel experimental investigations are described separately in another report.\textsuperscript{6}
1. **Introduction**

The behavior of a transversely stiffened plate girder subjected to pure bending can be described using the test data on measured web deflections and bending stresses shown in Fig. 1. This data was obtained from the test on specimen LB1, described in Ref. 6.

Plotted in Fig. 1a are lateral web deflection patterns measured at four different loads. The initial deflected configuration of the web is indicated for a load of zero kips and it can be seen that the maximum initial deflection was about one and a half times the thickness of the web. This situation is quite typical of welded girders with high web slenderness ratios. The figure indicates that the web deflections increased at a rather uniform rate in the upper half of the web, which was subjected to compressive bending stresses, while the deflections in the lower half of the web were somewhat reduced as load was increased due to the tensile stresses present in that region. Once again it can be stated that this behavior is typical of welded plate girders with high slenderness ratios.

The behavior of transversely stiffened plate girders subjected to bending is further illustrated by the curves of bending stress distribution in Fig. 1b. For the web, each plotted point represents the average of two values obtained from gages mounted on opposite sides of the web, therefore the curves indicate the web
membrane stresses. Also, the linear stress distributions predicted by conventional beam theory, that is, $\sigma = \frac{M y}{I}$, are shown in the figure by light lines. The measured tensile stresses in the lower portion of the web correspond very closely to those predicted by beam theory, however, due to the increasing lateral web deflections in the compression zone, a redistribution of compressive stresses from the web to the compression flange occurs. The stresses in a significant portion of the web between the neutral axis and the compression flange are essentially zero while the compression flange and a portion of the web adjacent to it carry a stress which exceeds that predicted by beam theory.

The effect of a longitudinal stiffener on the behavior of a girder subjected to pure bending can be explained with the aid of the web deflection and bending stress distribution data of Fig. 2. This data was obtained from specimen LB3, which was essentially identical to specimen LB1 except for the presence of a longitudinal stiffener located one-fifth of the web depth from the compression flange.  

The measured lateral web deflection patterns for four different loads are plotted in Fig. 2a. In comparing these web deflections with those plotted in Fig. 1a, a number of differences are apparent. The web of specimen LB3 was initially deformed in a single wave pattern while specimen LB1 (Fig. 1a) had an initial deflected configuration with two waves. The maximum initial deflection in
the web of LB3 was almost double the web thickness compared with a maximum initial deflection of about one and a half times the thickness for LB1. The magnitude and pattern of initial web deflections are random, however, they can be controlled to some extent by the amount of heat input during welding and by the welding sequence. The most significant difference between the behavior of the two girders is the extent of the increase in web deflections due to the applied loads. The maximum increase in deflection at a load of 120 kips for LB1 was 275% while the maximum increase for LB3 at the same load was only about 45%. This resulted from the fact that in specimen LB3 the web deflection growth under load was controlled by the longitudinal stiffener.

Further information on the influence of a longitudinal stiffener on bending strength can be obtained from a comparison of the stress distributions in specimen LB3 (Fig. 2b) and LB1 (Fig. 1b). Although the large initial web deflections of LB3 caused the web membrane stresses to deviate somewhat from beam theory (indicated by light lines in the figure), a redistribution of stress from the web to the compression flange of the type shown in Fig. 1b for specimen LB1 did not occur in specimen LB3. Beam theory could be used to predict the compression flange stresses in LB3 very accurately for the loads shown in Fig. 2b. Since the two girders were identical in every respect except for the presence of a longitudinal stiffener in specimen LB3, the stiffener must be credited with preventing an extensive stress redistribution.
The control of lateral web deflections in the compressed portion of the web by a longitudinal stiffener has been observed in a number of other tests on full size girders. The corresponding control of stress redistribution from the web to the compression flange is an important result of the use of a longitudinal stiffener and will be further discussed in Sect. 3.

2. Compression Flange Stability

The redistribution of stress from the web to the compression flange of a transversely stiffened plate girder subjected to bending has been described above (Fig. 1b). Because of this stress redistribution, Basler and Thürlimann reasoned that the bending strength of a plate girder is governed by the strength of the compression flange acting with a portion of the web as a column. It was assumed that the bending strength would be reached as a result of yielding or of instability of the "compression flange column". Three types of instability were considered; lateral buckling, torsional buckling and vertical buckling, the directions of which are indicated by the arrows in Fig. 3.

For the first of the three compression flange column buckling modes, lateral buckling, Basler and Thürlimann proposed the following formulas to estimate the lateral buckling stress: 
\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = 1 - \frac{\lambda^2}{4}, \quad \text{for } 0 \leq \lambda_l \leq \sqrt{2}
\]
\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_l = \frac{1}{\lambda^2}, \quad \text{for } \lambda > \sqrt{2}
\]

where \( \lambda_l = \frac{L}{r} \sqrt{\frac{\varepsilon_y}{2}}, \)

\( L \) is the effective unsupported length, \( r \) is the radius of gyration of the compression flange column and \( \varepsilon_y \) is the yield strain. In the derivation of Eq. 1, it was shown that one-sixth of the area of the web \( A_w \) acts with the compression flange, therefore the radius of gyration is given by \( r = \frac{I_f}{(A_f + A_w/6)} \), where \( I_f \) and \( A_f \) are the moment of inertia and the area of the compression flange, respectively.

Assuming that a longitudinal stiffener is located close enough to the compression flange to act with the flange in resisting lateral buckling, the effect of the stiffener on the lateral buckling stress of the compression flange column will now be explored. Lateral buckling of the compression flange is a function of the effective length, the material properties and the radius of gyration of the compression flange column according to Eqs. 1. The latter property is the only one which could be affected by the presence of a longitudinal stiffener.
Consider the three cases of Fig. 4, where the thickness of the web is assumed to be zero. The radius of gyration for each of the three sections is listed below.

Case I  \( r_1 = \frac{c}{\sqrt{3}} \)

Case II  \( r_2 = \frac{c}{\sqrt{3}} \left( 1 + \frac{\left( \frac{c_s}{c} \right)^2 A_s}{A_f} \right) \)

Case III \( r_3 = \frac{c}{\sqrt{3}} \left( 2 + \frac{\left( \frac{c_s}{c} \right)^2 A_s}{A_f} \right) \)

For a longitudinal stiffener to have a beneficial effect on the lateral buckling stress of the compression flange column, the radius of gyration of the column with a stiffener must be larger than that of the column without a stiffener. An examination of the cases listed above shows that for \( c_s < c \), \( r_1 \) is greater than \( r_2 \) and \( r_3 \); for \( c_s = c \), \( r_1 = r_2 = r_3 \) and for \( c_s > c \), \( r_2 \) and \( r_3 \) are greater than \( r_1 \). Thus, only when the width of a longitudinal stiffener exceeds the half-width of the compression flange can the stiffener increase the lateral buckling stress of the compression flange column. An increase in the size of the compression flange itself would obviously be a more economical way to increase the lateral buckling strength. Thus, it can be concluded that the lateral buckling strength is not affected by a longitudinal stiffener.
The compression flange buckling mode referred to by Basler and Thürlimann as torsional buckling is commonly called local buckling in beam and column analysis. By considering the compression flange as a long plate hinged at the flange-web junction and subjected to pure edge compression at its ends (Fig. 5), the following formulas for the torsional buckling stress were obtained:

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_t = \begin{cases} 
1 - 0.53 (\lambda_c - 0.45)^{1.36}, & \text{for } 0.45 \leq \lambda_c \leq \sqrt{2} \\
\frac{1}{\lambda_c^{2}}, & \text{for } \lambda_c > \sqrt{2}
\end{cases}
\]  

(2)

where \( \lambda_c = 1.61 \frac{c}{d} \sqrt{\frac{\sigma_y}{c_y}} \), \( c \) is one half the width of the compression flange and \( d \) is the thickness of the compression flange.

