Welded Columns and Flame-Cut Plates

A STUDY OF WELDED COLUMNS MANUFACTURED FROM FLAME-CUT PLATES

by

Richard Mc Falls
Lambert Tall

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Richard K. McFalls
and
Lambert Tall

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

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This report describes the work carried out during a research study into the strength of welded columns manufactured from flame-cut plates. Two column sizes were tested: 12H79 and 14H202. Both were manufactured by identical processes. Tests included residual stress measurement, tensile coupons, stub column tests, and pinned-end column tests at slenderness ratios of 30, 60, and 90. The pinned-end tests were conducted with rotation permitted about the weak axis of the shape.

The theoretical analysis is in two parts. The first portion is based on the tangent modulus theory. Tangent modulus load curves were developed for both "strong" and "weak" axis bending. The tangent modulus load curves were developed by a computer program which utilized the actual measured residual stress pattern found in the section. The second portion is an ultimate strength analysis of the pinned-end column assuming a sinusoidal deflected shape of the column. This analysis considered the actual residual stress pattern together with any value of initial eccentricity for bending permitted about the weak axis.
1. INTRODUCTION

1.1 Previous Work

Work by earlier investigators\(^{(1,2,3,4,5)}\) in the area of welded column strength has been concerned primarily with shapes manufactured from universal-mill (UM) plates. These shapes for the most part have been smaller than those most commonly used in construction in North America. H-shaped welded columns manufactured from universal-mill plates are characterized by fairly high compressive residual stresses at the flange tips. Results of tests on these shapes show that except for very heavy shapes at low slenderness ratios, the strength of this type of welded column is significantly less than corresponding rolled shapes\(^{(6)}\).

A pilot study at Lehigh University\(^{(4)}\) into the strength of columns built-up from flame-cut plates indicated that the more favorable residual stress distribution in these shapes could result in higher column strengths. This pilot study involved the testing of two such shapes: a 6" x 7" and a 9" x 10" welded H-shape.

1.2 Purpose and Scope

A large percentage of welded columns manufactured today are made up of flame-cut plates. The purpose of this
research program has been to investigate the strength of flame-cut columns of sizes which are both commercially available and representative of sizes ordinarily used in construction. Two shapes are studied here: 12H79 and 14H202. These two shapes were manufactured with automatic flame-cutting and submerged-arc welding equipment.

Various tests were conducted on these shapes: residual stress measurements, tensile coupon tests, stub column tests, and pinned-end column tests where the column was free to rotate about the so called "weak axis" of the cross-section.

The theoretical analysis is in two parts. The first part is based on the tangent modulus theory. Tangent modulus curves are developed for both strong and weak axis bending utilizing the actual measured residual stress pattern found in the section. The second part is an ultimate strength analysis of the pinned-end column assuming a sinusoidal deflected shape of the column.
2. **MANUFACTURE**

It has long been established that column strength is related to residual stresses\(^{(7,8)}\). The magnitude and distribution of residual stresses are determined largely by the method used to manufacture the column. Therefore, the method of manufacture has a direct bearing on the strength of a column. In order to include all of the major factors contributing to the column strength of these shapes, a brief outline of the manufacturing process is presented before going on to a discussion of the research program conducted at Lehigh University.

Wide plates are first "stripped" (flame-cut by automatic equipment; see Fig. 1) to appropriate widths for use as webs or flanges. Flange plates thinner than 3/4" are then placed in a "crimper" which pre-bows them so that a square section is obtained after welding. This process is not necessary for thick flange plates. Two flanges and a web are then placed on the "fit-up" table. A hydraulically operated "fit-up" gate (Fig. 2) is used at the head end of the flame-cut plates to obtain the correct section geometry. The plates are then tack-welded at the head end only and moved to the first welding unit (Fig. 3). Here the plates are preheated with torches and the two fillets on one side of the column are deposited by continuous submerged arc welding in one pass. On the exit
table from the first welding unit the shape is lifted by a moveable fork lift, rotated 180°, and placed on the entry table of the second welding unit (Fig. 4). Here the remaining two fillets are deposited. The section is then flame cut at each end to the correct length (Fig. 5). The final step in manufacturing takes place at the inspection and conditioning skids where any minor repair welding, if necessary, is accomplished (Fig. 6).

The manufacturing procedure described above is fully automatic. There are other procedures which are not fully automatic; in every case, however, the basic principles are the same.
3. TESTING PROGRAM

Table 1 presents a summary of the type and number of tests conducted on both shapes.

Tensile coupons were used as a routine check on the static yield stress level of the material making up the column shapes. ASTM specifications\(^{(9)}\) and recommendations were followed on all tests. All coupons were flat with the standard 8 in. gage length except for five round "\(0.505\)" coupons tested from the \(14H202\). The round coupons had a 2 in. gage length.

