Design of Laterally Unsupported Columns

TECHNIQUES FOR TESTING STRUCTURAL SUBASSEMBLAGES
WITH BRACED AND UNBRACED COLUMNS

by

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This work has been carried out as part of an
investigation sponsored jointly by the Welding
Research Council and the Department of the Navy
with funds furnished by the following:

American Iron and Steel Institute
Naval Ship Engineering Center
Naval Facilities Engineering Command

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Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

June 1970

Fritz Engineering Laboratory Report No. 329.2
ABSTRACT

Techniques and setups for testing full-size non-sway structural subassemblages with laterally braced and unbraced columns are described. The height of the subassemblages varied from approximately 20 to 33 feet and the width 20 to 37 feet. The columns were subjected to axial forces applied by a universal testing machine and also bending moments transmitted from the adjoining beams. These beams were loaded by two quarter-point loads through the use of gravity load simulators. Depending on the locations of the beams, the columns in the subassemblage were bent either in single curvature or in double curvature. In-plane deflections of the columns were recorded electronically while the out-of-plane deformations of the unbraced columns were recorded by means of depth gages. Electrical resistance gages were mounted on the beams and columns in a unique arrangement so as to allow various types of strains to be analyzed separately. These strains are: (1) normal strain due to axial force, (2) normal strain due to strong axis bending, (3) normal strain due to weak axis bending, (4) normal strain due to warping of section and (5) St. Venant shearing strain. Experimental behavior and sample test results are also described.
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1. INTRODUCTION

1.1 Definition of "Subassemblages"

A subassemblage is a structural system consisting of several column segments and their adjoining beams. It is an integral part of a planar building frame. The number of columns and beams that constitute a subassemblage varies considerably. It can merely be a system consisting of a beam and a column which is sometimes known as restrained column. A typical example is the top corner of a building frame (Fig. 1). On the other hand, a subassemblage can be very complex. A group of columns and beams that forms a portion of a building frame is an example of such a complex system. Typical examples are also shown in Fig. 1.

A subassemblage with columns that are parts of the exterior columns of a frame is hereafter known as exterior subassemblage. An interior subassemblage has only interior columns.

Under the action of transverse forces or unsymmetrical vertical loading, a structure usually sways sideway. It can be visualized that for geometrical compatibility of the entire frame, every subassemblage within the frame will sway. Such a subassemblage is known as 'sway' subassemblage. A 'non-sway' subassemblage refers to one that does not sway and it usually exists in frames under symmetrical vertical loading or in frames that are braced against sway.

1.2 Theoretical Bases

Within the last decade, there was considerable development on the Column Deflection Curve method which allows the complete load-
deformation behavior of individual beam-columns to be accurately predicted. (1) This work paved the way for the introduction of the concept of subassemblages for the plastic method of design of planar steel frames. (2, 3, 4) The concept of subassemblages assumes that the whole frame can be divided into a number of subassemblages each acted upon by the externally applied loads and the internal forces. The maximum load that each system can sustain is determined from its load-deformation behavior which in turn, depends on the response of its component members. The procedure to obtain the response for beams is well known. (5) Figure 2 is provided to illustrate the technique employed to find the strength and behavior of a subassemblage. The right hand side of this figure diagrammatically shows a system consisting of two columns with a beam framing into the columns at the joint j. The far end of each member is assumed pinned. An external moment is applied at the joint. As a result of the application of the external moment, the three component members react together to resist this moment. When the compatibility and equilibrium conditions are fully satisfied, the total resisting moment $M_j$ at a given joint rotation $\theta_j$ is given by

$$M_j \theta_j = \sum_m M_m \theta_j$$

where $M_m$ is the end moment of a member at end-rotation $\theta_j$. Figure 2(b) shows qualitatively the moment-rotation curves for the beam and the beam-columns of the subassemblage. The response curve for the joint is obtained by compounding the response curves of all its component members. The peak of the compounded curve represents the maximum resisting moment of the joint. Mathematically this maximum value is obtained by differentiating Eq. (1) with respect to end rotation $\theta_j$.
Furthermore, no theoretical solution is available for continuous beam-columns with joint restraints which, in reality, truly represent the practical columns in a rigid frame. For checking the likelihood of the occurrence of lateral-torsional buckling in a pinned-end beam-column (see Fig. 3), an empirical formula has been suggested: \( (13, 14, 15) \)
\[
\frac{P}{P_0} + \frac{C \frac{M_1}{M_0}}{1 - \frac{P}{P_e}} \leq 1.0
\]  

(3)

where

- \( P \) = axial load on the column,
- \( P_0 \) = maximum axial load which the column can support if no bending moments are present,
- \( P_e \) = elastic buckling load in the plane of bending,
- \( M_1 \) = numerically larger end moment,
- \( M_0 \) = maximum moment which the column can substain if no axial force is present, and
- \( C_m \) = end-moment correction factor depending on the end moment ratio \( q \).