According to Eqs. 2, torsional buckling of the compression flange is a function only of the compression flange dimensions and the material properties. The effect of web restraint on torsional buckling is negligible and was not considered in deriving these equations. Therefore, there is no possibility that a longitudinal stiffener will significantly increase the stress at which torsional buckling of the compression flange will occur.
Vertical movement of the compression flange is resisted by the bending rigidity of the flange plate and by the restraint offered to the flange by the web. A sudden movement of the flange into the web is referred to as vertical buckling of the compression flange. The appearance of a test girder after vertical buckling has occurred is shown in Fig. 6, taken from Ref. 4. Basler and Thürlimann neglected the flange rigidity and, by equating the transverse flange force components which result from curvature due to bending to the Euler buckling load of a transverse strip of the web, derived an expression for the limiting web slenderness ratio ($\beta = b/t$) below which vertical buckling would not be expected to occur prior to compression flange yielding. This limiting slenderness ratio, given in Eq. 3, varies with the yield point of the flange material and the magnitude of the tensile residual stresses at the flange-web junction ($\sigma_r$).

$$\beta_{\text{max}} = \frac{0.48E}{\sqrt{\sigma_y (\sigma_y + \sigma_r)}} \quad (3)$$

An examination of the results of the six bending tests described in Ref. 6 will be helpful in further discussing the vertical buckling problem. Since one of the original objectives of this test series was to investigate the effect of a longitudinal stiffener in increasing the resistance of the web to vertical buckling, the compression flange was designed so that neither local nor lateral buckling would occur prior to compression flange yielding.
With web slenderness ratios between 407 and 447, failure due to vertical buckling would surely be expected according to Eq. 3, especially in specimen LB1 which had no longitudinal stiffener. The results of the tests pertinent to this discussion of vertical buckling can be summarized as follows:

1) In each test, the ultimate load was reached as a result of general yielding of the compression flange.

2) The axial strain measured in the extreme fibers of the compression flange at ultimate load exceeded the yield strain in each test. Visual observations indicated that the compression flange was completely yielded at ultimate load.

3) Vertical buckling of the compression flange was observed in three tests (LB2, LB4 and LB6). In these cases testing was continued well beyond the ultimate load before vertical buckling occurred.

The test results described above seem to contradict the vertical buckling theory represented by Eq. 3. However, this is not necessarily the case. Since it was assumed that the bending rigidity of the compression flange plate could be neglected in deriving Eq. 3, the equation can only predict a value of the slenderness ratio for which the resistance of the web to vertical buckling becomes negligible. While it is true that the additional resistance to vertical buckling provided by the bending rigidity of the compression flange
will normally be very small, some bending rigidity will exist until the flange is completely yielded due to the bending moment acting on the girder. The photograph in Fig. 6 also indicates that the compression flange must be completely yielded for vertical buckling to occur; compression flange deformations of the magnitude shown in this figure could not occur in a steel plate without plastic hinges being developed.

Two conditions must be fulfilled before vertical buckling of the compression flange can occur: (1) the web must be slender enough to permit large lateral web deflections to develop so that the resistance to vertical buckling becomes negligible; (2) the compression flange must be completely yielded so that its bending rigidity also becomes negligible.

3. **Ultimate Bending Moment**

For transversely stiffened plate girders, the ultimate bending moment, defined as the highest static moment which a girder section can resist, is in some way related to the web slenderness ratio $\beta$. In girders with high web slenderness ratios, large lateral deflections will develop in the compression zone of the web, resulting in a redistribution of stress from the web to the compression flange (Sect. 1). The stresses in the compression flange and a portion of the web adjacent to the compression flange can then exceed the values predicted by the beam theory formula $\sigma = M y / I$, where $y$ is the distance from the neutral axis to the fiber
for which the stress $\sigma$ is being calculated and $I$ is the moment of inertia of the entire girder section. As the web slenderness ratio is increased, this stress redistribution becomes more pronounced, or, stated in a different way, a smaller portion of the web is effective in resisting bending stresses with the compression flange. For stocky webs, that is, for webs with low slenderness ratios, no stress redistribution will occur. For very low values of $\beta$, the stresses in the web between the neutral axis and the compression flange will reach the yield point $\sigma_y$, that is, the full plastic moment $M_p$ of the section will be developed.

In the following development, the ultimate bending moment $M_u$ will be non-dimensionalized by the yield moment $M_y$, which is defined as the moment required to initiate yielding in the compression flange, $M_y = \sigma_y S$. The general relationship between the ultimate bending moment and the web slenderness ratio can be summarized as follows:

For $\beta = \beta_A$, $\frac{M_u}{M_y} = \frac{M_p}{M_y}$;

For $\beta_A \leq \beta \leq \beta_0$, $\frac{M_p}{M_y} \geq \frac{M_u}{M_y} \geq 1$;

For $\beta = \beta_0$, $\frac{M_u}{M_y} = 1$;

For $\beta > \beta_0$, $\frac{M_u}{M_y} < 1$. 
\( \beta_A \) is defined as the highest slenderness ratio for which \( M_p \) can be developed and \( \beta_O \) is the highest slenderness ratio for which a linear stress distribution can be developed according to beam theory.

Basler and Thürlimann further defined \( \beta_O \) as the slenderness ratio at which, according to plate buckling theory, web buckling would occur when the applied moment reached \( M_y \). This slenderness ratio was expressed as \( \beta_O = 5.7 \sqrt{\sigma_y} \), which gives \( \beta_O = 170 \) for structural carbon steel with \( \sigma_y = 33 \) ksi. Based on Fig. 8 of Ref. 8, \( \beta_A \) was taken to be 53 for structural carbon steel. It was also proposed that, at the maximum slenderness ratio permitted by Eq. 3, a girder section consisting of the portion on the tension side of the neutral axis plus the compression flange acting with an effective width of the web equal to 30 times the web thickness should be considered available to resist bending moment. From Eq. 3, assuming \( \sigma_y = 33 \) ksi and \( \sigma_r = 16.5 \) ksi, the corresponding value of slenderness ratio is \( \beta_B = 360 \).

The ratio \( M_u/M_y \) is plotted against the web slenderness ratio in Fig. 7. As explained above, for \( \sigma_y = 33 \) ksi, \( M_u = M_p \) at \( \beta_A = 53 \), \( M_u = M_y \) at \( \beta_O = 170 \) and \( M_u = M_y \) of the reduced section of \( \beta_B = 360 \). The corresponding bending stress distributions are indicated in circles in the figure. The numerical values of \( M_u/M_y \) at \( \beta_A \) and \( \beta_B \) depend on the area ratio \( \rho = A_w/A_f \). Since a curve passing through the plotted points in Fig. 7 is essentially a straight line, Basler and Thürlimann assumed that the following linear \( M_u/M_y \) vs.
relationship would apply,

\[
\frac{M_u}{M_y} = 1 - 0.0005 \rho (\beta - 5.7/\sqrt{\sigma_{cr}})
\]  

(4)

Eq. 4 is plotted in Fig. 8 for various values of \( \rho \).

The influence of lateral or torsional buckling of the compression flange on the ultimate bending moment is incorporated in Eq. 4 simply by replacing \( M_y \) by the respective critical moment \( M_{cr} \). Since the stresses are related to moments by the expression \( \sigma = M/S \), the equation for ultimate bending stress becomes

\[
\sigma_u = \sigma_{cr} \left[ 1 - 0.0005 \rho (\beta - 5.7 \sqrt{E/\sigma_{cr}}) \right]
\]  

(5)

A reduction in the ultimate bending stress \( \sigma_u \) is required only when \( \beta > 5.7 \sqrt{E/\sigma_{cr}} \); this the reduction as a percentage of the critical stress \( \sigma_{cr} \) is 0.05 \( \rho (\beta - 5.7 \sqrt{E/\sigma_{cr}}) \).

The flange stress reduction formula (Eq. 5) was proposed by Basler and Thürlimann to compensate for the increase in compression flange stress above the beam theory stress due to stress redistribution in transversely stiffened plate girders with high web slenderness ratios. One of the points (Fig. 7, \( \beta_B = 360 \)) used to determine the reduction curve was derived using the vertical buckling equation (Eq. 3). However, it was concluded in Sect. 2 that the compression flange must be completely yielded before vertical buckling of the compression flange can occur. The bending strength
theory reviewed above is based on the assumption that the maximum moment which a girder section can resist is the moment required to initiate yielding of the extreme fibers, $M_y$. The use of point B in Fig. 7 to develop the flange stress reduction curve is therefore not justified.