The experimental or measured values of residual strains (and consequently residual stresses) were determined by the method of "sectioning"\(^{(7)}\). Since only the surface residual stresses can be measured by the sectioning method, the average of the two surfaces at each section was used as the residual stress in the section at that location. This method required the assumption that the residual stresses are uniformly distributed through the thickness; this appears to be valid for plate thicknesses less than approximately \(1\ 1/2\)". In very thick plates the variation of residual stress through the thickness is sufficient to affect the value of the tangent modulus load.\(^{(4)}\)

Two stub column tests were conducted on each size specimen. These tests were carried out in accordance with the stub column testing procedure described in Ref. 10.
A total of six pinned-end column tests were conducted in the program; three of each shape at slenderness ratios of 30, 60, and 90. All tests were conducted in a 5,000,000 lb. hydraulic-type universal testing machine.

The columns were tested with pinned-end supports in the "weak" axis direction and fixed-end supports in the "strong" axis direction by using end fixtures developed at Fritz Laboratory\(^{(11)}\). A detailed description of the instrumentation and the procedure for testing pinned-end columns can be found in Refs. 3 and 12.
4. **TEST RESULTS**

4.1 **Tensile Coupons**

From the coupon test results the average static yield strength of the flange and web of each cross-section was determined. Using a weighted average method, the static yield strength for the cross-section was calculated. The 12H79 shape was found to have a static yield strength of 36.7 ksi and that for the 14H202 was 34.7 ksi. This lower value for the 14H202 is probably due to the thicker plates making up the shape.

One interesting aspect of the results of these tests was that the coupons taken from the flanges of both shapes did not exhibit a "flat" yield plateau, whereas the web coupons had a "flat" yield plateau. Figure 7 shows typical flange and web coupon test results superimposed. Notice that the flange coupon has a slight positive modulus in the yield region while the web coupon has the flat yield region usually observed in A36 tensile coupons.

4.2 **Residual Stress Measurements**

The residual stress pattern observed in the 12H79 flame-cut shape is shown in Fig. 8 and that for the 14H202 flame-cut shape is shown in Fig. 9. Notice that for both shapes the residual stress patterns have the same general
shape; that is, the areas of tensile or compressive residual stress appear in the same locations and have the same relative size. The main difference between these two sizes is in the magnitudes of the residual stresses. The tensile residual stress at the flange tips average about 28 to 30 ksi for the lighter shape and about 18-21 ksi for the heavier shape. Maximum compressive residual stress in the flange is about 16 ksi in the lighter shape and about 12 ksi in the heavier shape. At the juncture of the flange and web, the effect of the thinner plates is clearly evident in the higher tensile residual stress in the flange of the 12H79. It can generally be said that for heavier shapes the weldment makes up a smaller percentage of the total area. The heat input per unit volume is reduced, and therefore, the magnitude of the residual stresses is lower.

For the 12H79 shape, measurements were made of the residual stresses present in the flange and web plates prior to welding. These are shown in Fig. 10. For both plates residual stresses due to flame cutting were measured somewhat in excess of the usual yield strengths of A36 steel. This increase of the yield strength is confined to the very edge of the material and may be caused by slight metallurgical changes in the grain structure caused by the flame cutting. Comparing Figs. 8 and 10, the effect of the fillet welds on the final residual
stresses can be seen. The web is nearly unaffected by welding since the heating pattern of welding is very similar to that of the flame cutting and no additional material flows. At the center of the flange, however, the material melts near the weld creating high tensile residual stresses there upon cooling. This in turn causes a reduction in the tensile residual stress at the flange tips and an increase in the compressive residual stresses in order to maintain equilibrium.

4.3 Stub Columns

A total of four stub column tests were made; two of the 12H79 shape and two of the 14H202. Figure 11 shows the results of one test on the 12H79 and Fig. 12, a test of a 14H202. The reason for the difference in the shape of these two curves near the upper portion is the fact that in the 12H79 local buckling occurred at a strain of about $19 \times 10^{-3}$, whereas the low width to thickness ratios of the flange of the 14H202 postponed local buckling to a much higher strain than is shown on the graph. As is common with welded stub columns, no yield plateau was observed in either shape. This could have been predicted from the tensile coupon tests just discussed.

The usual procedure in evaluating stub column test results is to use a yield stress level criteria defined by the stress at 0.5% strain. Using this criteria for these
stub columns, the yield stress is found to be 39.6 ksi for the 12H79 and 35.5 ksi for the 14H202, both of which are in excess of the yield strength determined by tensile coupons. This would be expected because the stub columns show the effect of the higher yield strength of the weld areas.

4.4 Pinned-End Columns

Six pinned-end column tests were conducted. The results of the three tests of 12H79 columns are shown in Fig. 13. The 14H202 results are shown in Fig. 14. Table II summarizes these results giving the values of centerline eccentricity and ultimate load for each column.

Generally, the curves are fairly flat in the range of deflections beyond the ultimate load. Load-deflection curves for rolled and welded columns (universal-mill plates) show a more rapid loss of load beyond the ultimate load. This indicates the possibility that flame-cut columns have better rotation capacities than their rolled and universal-mill welded counterparts. This aspect of flame-cut columns is beyond the scope of this report, but should be considered as the topic of a future investigation.

The curve for column 2B had a more pronounced loss of load beyond the ultimate load than the other five. Table II reveals that this column had the highest initial out-of-straightness, a fact which may have contributed to this more rapid loss of load.
5. TANGENT MODULUS ANALYSIS

The purpose of this analysis is three fold:

1. To develop tangent modulus curves for the two shapes tested taking into account the actual measured residual stress patterns.

2. To determine what value, if any, the use of the tangent modulus theory (The Engesser-Shanley Theory) has in predicting the ultimate strength of columns manufactured from flame-cut plates when the residual stresses are taken into consideration.

3. To examine the influence of various residual stress patterns on the tangent modulus curve of a given column shape and thus give an indication of the relative ultimate strengths resulting from these residual stress patterns.

5.1 The Tangent Modulus Equation

The stress at which a perfectly straight column will buckle according to the tangent modulus theory is given by:

\[ \sigma_{cr} = \frac{\pi^2 E (I_e/I)}{(KL/r)^2} \]  

(1)
where

\[ \sigma_{cr} = \text{Buckling stress} \]

\[ E = \text{Young's Modulus} \]

\[ I_e = \text{The moment of inertia of the elastic portion of the cross-section} \]

\[ I = \text{The moment of inertia of the total cross-section} \]

\[ KL/r = \text{Effective slenderness ratio} \]

This equation has been derived earlier in several papers.\(^{(7,13)}\)

In order to solve this equation it is necessary to obtain a relationship between \( \sigma_{cr} \) and \( I_e \). This can be achieved by assuming a value for \( \sigma_{cr} \), determining what portions of the cross-section have yielded under this applied stress, and then evaluating \( I_e \) for the remaining elastic portions. This procedure can be carried out quite easily and accurately by computer calculation.

A plot of the actual residual stress diagram is used in determining \( I_e \) for any applied stress. The cross-section is broken into small, equal segments. These segments should be about 1/4" for most shapes in order to get a smooth curve. The size of the segments in the web need not equal the size of the segments in the flange. From the diagram, the residual stress at the center of each segment is determined. This information together
with the dimensions of the column cross-section comprise the input data for the computer solution.

The computer solution uses the assumption that the residual stress pattern is symmetrical about both axes of the cross-section. This facilitates computation since only one-half of one flange and one-half of the web need to be read into the program to describe the entire shape. The residual stress patterns for the shapes shown in Fig. 8 and 9 verify this assumption.

A copy of this program and the variable notation can be found in reference 14. The program is written in WIZ computer language for use on a General Electric 225 Computer.

5.2 Tangent Modulus Load Curves

Figure 15 shows the tangent modulus load curve for weak axis bending in the 12H79 shape. Shown for reference with the curve are the results of the three column tests, a sketch of the residual stress pattern, and the weak axis curve for rolled shapes. The short dashed lines in Figs.15 and 17 show the ultimate strength predicted for each column tested. Although the ultimate strength analysis is not discussed in this chapter, the comparison is shown here for convenience.

The general shape of these tangent modulus curves for the 12H79 shape can be explained when the residual
stress pattern and the tangent modulus equation (Eq. 1) are considered. Note in each flange the two relatively large areas of high, constant compressive residual stress. As the column is loaded, these are the first areas to yield, and this yielding results in a fairly rapid reduction of the elastic moment on inertia ($I_e$) about the "y-y" axis.

This accounts for the flat portion of the tangent modulus curve in the slenderness ratio range of 120 down to about 80. As the column is loaded beyond this strain level, the curve again briefly approaches the Euler curve shape. This is caused by the steep slope of the tension portion of the residual stress diagram which slows the rate of yielding with respect to the rate of straining. In terms of the tangent modulus equation, $I_e$ decreases only slightly with a resulting increase in column strength as the slenderness ratio decreases. It is this action that leads to the conclusion that the tensile residual stresses at the flange-tips due to the flame-cutting cause increased column strength at lower slenderness ratios. This fact is quite important since most columns are designed at slenderness ratios less than 50.