The other two points at $\beta_A = 53$ and $\beta_O = 170$ (Fig. 7) were determined independently of the vertical buckling analysis, and since the reduction curve is a straight line through these two points, the reduction formula (Eq. 5) can still be used. Furthermore, the reduction formula can be extended beyond $\beta = 360$, since this is a limitation based on the vertical buckling analysis. This is partially verified by the test on specimen LBI, described in Ref. 6. The ultimate load predicted for this transversely stiffened girder specimen, which had a slenderness ratio of 444, was 156.4 kips. A flange stress reduction of 10.7%, according to Eq. 5, was used in calculating this theoretical ultimate load. The experimentally obtained ultimate load was 156.5 kips, almost exactly the value predicted. The applicability of Eq. 5 to girders with slenderness ratios above 360 will be further substantiated with the results of four other tests in Sect. 6.

In Sect. 1 it was shown that when a longitudinal stiffener is effective in controlling lateral web deflections, stress redistribution from the web to the compression flange is also controlled or prevented. A linear stress distribution in the girder section
results, and beam theory can then be used to predict the compression flange stresses. If this type of behavior can be maintained until the ultimate bending moment is reached, the longitudinal stiffener will have a significant and beneficial effect on the bending strength. Since no stress redistribution will occur, a reduction in the ultimate bending stress is not required, and the simple expression

\[ \sigma_u = \sigma_{cr} \]  

(6)
can be used to compute the ultimate bending stress. (In this equation \(\sigma_{cr}\) is the buckling stress for lateral or torsional buckling from Eqs. 1 or 2, whichever is lower). A longitudinal stiffener should be properly positioned and adequately proportioned so that the ultimate bending stress can be computed according to Eq. 6.

4. **Longitudinal Stiffener Requirements**

In order for a longitudinal stiffener to control web deflections and prevent stress redistribution from the web to the compression flange, it obviously must be located somewhere between the neutral axis and the compression flange. Although plate girder bending strength is not directly related to web buckling strength, the control of lateral web deflections by means of a longitudinal stiffener is similar to the problem of increasing web buckling strength by forcing a nodal line in the deflection pattern of the web. Thus, it is assumed that the optimum stiffener position from a web buckling viewpoint is also the most effective position for controlling web deflections. An analysis of the
stability of a longitudinally stiffened web panel subjected to pure bending has shown that the optimum stiffener position is between $\eta_1 = 0.2$ and $\eta_1 = 0.22$, depending on the degree of restraint offered to the web by the flanges. The web deflections of the other test specimens described in Ref. 6 confirm that the one-fifth depth position ($\eta_1 = 0.2$) is effective in controlling web deflections to the extent that the stress distribution in the web remains essentially linear. Therefore, the longitudinal stiffener position $\eta_1 = 0.2$ will be adopted in the following discussion. It should be noted that if extremely high web slenderness ratios are used (say $\beta \gg 450$), a single longitudinal stiffener will not be adequate to control web deflections in the entire region between the neutral axis and the compression flange. The problem of positioning and proportioning multiple longitudinal stiffeners is beyond the scope of this report, however.

In addition to the location requirement, an effective longitudinal stiffener must be proportioned so that it will control web deflections and stress redistribution for loads up to the ultimate load. The ratio of the width of the stiffener plate $c_s$ to its thickness $d_s$ must be kept low enough to avoid premature local buckling of the stiffener. If it is conservatively required that the stiffener stress reach the yield point before local buckling occurs and if the restraint offered to the stiffener by the web is neglected, the limiting width-thickness ratio for the stiffener plate is $^{10}$.
where $\sigma_y$ is the yield point of the stiffener material. For structural carbon steel with $\sigma_y = 36$ ksi, $(c_s/d_s)_{\text{max}} = 13$.

With regard to longitudinal stiffener rigidity, two requirements are suggested. The first is that the stiffener possesses the minimum rigidity required to form a nodal line in the deflected web up to the theoretical web buckling load (elastic). Based on a buckling analysis of a plate with a longitudinal stiffener at $\eta_l = 0.2$ and subjected to pure bending, at least two formulas for minimum stiffener rigidity are available.$^{11,12}$ The formula proposed by Massonnet is the simpler one and will be adopted in this work.$^{11}$ In the formula, given below, the required stiffener rigidity ratio $\gamma_{L^*}$ is expressed as a function of the aspect ratio $\alpha$ and the stiffener area ratio $\delta_L = A_s/A_w$

$$\gamma_{L^*} = 3.87 + 5.1\alpha + (8.82 + 77.6\delta_L)\alpha^2$$

where $\gamma_{L^*}$ is defined as $I_L/I_w$ and $I_w$, the moment of inertia of the web, is defined by $I_w = bt^3/12(1 - \nu^2)$. ($I_L$ is defined below).

Above the theoretical web buckling load, the stiffener should possess sufficient rigidity to control web deflections up to the ultimate load. A convenient method of ensuring that this is the case is by considering the stability of the stiffener acting with a portion of the web as a column in a manner analogous to the
compression flange column discussed in Sect. 2. Such a column is shown in Fig. 9a, along with the compression flange column and the linear stress distribution assumed for the girder section. Neglecting the thickness of the compression flange, the requirement for the lateral buckling stress of the longitudinal stiffener column is

\[
\left( \frac{\sigma_{cr}}{\sigma_y} \right)_L \geq 0.6 \left( \frac{\sigma_{cr}}{\sigma_y} \right)_C \quad (9)
\]

for \( \eta_L = 0.2 \), where \( (\sigma_{cr}/\sigma_y)_L \) is the lateral buckling stress of the compression flange column according to Eqs. 1 and 2. Eq. 9 ensures that the longitudinal stiffener column will not fail prior to the compression flange column. In Fig. 9b, this requirement is shown graphically. The stiffener column slenderness parameter \( \lambda_L \) is given by

\[
\lambda_L = \frac{a}{r_L} \sqrt{\frac{\sigma_y}{E}} \quad (10)
\]

(Separation of the two longitudinal stiffener functions is based on the assumption that the web slenderness ratio is high enough that the stiffened web plate buckles elastically before the longitudinal stiffener column buckles).

The section to be used in computing \( I_L \) and \( r_L \) is yet to be defined. It is customary to calculate the moment of inertia of unsymmetrical (one-sided) stiffeners with respect to an axis through the web-stiffener interface.\(^{13}\) In a few tests where
strain measurements were made to determine the width of the web plate which participates in the transverse bending of a longitudinal stiffener, it was determined that the mean effective width is approximately 20 t.\textsuperscript{14} The moment of inertia $I_s$ calculated using the customary method is greatly exaggerated compared with the value obtained using an effective width of the web with the stiffener. The stiffener section properties $I_L$, $A_{sL}$ and $r_L$ to be used in Eqs. 8-10 should be computed for a section consisting of the stiffener plate and 20t of the web.

It should be noted that if a longitudinal stiffener fails to fulfill the proportioning requirements given by Eqs. 8-10, there is no justification in assuming that a stress redistribution from the web to the compression flange will be prevented and that the ultimate bending stress can be computed according to Eq. 6. In the case of an inadequately proportioned stiffener, the ultimate bending stress should be calculated from Eq. 5, which is primarily intended for transversely stiffened plate girders. This will be verified experimentally in Sect. 6.

5. **Transverse Stiffener Requirements**

A longitudinal stiffener, in performing its role of controlling web deflections, will subject the transverse stiffeners to concentrated forces at the intersection of the two stiffeners. If the longitudinal stiffener were removed from the web, the deflected shape of the web would approximate a sine curve between
transverse stiffeners. Therefore, it will be assumed that the longitudinal stiffener is subjected to a sinusoidal load by the web, as shown in Fig. 10a. The reactions at the ends of the stiffener are \( R = \frac{P_0 a}{\pi} \) and the moment at midspan is \( M_L = \frac{P_0 a^2}{\pi^2} \). If it is conservatively required that the bending stress in the longitudinal stiffener reach the yield point, the corresponding value of \( P_0 \) is \( P_0 = \sigma_y S_L \frac{\pi^2}{a^2} \), where \( S_L \) is the section modulus of the longitudinal stiffener.

A transverse stiffener, at its intersection with the longitudinal stiffener, will be subjected to a concentrated force \( 2R \) from the two adjacent longitudinal stiffener spans (Fig. 10b) if these spans are assumed to be simply supported. Using \( \eta_1 = 0.2 \) for the position of the longitudinal stiffener, the maximum moment in the transverse stiffener can be determined as \( M_T = 8 \frac{\pi \sigma_y S_L}{25 a} \). The maximum bending stress in the transverse stiffener is permitted to reach \( \sigma_y \), resulting in the expression \( S_T = \frac{8\pi}{25} \frac{S_L}{a} \), where \( S_T \) is the required section modulus of the transverse stiffener. Since the fraction \( \frac{8\pi}{25} \) is very close to unity, the simple formula

\[ S_T \geq S_L / \alpha \]  

is obtained for the required section modulus of the transverse stiffener.
Due to the conservative assumptions made in the above derivation, Eq. 11 will result in a conservative design. However, the resulting transverse stiffeners will normally not have to be larger than they would be if designed by other available criteria. If a longitudinal stiffener exceeds its minimum rigidity requirements, the value of $S_T$ required by Eq. 11 can be reduced by multiplying it by the ratio of the required longitudinal stiffener rigidity to the rigidity actually supplied.

6. Correlation With Test Results

Two series of tests are available to substantiate the expressions which have been developed in this section for computing the bending strength of longitudinally stiffened plate girders and for proportioning the stiffeners of these girders. The first series is described in Ref. 6. The principal specimen parameters, the stiffener properties and the correlation of the test results with theory are summarized in Table 1. For four of the five specimens, the longitudinal stiffener width-thickness ratio exceeded the maximum value of 13 permitted by Eq. 7. In the case of specimen LB4, neither of the longitudinal stiffener rigidity requirements given by Eqs. 8 and 9 were fulfilled either. Therefore, the ultimate bending stresses for specimens LB1 to LB4 have been computed using Eq. 5. The resulting flange stress reductions varied from 10.5

* The test specimens were designed before the longitudinal stiffener requirements presented in this report had been developed therefore, the proportions of the longitudinal stiffeners were based on other considerations.
to 10.7% for the four specimens. The theoretical ultimate loads $P^u_{th}$, the experimentally obtained loads $P^e_{ex}$ and the $P^e_{ex}/P^u_{th}$ ratios are listed at the bottom of Table 1. The excellent correlation between theory and test results confirms the applicability of Eq. 5 to girders with inadequately proportioned longitudinal stiffeners and provides further evidence that this equation can be successfully used for slenderness ratios up to 450, as tentatively concluded in Sect. 3.

The longitudinal stiffener of specimen LB6 did fulfill the requirements of Eqs. 7-9, therefore the theoretical ultimate load $P^u_{th}$ was computed using the ultimate bending stress given by Eq. 6. The resulting value of $P^u_{th}$ agrees with the experimentally obtained ultimate load within 4%. The percent reduction in ultimate bending stress shown in parentheses in Table 1 for specimen LB6 is the value which would have been applicable if no longitudinal stiffener were present or if an inadequately proportioned stiffener had been used.

The second series of tests, performed by Longbottom and Heyman, is described in detail in Ref. 15. The girder parameters, stiffener properties and test results are summarized in Table 1. All of the girders in this series had longitudinal stiffeners of sufficient proportions to fulfill the requirements of Eqs. 7-9. In addition, girders E and 4 both had a second longitudinal stiffener located at mid depth ($\eta_2 = 1/2$) which was not considered
in calculating the values in Table 1. Since the longitudinal stiffener requirements were fulfilled, the reduction in ultimate bending stress listed in parentheses in the table for each of the four girders was not used in calculating $P_u^{th}$; it is listed so that the magnitude of the increase in bending strength due to the longitudinal stiffener can be noted. The correlation of the experimental results with theory is very good, and indicates that when a properly proportioned longitudinal stiffener is used, Eq. 6 provides a reliable estimate of the bending strength.

7. Summary

The behavior of a longitudinally stiffened plate girder subjected to pure bending has been described. It has been shown that the redistribution of stress from the compressed portion of the web to the compression flange, which occurs in transversely stiffened plate girders with high web slenderness ratios, can be eliminated by a longitudinal stiffener.

The various compression flange buckling modes have been reviewed. It has been concluded that a longitudinal stiffener does not significantly increase the stress at which lateral or torsional buckling of the compression flange occurs. Based on observations of test girders, it has also been concluded that vertical buckling of the compression flange can not occur until the compression flange is completely yielded.
The ultimate strength theory for transversely stiffened plate girders has been reviewed. It has been suggested that the restriction on maximum web slenderness ratio (Eq. 3), based on a vertical buckling analysis, is an unnecessary one since vertical buckling can only be expected to occur after the ultimate moment has been attained. It has also been proposed that the flange stress reduction formula (Eq. 5) can be applied to plate girders with slenderness ratios greater than 360. This formula has been checked experimentally only for slenderness ratios up to 450, however.

Finally, a method of predicting the bending strength of longitudinally stiffened plate girders has been presented, as well as requirements for proportioning both longitudinal and transverse stiffeners in girders subjected to pure bending. Ultimate load predictions based on this method agree quite closely with the ultimate loads obtained in tests on five girders with stiffeners which fulfilled the suggested proportioning requirements, indicating that the method provides a reasonable estimate of the bending strength of longitudinally stiffened plate girders.
1. Introduction

The type of shear panel which will be discussed in this paper is shown in Fig. 11. The panel consists of a rectangular portion of the web bounded by the flanges and transverse stiffeners. It is assumed that the moment present on any section in the panel is small so that the shear strength of the panel can be studied independently.

It has been well established that plate buckling theory is not adequate to predict the shear strength of a transversely stiffened plate girder panel. Test results indicate that the ultimate shear force which a panel can sustain is considerably higher than the critical shear force calculated according to buckling theory. For one series of tests the ratio of the ultimate shear force to be critical shear force varied from about 2 to 4.4

An element subjected to pure shear stresses $\tau$ is shown at the left of Fig. 12a. These stresses correspond to the principal stresses shown at the right of the figure, where the tensile principal stress $\sigma_1$ is numerically equal to both the compressive principal stress $\sigma_2$ and the shear stress $\tau$. The state of stress shown in Fig. 12a is the type usually assumed in simple beam theory; in the following discussion it will be referred to as "beam action
shear. As the shear force on a plate girder panel is increased, a stage will be reached where the compressive stress $\sigma_2$ can no longer increase because the web deflects laterally. For an ideal panel which is initially perfectly plane, this stage occurs when the shear force reaches the critical value predicted by plate buckling theory. The stress in the direction of the tension diagonal continues to increase as the applied shear force increases beyond the critical shear force. A field of tension stresses $\sigma_t$ of the type shown on the element in Fig. 12b develops and is the source of the post-buckling shear strength of the panel. This state of stress is termed "tension field action shear".

The ultimate shear strength of a panel is the sum of the beam action and tension field action shear forces and will be reached when the combination of the two states of stress shown in Fig. 12 fulfills the yield condition. Tension field action can develop only if the panel framing members, that is, the flanges and transverse stiffeners, which serve to anchor the tension field stresses, have sufficient strength and rigidity. After the static ultimate shear force is reached, the panel yields and large shear deformations result. Final failure occurs when these shear deformations become so pronounced that one of the flanges bends into the web. This process can be traced on the load-deflection curve for test girder LS2 in Fig. 13, where a substantial yield plateau was obtained before unloading and failure occurred. The extent of the shear deformations in this girder is evident in
Fig. 14, a photograph of girder LS2, taken after completion of the destruction test.

In 1961, Basler presented a shear strength theory for plate girders which incorporated both beam action and tension field action. A number of tests of full size girders have demonstrated that this theory can be used successfully to approximate the shear strength of transversely stiffened plate girders. In the following section, this theory will be used as a basis for developing a method of predicting the shear strength of longitudinally stiffened plate girders. The applicability of the method will be checked with the results of seven shear tests on longitudinally stiffened plate girders.

2. Ultimate Shear Force

Shear tests on longitudinally stiffened plate girders have shown conclusively that each individual subpanel can develop its own tension field independently of adjacent subpanels. Photographs of test girders (for example, Fig. 14) provide visual evidence of this fact. In the following development of a method of predicting the shear strength of longitudinally stiffened girder panels, the fundamental assumption that each subpanel will develop its own shear strength is based on this experimental evidence. The effectiveness of the method in predicting shear strength will be compared with test results later.