Figure 16 shows the tangent modulus load curve for the 12H79 shape with rotation permitted about the "strong" or "x-x" axis of the section. Comparison with the strong axis curve for rolled shapes shows that at high slenderness ratios the flame-cut welded column has a much lower strength
than its rolled counterpart. At the more important lower slenderness ratios, however, the strength of the flame-cut column is more nearly equal to that of the rolled column. A similarity exists in the shape of the tangent modulus curves for both strong and weak axis bending. The residual stress pattern for this shape is such that the column behaves very much like a square box-shaped column.

Figure 17 shows the tangent modulus load curve for the 14H202 shape allowing rotation about the "weak" axis. Again the weak axis curve for rolled shapes, the residual stress pattern, and the results of the three column tests on this shape are shown for comparison. The general shape of this curve is similar to that of the lighter 12H79 column, and the reasons for this are the same. The strength at high slenderness ratios is higher in the heavier shape because of the lower compressive residual stresses. Note that the flat portion of the curve in the high slenderness ratio region extends down to a lower value of L/r. This indicates that a slightly higher percentage of the total area is included in the zone of nearly constant compressive residual stress in the flange. Also in the range of L/r less than 50 the tangent modulus curve is slightly higher than that for the 12H79 shape despite the fact that the magnitude of the tensile residual stress at the flange tips is somewhat lower. This suggests that for heavier shapes the lower magnitude of residual stress at the flange tips
is still sufficient to keep the flange tips elastic at high loads while the thicker flanges increase the load capacity.

The tangent modulus load curve for the 14H202 with rotation permitted about the strong axis is shown in Fig. 18. Here, as in the 12H79 shape, the column strength is nearly identical for both axes of bending; analogous to a square box-shaped column. Comparing this curve with Fig. 16, it can be seen that the heavier shape compares more favorably with the strong axis approximation for rolled shapes but is still considerably below it. However, in making this comparison with fairly heavy shapes, it should be emphasized that these approximate curves are based on tests of light, rolled, WF shapes. No tests of comparable rolled WF sections are available to lend real validity to the comparison.

Based on this tangent modulus analysis, it appears that relative to column strength the designations "strong" and "weak" axes have little meaning for flame-cut welded columns since there appears to be very little difference in strength between the "x-x" and "y-y" axes.

Comparing the curves of Figs. 15 and 17 with the three test results shown in these figures, it can generally be said that the tangent modulus load is a fairly good prediction of the ultimate strength of flame-cut welded columns.
having allowable values of initial eccentricity. The tangent modulus theory is not an ultimate strength theory but merely predicts the load at which a perfectly straight column will buckle. The reason that this theory agrees well with the ultimate strength of these columns is simply that some initial eccentricity always is present in manufactured columns. It will be shown in Part 7 of this paper that these initial eccentricities are sufficient to reduce the ultimate strength by a small amount, into the region of the tangent modulus load.
6. **INFLUENCE OF RESIDUAL STRESSES ON TANGENT MODULUS LOAD**

The computer program discussed earlier was used to plot tangent modulus load curves resulting from various residual stress distributions. The purpose of this is to indicate how the tangent modulus load curve is influenced by changes in the residual stress pattern. Three residual stress patterns for both size shapes were selected in addition to the patterns measured in the flame-cut welded columns tested. It is felt that the patterns selected represent the full range of residual stress distributions found in H-shaped cross-sections.

6.1 **Weak Axis - 12H79**

Figure 19 shows the four tangent modulus load curves corresponding to the four residual stress patterns considered with bending permitted about the "weak" axis of the shape. The four patterns are shown in the figure, with pattern 1 representing the actual measured pattern found in the 12H79 test specimens. Pattern 2 indicates this shape with higher tensile residual stresses at the flange tips than found in the actual pattern (1). This type of pattern might result from welding a bead along the flange tips. From the corresponding tangent modulus load curve the effect of this change in residual stress is seen to be an increase in column strength for low slenderness ratios.
Pattern 3 shows a small portion of the flange tips to have compressive residual stress and the remaining interior of the flange to have tensile residual stress. Such a pattern generally is found in rolled steel columns. The tangent modulus load curve for pattern 3 is lower than those for 1 & 2 indicating a slightly lower column strength at all slenderness ratios for "weak axis" bending. Pattern 4 has slightly more than one-half of the flange under the influence of compressive residual stress. This type of pattern is generally found in welded columns manufactured from universal mill plate and will considerably reduce column strength as shown by the lowest tangent modulus load curve.