A longitudinally stiffened panel with separate tension fields
in subpanels "1" and "0" is shown in Fig. 15. The subpanel dimensions are \(a_1, b_1\) and \(a_0, b_0\), where \(b_0 = b - b_1\). Using the notation \(\eta_1 = b_1/b\), the corresponding subpanel slenderness ratios are \(\beta_1 = b_1/t = \eta_1 \beta\) and \(\beta_0 = b_0/t = \beta (1 - \eta_1)\), while the subpanel aspect ratios are \(\alpha_1 = a_1/b_1 = \alpha/\eta_1\) and \(\alpha_0 = a_0/b_0 = \alpha/(1 - \eta_1)\). The subpanel shear strengths will be designated \(V_{ul}\) and \(V_{uo}\).

The ultimate shear force of the longitudinally stiffened panel is assumed to be the sum of the shear strengths of the two subpanels,

\[
V_u = V_{ul} + V_{uo} \quad (12)
\]

Dividing both sides of Eq. 12 by \(\tau_y b_1t\) and using the definitions \(\eta_1 = b_1/b\) and \(1 - \eta_1 = b_0/b\), Eq. 13 is obtained,

\[
\frac{V_u}{\tau_y b_1t} = \frac{V_{ul}}{\tau_y b_1t} \eta_1 + \frac{V_{uo}}{\tau_y b_0t} (1 - \eta_1) \quad (13)
\]

The quantity \(\tau_y b_1t\) is defined as the elastic shear force \(V_p\) for the entire panel while the subpanel shear forces are given by \(V_{pl} = \tau_y b_1t\) and \(V_{p0} = \tau_y b_0t\). Thus, Eq. 13 can be written

\[
\frac{V_u}{V_p} = \frac{V_{ul}}{V_{pl}} \eta_1 + \frac{V_{uo}}{V_{p0}} (1 - \eta_1) \quad (14)
\]

To simplify the notation, the definitions \(V_{ul}/V_{pl} = (V_u/V_p)_1\) and \(V_{uo}/V_{p0} = (V_u/V_p)_0\) will be used, resulting in the basic equation for the ultimate shear force for a longitudinally stiffened panel in the non-dimensional form,

\[
\frac{V_u}{V_p} = (\frac{V_u}{V_p})_1 \eta_1 + (\frac{V_u}{V_p})_0 (1 - \eta_1) \quad (15)
\]
The approach used by Basler will be adopted to evaluate the components \( \left( V_u/V_p \right)_1 \) and \( \left( V_u/V_p \right)_0 \) in Eq. 15. For subpanel "1", the ultimate shear force \( V_{u1} \) is equal to the sum of the beam action contribution \( V_{\tau 1} \) and the tension field contribution \( V_{\sigma 1} \):

\[
V_{u1} = V_{\tau 1} + V_{\sigma 1}
\]  

(16)

The beam action contribution is defined as the shear force carried by the web at the theoretical web buckling stress \( \left( \tau_{cr} \right)_1 \):

\[
V_{\tau 1} = \left( \tau_{cr} \right)_1 b_1 t
\]  

(17)

The tension field contribution is the vertical component of the tension field force \( F_1 \). Using the tension field geometry shown in Fig. 15, \( F_1 = \sigma_{t1} b_1 t \), where \( \sigma_{t1} \) is the tension field stress. The vertical component of \( F_1 \) is

\[
V_{\sigma 1} = \sigma_{t1} b_1 \frac{t}{2} \cdot \frac{1}{\sqrt{1 + \sigma^2 / \eta^2_1}}
\]  

(18)

By substituting the values of \( V_{\tau 1} \) and \( V_{\sigma 1} \) from Eqs. 17 and 18 into Eq. 16 and non-dimensionalizing the resulting expression with \( V_{p1} = \tau_y b_1 t \) and \( \tau_y = \sigma_y / \sqrt{3} \), the component \( \left( V_u/V_p \right)_1 \) can be obtained. A similar expression for \( \left( V_u/V_p \right)_0 \) can also be derived using the process described above:

*A discussion of various tension field geometries and their effect on the value of the tension field contribution to the ultimate shear force is included in Ref. 17.*
\[
\left( \frac{V_u}{V_p} \right)_1 = \left( \frac{\tau_{cr}}{\tau_y} \right)_1 + \frac{3}{2} \left( \frac{\sigma_t}{\sigma_y} \right)_1 \frac{1}{\sqrt{1 + \left( \frac{\alpha}{\eta_1} \right)^2}}, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_1 \leq 1,
\]

\[
\left( \frac{V_u}{V_p} \right)_0 = \left( \frac{\tau_{cr}}{\tau_y} \right)_0 + \frac{3}{2} \left( \frac{\sigma_t}{\sigma_y} \right)_0 \frac{1}{\sqrt{1 + \left( \frac{\alpha}{(1-\eta_1)} \right)^2}}, \quad \text{for} \quad \left( \frac{\tau_{cr}}{\tau_y} \right)_0 \leq 1,
\]

where the limit on the \( \tau_{cr}/\tau_y \) ratios is based on the assumption that no tension field action will be developed if the critical shear stress exceeds the shear yield stress.

The ultimate subpanel shear forces will be reached when the combination of beam action and tension field action stresses satisfy the yield condition. An approximate form of Mises' yield condition could be used for this purpose.\(^2\) However, it has been pointed out that this approximate form is more conservative for higher aspect ratios.\(^17\) The subpanels of a longitudinally stiffened panel will usually have quite high aspect ratios. Therefore, to avoid having the predicted shear strength according to Eq. 14 be excessively conservative, the use of Mises' yield condition in its exact form is desirable. According to Ref. 17, the corresponding \( \sigma_t/\sigma_y \) ratios are
These equations can be simplified by substituting the values of \( \sin(2\theta_1) \) and \( \sin(2\theta_0) \), where \( \theta_1 \) and \( \theta_0 \) are the angles of the two subpanel diagonals:

\[
\left(\frac{\sigma_t}{\sigma_y}\right) = \sqrt{1 + \frac{1}{3} \left(\frac{\tau_{cr}}{\tau_y}\right)^2 \left\{\frac{3}{2} \sin(2\theta_1)\right\}^2 - \frac{\sqrt{3}}{2} \left(\frac{\tau_{cr}}{\tau_y}\right) \sin(2\theta_1)}
\]

\[
\left(\frac{\sigma_t}{\sigma_y}\right) = \sqrt{1 + \frac{1}{3} \left(\frac{\tau_{cr}}{\tau_y}\right)^2 \left\{\frac{3}{2} \sin(2\theta_0)\right\}^2 - \frac{\sqrt{3}}{2} \left(\frac{\tau_{cr}}{\tau_y}\right) \sin(2\theta_0)}
\]

(20)

Provision was made in the shear strength theory for transversely stiffened girders for panels with low slenderness ratios to develop a shear strength greater than the plastic shear force \( V_p \) because of strain hardening. In a longitudinally stiffened panel, it is possible for one subpanel to have a low slenderness ratio while the other subpanel does not. For example, consider a girder having \( \beta = 300 \) and \( \eta_1 = 0.2 \); the subpanel slenderness ratios are \( \beta_1 = 60 \) and \( \beta_0 = 240 \). In this case, subpanel "1" would be well into the strain-hardening range before its fails. However, there is no assurance that subpanel "0" could tolerate the associated shear deformations without having one of the subpanel
boundary members fail. Therefore, to fulfill the requirement of compatibility of subpanel deformations, it is necessary to eliminate the possibility of a subpanel reaching the strain-hardening range:

\[
\left( \frac{V_u}{V_p} \right)_1 = 1, \text{ for } \left( \frac{\tau_{cr}}{\tau_y} \right)_1 > 1, \\
\left( \frac{V_u}{V_p} \right)_0 = 1, \text{ for } \left( \frac{\tau_{cr}}{\tau_y} \right)_0 > 1,
\]

(22)

The values of the critical shear stresses of the subpanels are given by

\[
\tau_{cr} = \tau_{cri}, \text{ for } \tau_{cri} \leq 0.8 \tau_y \\
\tau_{cr} = \sqrt{0.8 \frac{\tau_y}{\tau_{cri}}}, \text{ for } \tau_{cri} > 0.8 \tau_y \\
\tau_{cri} = k \left( \frac{\pi^2 E}{12(1-\nu^2)} \right) \left( \frac{1}{B} \right)^2
\]

(23a, 23b, 23c)

The subpanel buckling coefficient \( k \) is determined assuming simply supported edges, since the degree of fixity along the subpanel borders will vary considerably and would be difficult to evaluate even for a specific web, flange and longitudinal stiffener size.

From Ref. 17, the \( k \)-values are given by
\[ k_1 = 4.00 + 5.34/\alpha_1^2, \text{ for } \alpha_1 \leq 1 \]
\[ k_1 = 5.34 + 4.00/\alpha_1^2, \text{ for } \alpha_1 > 1 \]
\[ k_0 = 4.00 + 5.34/\alpha_0^2, \text{ for } \alpha_0 \leq 1 \]
\[ k_0 = 5.34 + 4.00/\alpha_0^2, \text{ for } \alpha_0 > 1 \]  

(24)

It should be noted that since the flanges will always restrain the web to some extent, the use of Eq. 24 to determine the subpanel \( k \)-values should result in a conservative estimate of the subpanel shear strengths.

As an example, the non-dimensionalized ultimate shear forces of the subpanels given by Eqs. 19 and 22 are plotted in Fig. 16 for \( \alpha = 1.5, \eta_1 = 0.33 \) and \( \varepsilon_y = 0.0012 \). Also shown in the figure for comparison purposes is the \( V_u/V_p \) curve for the same panel if the longitudinal stiffener were not used. For this case, \( (V_u/V_p)_1 \) is greater than \( (V_u/V_p)\text{ unstiffened} \) for the entire range of \( \beta \)-ratios while \( (V_u/V_p)_0 \) is less than \( (V_u/V_p)\text{ unstiffened} \) for \( \beta > 230 \). The points of discontinuity on each curve represent the transition from the elastic range to the inelastic range, that is, \( \tau_{cr} = 0.8 \tau_y \). According to Eq. 15, the ultimate shear force for the stiffened panel for a particular value of \( \beta \) is obtained by multiplying the corresponding values of \( (V_u/V_p)_1 \) and \( (V_u/V_p)_0 \) by \( \eta_1 \) and \( (1 - \eta_1) \), respectively, and multiplying the sum of these two products by the plastic shear force \( V_p \).
Figures 17, 18 and 19 show the results of such a calculation for $\eta_1 = 0.2$, $\eta_1 = 0.33$ and $\eta_1 = 0.5$, respectively. In each figure, the ultimate shear strength curves are plotted for three values of aspect ratio; $\alpha = 0.75$, $\alpha = 1.0$ and $\alpha = 1.5$. For $\eta_1 = 0.2$ (Fig. 17), the transition points for $(V_u/V_p)_1 = 1$ occur near $\beta = 380$ and those for $(\tau_{cr})_1 = 0.8 \tau_y$ occur near $\beta = 150$. (The point where $(\tau_{cr})_1 = 0.8 \tau_y$ occurs above $\beta = 400$ for $\eta_1 = 0.2$). Three transition points are shown on the curves for $\eta_1 = 0.33$ in Fig. 18: $(\tau_{cr})_1 = 0.8 \tau_y$ at $\beta \approx 295$, $(V_u/V_p)_1 = 1$ at $\beta \approx 240$ and $(\tau_{cr})_0 = 0.8 \tau_y$ at $\beta \approx 160$. Since the two subpanels are the same size when $\eta_1 = 0.5$, only the transition points for $(\tau_{cr})_1 = (\tau_{cr})_0 = 0.8 \tau_y$ are shown in Fig. 19. The ultimate shear force ratio $V_u/V_p$ for a longitudinally stiffened panel can be determined by selecting the figure for the proper $\eta_1$ value and reading off the ordinate where the slenderness ratio $\beta$ intersects the proper aspect ratio curve.

The optimum stiffener position $\eta_1$ varies with the slenderness ratio. For the lower range of $\beta$-ratios, $\eta_1 = 0.5$ gives the highest value of $V_u/V_p$; for $220 \leq \beta \leq 280$, $\eta_1 = 0.33$ is the optimum position and for $\beta > 280$, $\eta_1 = 0.2$ gives the highest $V_u/V_p$ value. This situation is shown in Fig. 20, where the ultimate shear strength curves for longitudinally stiffened panels having $\alpha = 0.75$, $1.0$ and $1.5$ are shown. Only three stiffener positions ($\eta_1 = 0.2$, $\eta_1 = 0.33$ and $\eta_1 = 0.5$) are considered in the figure and for each value of $\beta$,
the $\frac{V_u}{V_p}$ curve shown is for the stiffener position which gives
the highest value of $\frac{V_u}{V_p}$.

Also shown in Fig. 20 are the ultimate shear strength curves
for unstiffened panels with $\alpha = 0.75, 1.0$ and $1.5$, so the increase
in shear strength due to the longitudinal stiffener can be seen
graphically. A better indication of this, however, can be obtained
from Fig. 21, where the ratio of the shear strength of the stiff-
ened panel to the shear strength of the unstiffened panel $\Delta$ is
plotted against the slenderness ratio. Again, only three values
of aspect ratio and three stiffener positions are considered. The
efficiency ratio $\Delta$ is 1.0 until the shear strength of the unstiff-
ened panel becomes less than $V_p$. This occurs for $90 < \beta < 220$,
and below this range of $\beta$-values, no advantage is gained by using
longitudinal stiffeners. A peak in the efficiency curves is
reached at the highest $\beta$-value for which $(\frac{V_u}{V_p})_{\text{stiffened}} = 1.0$.
At about $\beta = 220$, an abrupt transition occurs as the optimum
stiffener position changes from $\eta_1 = 0.5$ to $\eta_1 = 0.33$, and a
similar transition occurs at about $\beta = 280$ when the optimum $\eta_1$-
value changes from 0.33 to 0.2. Of the three aspect ratios
considered, $\alpha = 1.5$ provides the greatest increase in shear
strength due to a longitudinal stiffener with an increase of over
10% for the entire range $190 < \beta < 400$ and a maximum increase of
47% at $\beta = 155$. Similar efficiency curves could be prepared using
Eqs. 15, 19, 21, 22, 23 and 24, to include more stiffener positions.
and a larger number of aspect ratios, so that for any $\alpha$ - value the optimum $\eta_1$ - value could be determined more accurately and for these $\alpha$ and $\eta_1$ values the increase in shear strength due to a longitudinal stiffener could also be determined.

3. **Longitudinal Stiffener Requirements**

A longitudinal stiffener must fulfill three requirements if the ultimate shear strength of the stiffened panel is to be attained:

1) It must be rigid enough to force a nodal line in the deflected web so that separate tension fields will form in the subpanels.

2) It must have sufficient area to transfer the horizontal components of the tension fields from one side of a panel to the other (see Fig. 15).

3) It should be proportioned according to Eq. 7 to avoid premature local buckling.

The first requirement can be satisfied by providing the minimum stiffener rigidity $\gamma_L^*$ obtained from a web buckling analysis. Unfortunately, formulas for $\gamma_L^*$ are not available for the various stiffener positions considered in this discussion. However, charts have been published in Ref. 18 to determine $\gamma_L^*$ for $\eta_1 = 0.2, 0.25, 0.33, 0.4$, and $0.5$ and for $0.7 \leq \alpha \leq 3.8$. Curves plotted from data obtained from these charts are shown in Fig. 22 for $\eta_1 = 0.2, 0.33$ and $0.5$ and the values of $\gamma_L^*$ for these same $\eta_1$ - values
and $\alpha = 0.75, 1.0$ and $1.5$ are listed in Table 2.