6.2 Strong Axis - 12H79

Figure 20 shows the four tangent modulus load curves corresponding to the same four residual stress patterns as used in Fig. 19 but with rotation permitted only about the "strong" axis of the shape. The relative positions of the four curves have changed from Fig. 19. The curve corresponding to pattern 3 now has the highest tangent modulus load, while the other three curves fall within a very narrow band below it. The greater strength of pattern 3 is a result of the large areas of tensile residual stress in each flange and the position of these areas away from the neutral axis. Another important feature is that for
low slenderness ratios the strong-axis bending curves have similar strengths, while those for weak-axis bending are widely separated. Therefore, the effect of residual stresses on column strength is greater when bending occurs about the weak axis.

6.3 Weak Axis - 14H202

The four tangent modulus load curves corresponding to the four residual stress distributions considered for the case where bending is permitted about the "weak" axis of the shape are shown in Fig. 21. The four patterns are shown in the figure with pattern 1 representing the actual measured residual stress pattern found in the 14H202 test specimens. Patterns 2, 3, and 4 match patterns 2, 3, and 4 used for the 12H79 shape but vary slightly because of the thicker plates of the 14H202 shape. The four curves have the same relative positions as those in Fig. 19. However, there are two significant differences between the two figures. For the 14H202 shape the increase in strength of pattern 1 compared to pattern 3 is greater. Secondly, the curve for pattern 2 shows a greater increase in strength relative to pattern 1 over a wider range of L/r. From this, it appears that for weak axis bending the effect of residual stresses on the tangent modulus load does not diminish for heavier columns. Furthermore, for some types of residual stress patterns, the effect of residual stresses on column strength may be increased.
6.4 **Strong Axis - 14H202**

Figure 22 shows the tangent modulus curves corresponding to the same four residual stress patterns shown in Fig. 21. Here rotation is permitted about the "strong" axis of the shape. Notice that the relative position of the curves is identical with the 12H79 curves shown in Fig. 20. The influence of the residual stress patterns on the tangent modulus load curves again appears less pronounced for "strong" axis bending than for "weak" axis bending as was observed in the 12H79 shape. Here too pattern 3 with the largest percentage of flange area subjected to tensile residual stress had the highest strength of the four patterns considered.

6.5 **Other Measured Patterns**

Residual stress measurements were made on rolled and universal-mill welded H-shapes in the course of earlier column investigations. For each of these two types of columns two shapes were chosen which most nearly corresponded to the two flame-cut welded shapes included in this investigation. Tangent modulus curves were developed for these residual stress patterns using the computer program discussed earlier.

The tangent modulus load curves for these two types of columns are compared with those for the flame-cut welded columns in Figs. 23 and 24. In these two graphs the abscissa
is non-dimensionalized in order that columns of slightly varying yield strengths may be plotted on the same scale. This was necessary because the flame-cut welded and the rolled shapes were of A36 steel, whereas the universal-mill welded shapes were of A7 steel.

Figure 23 shows the curves which were developed for rotation permitted about the 'weak' axis of the shape. The curves for rotation permitted about the 'strong' axis are shown in Fig. 24.

From these tangent modulus load curves it appears that the flame-cut welded columns have the greatest weak-axis strength, while the rolled columns have the greatest strong-axis strength. The observation made above concerning the theoretical residual stress patterns still holds true here. That is, the effect of residual stress on column strength is more pronounced for weak axis bending. Also for strong axis bending at low slenderness ratios, there is almost no difference in column strength among the different types of columns (and therefore different residual stresses).

In both of these figures the CRC curve is shown for comparison. It is clear that the CRC curve does not represent a good average behavior of these columns. In fact, some of the tangent modulus load curves are considerably below the CRC curve. Because the CRC curve forms
the basis for the AISC design formula used for all steel columns, the factor of safety for some heavier columns may be below a desirable minimum.

It was noted earlier in this paper that pinned-end column tests have not been conducted on heavy rolled and universal-mill welded columns. It now appears that these would be valuable in determining the relationship of tangent modulus load curves to the actual ultimate strength of the column. It has been shown in this paper that the tangent modulus load curves for flame-cut welded columns approximate very closely the ultimate strength of the column. This may or may not be true for other types of heavy columns, but because of the poor comparison of these tangent modulus curves to the CRC curve, a study into this problem seems justified. Only then can the present design methods be evaluated in relation to actual column behavior for a wide range of column sizes.
7. THEORETICAL ULTIMATE STRENGTH ANALYSIS

A computer program was developed to calculate the load-deflection curve for any H-shaped column when bending about the "weak" axis is permitted. The program computes the ultimate strength of the entire cross-section (that is, flanges and web) considering any residual stress distribution. For this analysis the actual measured residual stress patterns were used to calculate load-deflection curves for the 12H79 and 14H202 columns tested. A computer program was recently also developed at the University of Michigan for predicting the ultimate strength of metal columns \(15\). Their program considers deflection and initial eccentricities at several points along the column length. The computer program of this report considers deflections at the column mid-height only.