The horizontal components of the two subpanel tension field forces (Fig. 15) are

$$
\begin{align*}
F_{ho} &= \sigma_{t0} t \frac{b_0}{2} \cos \theta_0 = \sigma_{t0} t \frac{b_0}{2} \frac{\alpha_0}{\sqrt{1 + \alpha_0^2}} \\
F_{hl} &= \sigma_{t1} t \frac{b_1}{2} \cos \theta_1 = \sigma_{t1} t \frac{b_1}{2} \frac{\alpha_1}{\sqrt{1 + \alpha_1^2}}
\end{align*}
$$

The following approximate expressions can be used for the tension field stresses $\sigma_{t0}$ and $\sigma_{t1}$:

$$
\begin{align*}
\frac{\sigma_{t0}}{\sigma_y} &= 1 - \left( \frac{T_{cr0}}{T_y} \right) \\
\frac{\sigma_{t1}}{\sigma_y} &= 1 - \left( \frac{T_{cr1}}{T_y} \right)
\end{align*}
$$

Unless the longitudinal stiffener is at midheight ($\eta_1 = 0.5$), the horizontal components of the tension field forces in the two subpanels will be different in magnitude. These forces will be at least partially anchored by the tension fields in the adjacent subpanels. The adjacent panels are assumed to be the same size (same $\alpha$ and $\beta$-ratios) and to have the same stiffener location (same $\eta_1$-ratio). If it is further assumed that the horizontal components are all applied at the corners of the subpanels (see Fig. 15), the longitudinal stiffener will be required to carry the
difference between the two horizontal components, thus the force on the longitudinal stiffener $F_L$ is

$$F_L = F_{ho} - F_{hl}$$  \hspace{1cm} (27)

A longitudinal stiffener is a compression member and should be proportioned according to Eq. 7, as already pointed out. In addition, the longitudinal stiffener force $F_L$ should be modified if the stiffener slenderness ratio $\ell/r$ is large enough to cause premature lateral buckling. Thus, the required longitudinal stiffener area $A_{sL}^*$ is $F_L/(\sigma_{cr} \ell)$, where $(\sigma_{cr} \ell)$ is the lateral buckling stress of the stiffener as determined from Eq. 1. Therefore, Eq. 27 becomes

$$A_{sL}^* = \frac{F_{ho} - F_{hl}}{(\sigma_{cr} \ell)}$$  \hspace{1cm} (28)

It is convenient to specify stiffener area requirements in non-dimensional form by dividing by the web area $A_w = bt$. Defining $\delta_L^*$ as the ratio $A_{sL}^*/A_w$ and using Eqs. 25 and 26,

$$\delta_L^* = \left( \frac{\alpha_o}{2 \ell + \alpha_o^2} \right) \left[ 1 - \left( \frac{\ell}{\alpha_o^2} \right) \right] - \frac{\alpha_1}{2 \ell + \alpha_1^2} \left[ 1 - \left( \frac{\ell}{\alpha_1^2} \right) \right] \frac{\sigma_y}{(\sigma_{cr} \ell)}$$  \hspace{1cm} (29)

As pointed out in the discussion of longitudinal stiffener requirements to develop the bending strength, an effective width of 20 times the web thickness can be assumed to act with a one-sided stiffener in resisting axial force or lateral bending. Therefore the stiffener area and section properties used in establishing the
stiffener proportions can be computed for a "T" section consisting of the one-sided stiffener and 20t of the web.

4. Transverse Stiffener Requirements

The transverse stiffeners must have sufficient area to transfer the vertical component of the tension field force from the top of a subpanel to the bottom (see Fig. 15). A stiffener must therefore have sufficient area to carry the larger of the vertical components of the two subpanel tension field forces. With the longitudinal stiffener located between middepth ($\eta_l = 0.5$) and the compression flange, subpanel "0" will have the larger vertical component. Thus, the transverse stiffener force can be expressed (see Fig. 15) as

$$ F_T = \sigma_{to} t \frac{t_o}{2} \sin \theta_o = \sigma_{to} t \frac{b_o}{2} \frac{1}{\sqrt{1 + \alpha_0}} $$

(30)

The tension field stress $\sigma_{to}$ can be evaluated from Eq. 26.

The required transverse stiffener area $A_{st}^* = F_T/\sigma_y$, and in non-dimensional form, Eq. 20 becomes

$$ \frac{\delta}{T^*} = \frac{A_{st}^*}{A_w} = \frac{1}{2\sqrt{1 + \alpha_0}^2} \left[ 1 - \left(\frac{T_{cr}}{T_y}\right)_0 \right] $$

(31)

Transverse stiffeners are compression members and, like the longitudinal stiffeners, should be proportioned according to Eq. 7 to avoid premature local buckling. The transverse stiffener area requirement may have to be modified for large stiffener $L/r$
ratios. When this modification is necessary, it can be accomplished in the same manner as for the longitudinal stiffeners as discussed in the preceding section.

5. Correlation With Test Results

The shear strength theory developed in this section can be checked experimentally with the results of the shear tests on longitudinally stiffened plate girders described in Ref. 6. The principal specimen parameters, stiffener properties and ultimate loads for the seven tests are summarized in Table 3.

The required transverse stiffener area ratios $\delta_T^*$ listed in Table 3 have been computed from Eq. 31. The requirements were satisfied for all tests except test LS4-T1, where only 91% of $\delta_T^*$ was supplied.

The longitudinal stiffener rigidity requirements (from Table 2) were exceeded for all seven tests; however, the longitudinal stiffener area requirements (from Eq. 29) were fulfilled in only four tests. For test LS3-T1, only 76% of $\delta_L^*$ was supplied, while for tests LS3-T2 and LS4-T1, 95% and 87% of $\delta_L^*$, respectively, were supplied. However, a longitudinal stiffener failure was observed in only one of these cases. In test LS3-T1, just before the ultimate load was reached, a sudden increase in lateral stiffener deflection was observed, resulting in a rapid drop in the applied load. In spite of this failure, the panel was still able to maintain a load
higher than that predicted by the theory. The behavior of the longitudinal stiffeners in the three tests where $\delta_L < \delta_L^*$ is a good indication that the stiffener area requirement given by Eq. 29 is conservative.

In the last section of Table 3 the theoretical ultimate loads predicted by the theory, the experimentally obtained ultimate loads and the correlation ratio $P_u^\text{ex}/P_u^\text{th}$ are listed. For the seven tests this ratio varied from 1.00 to 1.18, with a mean value of 1.10. While it is difficult to establish a trend from seven tests, it could be postulated that the theory is about 10% conservative, with experimental scatter for the seven tests ranging from -10% to +8% from this point. There does not appear to be any tendency for the theory to be more conservative for some values of $\alpha$ and $\eta_1$ than for others.

The conservative nature of the shear strength theory could be attributed to two factors. The first of these is the assumption that the buckling coefficient for the subpanels should be that associated with a plate with simply supported edges. It has been pointed out previously that the flanges will exert some restraint on the web plate. However, the extent of this restraint is difficult to establish so the assumption of simply supported edges was intentionally made to be conservative. The other factor which could contribute to the conservativeness of the theory is the type of tension field geometry which was assumed. The model
which was used, that of a diagonal tension field with width $b/2$, was selected because it appeared to approximate the behavior of transversely stiffened plate girders rather closely. Observations of the tension fields which developed in the longitudinally stiffened girder tests indicated that the model was also a reasonable approximation for the range of $\alpha$ and $\eta_1$ ratios which was tested. For this reason, it can be concluded that the use of buckling coefficients for simply supported panels is the main reason for the conservative nature of the shear strength theory developed in this report.

6. Summary

A method of predicting the shear strength of longitudinally stiffened plate girders has been developed. The method is based on the assumptions that the subpanels formed by a longitudinal stiffener can develop independent tension fields, that the shear strength of a subpanel can be estimated using a diagonal tension field with a width of one-half the subpanel depth and that the ultimate shear strength of a stiffened panel is equal to the sum of the shear strengths of the subpanels. Transverse and longitudinal stiffener requirements have also been established based on the assumed tension field geometry.

Based on the theory, the shear strengths of panels with aspect ratios equal to 0.75, 1.0 and 1.5 and with longitudinal stiffeners located 0.2, 0.33 and 0.5 times the web depth from the
compression flange have been computed and presented graphically. The optimum stiffener position was found to vary with the web slenderness ratio and the increase in shear strength due to a longitudinal stiffener was found to vary with both aspect ratio and slenderness ratio. According to the theory, an increase in shear strength of over 40% can be obtained for some values of aspect ratio and slenderness ratio and a minimum increase of 10% can be attained for slenderness ratios from 100 to 400 if the proper aspect ratio and stiffener position are used.