7.1 Development

The following assumptions were made in the development of the ultimate strength analysis:

1. Each fiber in the cross-section has an idealized elastic-plastic stress-strain curve.

2. The initial and final deflected shape is described by a half-sine wave.

3. A fiber having yielded in compression may be reloaded in tension due to bending of the column under load.
4. The residual stresses are uniform through the thickness of the plates in the shape.

5. Sections originally plane remain plane through the range of deflections considered.

From assumption 2 and referring to Fig. 25, the deflection of the column \( u \) is a function of the distance from the column end \( z \) given by

\[
u(z) = V \sin \frac{\pi z}{L}
\]

where \( V \) is the deflection at the mid-height due to loading. The slope at any point along the column is

\[
\frac{du}{dz} = \frac{\pi}{L} V \cos \frac{\pi}{L} z
\]

and the curvature at any point is

\[
\frac{d^2u}{dz^2} = -\frac{\pi^2}{L^2} V \sin \frac{\pi}{L} z = -\phi
\]

then

\[
\phi = \frac{\pi^2}{L^2} V \sin \frac{\pi}{L} z
\]

Considering the curvature at mid-height only \( (z=L/2) \)

\[
\phi = \frac{\pi^2}{L^2} V \tag{2}
\]

This simplifies the analysis. By selecting any value of mid-height deflection, the curvature of the column at mid-height is found directly. The residual stress pattern, the axial component of strain and the curvature
completely define the total strain in the cross-section. It can be seen then that by arbitrarily choosing a value of \( V \), an iteration procedure can be used to find the axial strain which produces equilibrium in the cross-section. If \( P \) is the axial load, \( e_1 \) the initial mid-height deflection, and \( v \) the mid-height deflection due to load; then for equilibrium

\[
P(e_1 + v) = M
\]

(3)

where \( M \) is the internal moment due to bending of the column. The iteration is carried out until this equation is satisfied. It can be shown by virtual work that the lowest value of \( P \) that satisfies Eq. 3 will be the correct value.

7.2 Load - Deflection Curves

A computer program was used to carry out the iteration procedure described above. Initial eccentricities used in the analysis range from zero up to approximately the maximum allowed under the AISC code(16). Because the lowest value of \( P \) was required, the iteration was begun at a load well below the expected ultimate load so that the first point of equilibrium reached would be the correct value.

The set of curves developed in this manner are shown in Figs. 26 and 31. The six sets of curves were chosen to match the six pinned-end column test specimens used in the
testing program. Each column size is shown with a range of allowable initial eccentricities. Theoretically, an initially straight column will begin to deflect at the tangent modulus load. This load is designated as $P_t$ and is shown in each figure as the starting point for the curve where $e_i = 0.00"$. The value of $P_t$ was calculated in part 5 of this paper.

Figures 26, 27, and 28 show the load-deflection curves for the three 12H79 columns tested while Figs. 29, 30, and 31 represent those for the three 14H202 columns. Each figure contains a tabular list of the initial eccentricity ($e_i$) with the corresponding value of the ratio of initial eccentricity to flange width ($e_i/b$).

The curves tend to approach a common tangent at deflections beyond the ultimate load indicating that the value of initial eccentricity has little effect in this region. The ultimate strength decreases with increasing $e_i$ and the effect of $e_i$ becomes more pronounced as the slenderness ratio of the column increases.

For each cross-sectional shape the ratio of the ultimate load at zero initial eccentricity to the tangent modulus load increases with increasing $L/r$. This is particularly interesting because Fujita\(^1\) found the opposite to be true in his analysis of welded H-shaped columns manufactured from universal-mill plates. He assumed an idealized
linear residual stress pattern of \(-1/2\) at the flange tips and \(+1/2\) at the center of the flange. He used a similar procedure to determine the ultimate load of these columns with bending permitted about the weak axis. The difference between Fujita's conclusions and those of this paper can be explained in terms of the very different tangent modulus curves that exist for these two different types of columns. The curve corresponding to pattern 4 in Fig. 19 would be similar to that resulting from a column of the type investigated by Fujita. This tangent modulus curve has relatively low values at low slenderness ratios compared to the curve for pattern 1 (the flame-cut welded column). Because there is not a large difference between the theoretical ultimate load curves for these two types of welded columns, Fujita's conclusion becomes obvious. This also explains why he found that the tangent modulus theory was too conservative for predicting the ultimate strength of welded columns manufactured from universal-mill plate. It was shown in part 5 of this paper that the tangent modulus theory is indeed a good approximation of the ultimate strength of flame-cut welded columns.