The results of seven shear tests on longitudinally stiffened plate girders have been summarized and compared with the shear strength theory. The test results indicate that the stiffener proportioning requirements are conservative and that the ultimate shear strength predicted by the theory provides a conservative estimate of actual girder shear strength.
NOMENCLATURE

1. Lower Case Letters
   a: panel width or distance between transverse stiffeners
   b: panel depth or distance between flanges; with subscript "l", distance from compression flange to longitudinal stiffener
   c: half of flange width; with subscript "s", width of a longitudinal stiffener
   d: flange thickness; with subscript "s", thickness of a longitudinal stiffener
   k: buckling coefficient
   l: effective length of a column
   r: radius of gyration
   t: web thickness

2. Capital Letters
   A: area
   E: modulus of elasticity, 30,000 ksi.
   F: force
   I: moment of inertia
   M: bending moment
   P: test load on a girder
   S: section modulus
   V: shear force
3. **Greek Letters**

- $\alpha$: panel aspect ratio, $a/b$
- $\beta$: web slenderness ratio, $b/t$
- $\gamma$: stiffener rigidity ratio, $12(1 - \nu^2) I_s/bt^3$
- $\delta$: stiffener area ratio, $A_s/A_w$
- $\epsilon$: strain
- $\eta$: longitudinal stiffener position, $l = b_1/b$
- $\lambda$: column buckling parameter
- $\nu$: Poisson's ratio, 0.3
- $\rho$: ratio of web area to flange area, $A_w/A_f$
- $\sigma$: normal stress
- $\tau$: shear stress

4. **Subscripts**

- $\text{cr}$: critical
- $\text{cri}$: ideal critical
- $f$: flange
- $\ell$: lateral buckling
- $p$: plastic
- $s$: stiffener
- $t$: tension, torsional buckling
- $u$: ultimate
- $w$: web
- $y$: yield
- $0$: subpanel "0"
1: subpanel "1"

L: centerline

L: longitudinal stiffener

T: transverse stiffener

σ: as carried in tension

τ: as carried in shear

5. **Superscripts**

ex: experimental

th: theoretical

*: required
TABLES AND FIGURES
Table 2  Longitudinal Stiffener Rigidity Requirements (Ref. 18)

<table>
<thead>
<tr>
<th>α</th>
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Table 3  Correlation of Shear Strength Theory With Test Results (Ref. 6)

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Table 1 Correlation of Bending Strength Theory With Test Results
Specimen LBI
\( a = 1.0, \beta = 444 \)
\( P_{cr} = 15.1k \)
\( P_{u} = 156.5k \)

Specimen LB3
\( a = 1.0, \beta = 447 \)
\( P_{cr} = 81.4k \)
\( P_{u} = 150.0k \)

(a) Lateral Web Deflections
(b) Stress Distributions

Fig. 1 Test Measurements on Specimen LBI

Fig. 2 Test Measurements on Specimen LB3
Fig. 3 Compression Flange Column Buckling Modes

CASE I
\[ A = A_f = 2cd \]
\[ I = \frac{2}{3}c^3d \]

CASE II
\[ A_f = 2cd \]
\[ A_s = 2c_s d_s \]
\[ A = 2cd + 2c_s d_s \]
\[ I = \frac{2}{3}(c^3d + c_s^3d_s) \]

CASE III
\[ A_f = 2cd \]
\[ A_s = c_s d_s \]
\[ A = 2cd + c_s d_s \]
\[ I = \frac{1}{3}(2c^3d + c_s^3d_s) \]

Fig. 4 Section Properties of Various Compression Flange Columns
Assumed Hinge

(a) Assumed Model

Compression Flange

(b) Loading

Fig. 5 Torsional Buckling Model

Fig. 6 Vertical Buckling of Compression Flange (Test G4-T2, Ref. 4)
Fig. 7 Ultimate Bending Moment as Influenced by Web Slenderness Ratio

Fig. 8 Flange Stress Reduction Curves
Fig. 9 Lateral Buckling of Longitudinal Stiffener Column

Fig. 10 Derivation of Transverse Stiffener Requirements
Fig. 11 Typical Shear Panel

(a) Beam Theory Shear Stress

(b) Tension Field Stress

Fig. 12 Stress States in Plate Girder Web
Fig. 13 Load-Vs-Centerline Deflection Curve for Girder LS2

Fig. 14 Girder LS2 After Testing
Fig. 15 Tension Field Model for Longitudinally Stiffened Panel

$\alpha = \alpha_1 = \alpha_0$

Fig. 16 Subpanel Ultimate Shear Forces, $\alpha = 1.5$ and $\eta_1 = 0.33$
Fig. 17 Shear Strength Curves for $\eta_1 = 0.2$

Fig. 18 Shear Strength Curves for $\eta_1 = 0.33$
Fig. 19 Shear Strength Curves for $\eta_1 = 0.5$

Fig. 20 Shear Strength Curves Using Optimum Stiffener Position
Fig. 21 Increase in Shear Strength Due to the Use of a Longitudinal Stiffener

Fig. 22 Longitudinal Stiffener Rigidity Requirements
REFERENCES

1. Basler, K. and Thürlimann, B.

2. Basler, K.

3. Basler, K.

4. Basler, K., Yen, B. T., Mueller, J. A. and Thürlimann, B.
   WEB BUCKLING TESTS ON WELDED PLATE GIRDERs,
   Bulletin No. 63, Welding Research Council, Sept., 1963

5. American Institute of Steel Construction, Inc.
   SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS, AISC, New York, 1963

6. D'Apice, M. A., Fielding, D. J. and Cooper, P. B.
   STATIC TESTS ON LONGITUDINALLY STIFFENED PLATE GIRDERs,
   Fritz Engineering Laboratory Report No. 304.8, May, 1966

7. Lay, M.
   SOME STUDIES OF FLANGE LOCAL BUCKLING IN WIDE-FLANGE SHAPES,
   Fritz Engineering Laboratory Report No. 297.10, July, 1964

8. Haaijer, G. and Thürlimann, B.
   ON INELASTIC BUCKLING IN STEEL, Proc. ASCE, Vol. 84 (EM2), April, 1958, Paper No. 1581

   THE BUCKLING OF A PLATE GIRDER WEB UNDER PURE BENDING WHEN REINFORCED BY A SINGLE LONGITUDINAL STIFFENER,

10. Ostapenko, A.
    LOCAL BUCKLING, Chapter 13 of "Structural Steel Design", The Ronald Press, 1964
11. Massonnett, C.
ESSAIS DE VOILEMENT SUR POUTRES À ÂME RAIDIE,

12. Deutscher Normenausschuss
DIN 4114, Part 1 and 2, Beuth-Vertrieb G.m.b.H.,
Berlin W15 and Köln, 1952

13. American Association of State Highway Officials
STANDARD SPECIFICATION FOR HIGHWAY BRIDGES, 8th

14. Massonnett, C.
STABILITY CONSIDERATIONS IN THE DESIGN OF STEEL
PLATE GIRDERS, Proc. ASCE, Vol. 86 (ST1), Jan.,
1960, p. 71

15. Longbottom, E. and Heyman, J.
EXPERIMENTAL VERIFICATION OF THE STRENGTHS OF PLATE
GIRDERS DESIGNED IN ACCORDANCE WITH THE REVISED
BRITISH STANDARD 153: TESTS ON FULL SIZE AND MODEL
Part III, 1956, p. 462

WELDED CONSTRUCTIONAL ALLOY STEEL PLATE GIRDERS,
Proc. ASCE, Vol. 90 (ST1), Feb., 1964, p. 1

17. Cooper, P. B.
BENDING AND SHEAR STRENGTH OF LONGITUDINALLY STIFF-
ENED PLATE GIRDERS, Fritz Engineering Laboratory
Report No. 304.6, Sept., 1965

18. Klöppel, K. and Scheer, J. S.
BEULWERTE AUSGESTEIFTER RECHTECKPLATTEN, Verlag von
Wilhelm Ernst & Sohn, Berlin, 1960
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