7.3 Comparison With Test Results

From each of the six sets of theoretical load-deflection curves just discussed, the curve whose initial eccentricity most closely matched that measured in the column
test specimens was compared with the test result. These comparisons are shown on Figs. 32 to 37. In each curve the result of the pinned-end column test is shown as a series of test points connected by solid straight lines. The theoretical load-deflection curve is shown by the long dashed curve. The short dashed portion near the end of these curves represents extrapolation of the theoretical curve. In all cases the two values of ultimate load ratio \(\frac{P_u}{P_y}\) are given in the figure.

In most cases the correlation of the values for \(\frac{P_u}{P_y}\) is good. The average difference between the theoretical value and the test result was about 5%. The largest differences occurred for the 12H79 shape at \(L/r=60\) and the 14H202 at \(L/r=90\). In both cases the predicted value was more than 10% above the test result. It is interesting to note that in part 5 these same two test results were also well below the predicted tangent modulus load.

Except for the shortest columns, the ultimate strength theory predicts the shape of the load deflection curve accurately. It is felt that the results of the short columns do not agree well because of the assumption of the idealized stress-strain curve. In very short columns considerable yielding occurs without a flat yield plateau as shown in the stub-column test results of Figs. 11 and 12.
Generally, the accuracy of this theory is good considering the simplifying assumptions made in its derivation. In order to obtain better than 5% agreement with the test results, a much more refined analysis would have to be made. Present methods of testing pinned-end columns are such that the scatter of results is usually about 5%.

7.4 Stress Distribution Under Load

From the results of the computer solution for ultimate strengths, it was possible to determine the theoretical distribution of stresses at mid-height in the cross-section at the various stages of loading. To demonstrate this, the L2H79 shape with $L/r=30$ and $e_1=0.015"$ was selected as an example. Bending was permitted about the "weak" axis. The stress patterns in the flange of this column are shown in Figs. 38 and 39.

Figure 38 shows two patterns at very low values of deflection, while Fig. 39 shows six patterns at higher values of deflection. The patterns are presented in two figures merely to make the changes in the patterns more observable.

At any given load the net area enclosed between the stress pattern and the base line is a measure of the load on the cross-section. In this case the ultimate load was .95 P/Py. The stress pattern for this load is shown by
the heavy solid line in Fig. 39. It is evident from the pattern that at this load the enclosed area is a maximum as would be expected. Beyond the ultimate load, all of the flange is yielded except for a small portion at the end of the convex side of the flange. As the deflections continue to increase this area of elastic material increases causing a decrease in compressive load and an increase in internal moment. When the deflections become sufficiently large, the convex edge of the flange yields in tension. Because the program does not consider local buckling or strain hardening, the stress pattern will continue in this manner at higher deflections. These patterns approximate to a close degree the stresses measured in the flange at mid-height during the pinned-end column tests.
8. SUMMARY

This paper describes the results of a preliminary study of welded steel columns manufactured from flame-cut plates. Tests were conducted on two sizes of columns: 12H79 and 14H202, manufactured from A36 steel plate. The conclusions reached on the basis of these tests, as well as the theoretical analyses are the following:

1. The tensile residual stresses at the flange tips are a direct result of the flame cutting of the plates. Welding reduces these somewhat, but not enough to change the favorable effect on column strength. (Section 4.2)

2. The magnitude of residual stresses in heavier shapes of flame-cut welded columns is lower than those in lighter shapes, but the column strength is slightly improved. Therefore, the exact relationship of residual stress magnitude and column geometry to column strength is not yet clearly defined. (Sections 4.2 and 5.2)

3. Both the test results and the tangent modulus analysis show that welded columns manufactured from flame-cut plates have high weak-axis column strengths in the important lower slenderness
ratios. This improved column strength is caused by the tensile residual stresses at the flange tips. (Section 5.2)

4. The tangent modulus analysis shows that the strength of these flame-cut welded columns is nearly identical about the "strong" and "weak" axes of the shape. (Section 5.2 and Figs. 15-18)

5. The tangent modulus theory appears to give a good approximation of the ultimate strength of flame-cut welded columns. This occurs because the ultimate strength is reduced slightly due to the allowable values of initial out-of-straightness commonly found in steel columns. (Section 5.2 and Figs. 15 and 17)

6. Pinned-end column strength increases with the amount of area in the flanges influenced by tensile residual stress and the distance of this area from the neutral axis of the cross-section. (Section 6.2 and 6.4)

7. The influence of residual stress on column strength is greater for "weak" axis bending than for "strong" axis bending. (Sections 6.2, 6.4, 6.5, and Figs. 19-24)

8. From a tangent modulus analysis it appears that the CRC column strength curves do not
represent the average behavior of heavier H-shaped columns of all types. However, the agreement of these tangent modulus curves with the true ultimate strength of heavy columns still remains to be determined. Future studies should include selected heavy column tests to determine the relationship of ultimate strength to tangent modulus load for these columns. (Section 6.5 and Figs. 23 and 24)

9. The ultimate strength of these flame-cut welded columns with rotation permitted about the "weak" axis decreases with increasing initial eccentricity. Also, this effect of initial eccentricity increases with the slenderness ratio of the column. (Figs. 26-31)

10. For a perfectly straight column, the ratio of ultimate load to tangent modulus load increases with increasing slenderness ratio. (Section 7.2 and Figs. 26-31)
9. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>Flange width</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus for steel</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Initial eccentricity measured at mid-height of a column</td>
</tr>
<tr>
<td>I</td>
<td>Moment of inertia of entire cross-sectional area</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Moment of inertia of elastic portion of cross-section</td>
</tr>
<tr>
<td>L</td>
<td>Length of pinned-end column</td>
</tr>
<tr>
<td>M</td>
<td>Internal moment due to deflecting column</td>
</tr>
<tr>
<td>P</td>
<td>Load on column</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Ultimate load on the column</td>
</tr>
<tr>
<td>$P_y$</td>
<td>Load on column causing a fully-yielded cross-section</td>
</tr>
<tr>
<td>r</td>
<td>Radius of gyration</td>
</tr>
<tr>
<td>u</td>
<td>Deflection of column at any point along the length</td>
</tr>
<tr>
<td>V</td>
<td>Deflection of column at mid-height</td>
</tr>
<tr>
<td>z</td>
<td>Distance along column length</td>
</tr>
<tr>
<td>$\sigma_{cr}$</td>
<td>Buckling stress</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Yield stress</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Curvature of column due to deflection</td>
</tr>
</tbody>
</table>
10. ACKNOWLEDGEMENTS

This report presents the results of a preliminary research study into the strength of welded steel columns manufactured from flame-cut plates.

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<th>Tensile Coupons</th>
<th>Stub Column Tests</th>
<th>Column Tests (Pinned End)</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>8&quot; Flat</td>
<td>505</td>
<td></td>
</tr>
<tr>
<td>12H79</td>
<td>3</td>
<td></td>
<td>13</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>14H202</td>
<td>2</td>
<td></td>
<td>5</td>
<td>5</td>
<td>2</td>
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TABLE II

Column Test Results

<table>
<thead>
<tr>
<th>Column</th>
<th>Size</th>
<th>L/r</th>
<th>$\xi_e$ (in)</th>
<th>e/b</th>
<th>R/Py</th>
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<tbody>
<tr>
<td>1A</td>
<td>12H79</td>
<td>30</td>
<td>.015&quot;</td>
<td>.001</td>
<td>0.97</td>
</tr>
<tr>
<td>1B</td>
<td>12H79</td>
<td>60</td>
<td>.060&quot;</td>
<td>.005</td>
<td>.76</td>
</tr>
<tr>
<td>1C</td>
<td>12H79</td>
<td>90</td>
<td>.005&quot;</td>
<td>.0004</td>
<td>.68</td>
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<tr>
<td>2A</td>
<td>14H202</td>
<td>30</td>
<td>.110&quot;</td>
<td>.007</td>
<td>.97</td>
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<tr>
<td>2B</td>
<td>14H202</td>
<td>60</td>
<td>.145&quot;</td>
<td>.009</td>
<td>.84</td>
</tr>
<tr>
<td>2C</td>
<td>14H202</td>
<td>90</td>
<td>.07&quot;</td>
<td>.004</td>
<td>.64</td>
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Fig. 16  Strong Axis Tangent Modulus Curve - 12H79

\[ \sigma_{CR} = \frac{\pi^2 E (l_e/l)_x}{(L/r)^2} \]
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\[ \lambda = \sqrt{\frac{\sigma_y}{\pi^2 E}} \cdot \frac{1}{r} \]
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Fig. 33  Comparison of Theory and Test 12H79 L/r = 60
Fig. 34  Comparison of Theory and Test 12H79 \( L/r = 90 \)

Predicted \( \frac{R_u}{R_y} = 0.71 \)

Test \( \frac{R_u}{R_y} = 0.68 \)

\( \theta_b = 0.0012 \) (Theory)

\( \theta_b = 0.0004 \) (Test)

Fig. 35  Comparison of Theory and Test 14H202 \( L/r = 30 \)

Predicted \( \frac{R_u}{R_y} = 0.97 \)

Test \( \frac{R_u}{R_y} = 0.97 \)

\( \theta_b = 0.004 \) (Test)

\( \theta_b = 0.006 \) (Theory)
Fig. 36  Comparison of Theory and Test 14H202 L/r = 60

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Fig. 39  Stress Distribution Under Load
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