Equation (3) only indicates a probable buckling load for an unbraced pinned-end beam-column. The behavior of this beam-column before buckling may be accurately predicted by the CDC method as mentioned earlier. Its behavior after buckling is not yet fully understood.

1.3 Experimental Bases

Two major studies have been made to compare the theoretical in-plane behavior of structural members with experimental results.\(^{(16,17)}\) The first dealt with the response of individual beam-columns bent in four different configurations as shown in Fig. 4a.\(^{(16)}\) The second study was concerned with the behavior of braced restrained beam-columns.\(^{(17)}\) Each beam-column consisted of a column with an adjoining beam welded to each end of the column (Fig. 4b). The experimental results were compared with the theoretical predictions for both studies. Not only was the theoretical maximum strength reached, but that the complete load-deformation relationship of the columns correlated exceptionally well with the theory.
Other tests on small-scale restrained beam-columns are described in Refs. 18 and 19. These tests had column bent in single curvature and under relatively low axial load. Tests on small-scale non-sway subassemblages are reported in Refs. 20 and 21. The moments acting on each column end was first applied through beams and then the column was axially loaded to failure. To a limited extent, all these tests help to throw some light on the behavior of simple structural systems. But they do not by any means provide a complete experimental verification of the theory discussed in Section 1.2. How a group of laterally braced continuous columns collectively resists externally applied moments is unknown.

Recent tests on W beam-columns, unbraced against out-of-plane deformation and loaded into the inelastic range, have been reported in Refs. 22 and 23. The majority of these columns failed by combined bending and twisting with significant reduction in the in-plane moment and rotation capacities. A restrained beam-column, unbraced in the weak direction was also tested. Its behavior did not differ much from that of a similar but braced specimen as reported in Ref. 17 although its strength was slightly reduced.

There has not been any test done on continuous beam-columns with joint restraints and without bracing between the joints. A physical understanding of the effects of joint restraints on the response of a continuous beam-column after lateral-torsional buckling is essential at the present stage when no theoretical prediction is available.

1.4 Objectives of Tests

The objectives of the tests are to study the following aspects of the behavior of non-sway subassemblages:
1. To provide experimental confirmation of the procedure for the design of laterally braced columns.

2. To study the behavior of single and double curvature braced columns loaded through beams.

3. To study the manner in which two columns, above and below a joint, share the applied moment.

4. To study the behavior of subassemblages with laterally unbraced columns.

5. To observe the failure modes of subassemblages subjected to a programmed application of load. That is, an axial load of predetermined magnitude is first applied on the columns, then, while maintaining this axial load, moments are applied incrementally on the columns through beams.

The subassemblage tests were designed to study the above aspects of the behavior of non-sway subassemblages. To achieve this requires the proper design of testing arrangements and instrumentation that would yield useful experimental data.
2. DESIGN OF TESTING ARRANGEMENTS

2.1 Types of Test Specimens

The basic subassemblages used in this investigation are the interior and the exterior subassemblages shown in Fig. 1. The interior subassemblage consists of three columns and four adjacent beams two of which framing into the upper joint and two into the lower joint. The exterior subassemblage consists of three columns and two adjacent beams, one framing into the upper joint and the other into the lower joint. Also illustrated in the same figure are the critical loading conditions for these two types of subassemblages: checkerboard gravity loading pattern for interior subassemblage and full gravity loading for exterior subassemblage. Under checkerboard gravity loading, columns of the interior subassemblage will be bent in single curvature configuration. On the other hand, under full gravity loading, columns of the exterior subassemblage will be bent in double curvature configuration.

At the upper or the lower joint of the interior subassemblage, it is likely that the girder that is loaded by the full factored gravity load will transmit more moment to the joint than the girder that carries only the factored dead load. The difference in the girder moments will be resisted by the columns above and below the joint and also by the restraining beam with factored dead load. The collective resistance of a restraining beam and a column to an applied joint moment has previously been studied in depth. Since the present investigation is primarily concerned with the response of columns, the two restraining beams in the
interior subassemblages were deliberately left out. Also the rotational restraint at the far end of each component member has been omitted in order to simplify the tests. Thus the shape of the interior subassemblages under investigation is as shown in Fig. 5a.

The columns in the exterior subassemblages are usually bent in double curvature with the point of inflection near their midheight. The structural action of a double curvature column can therefore be represented approximately by a column with half of the actual height and with its far end pinned. This was done for the upper and lower columns of the exterior test subassemblages whose shape is shown in Fig. 5b.

2.2 Design Details of Testing Arrangements

An overall view of the setup for testing interior subassemblages (hereafter referred to as single curvature subassemblages) is shown in Figs. 6 and 7 and that for testing exterior subassemblages (hereafter referred to as double-curvature subassemblages) is shown in Figs. 8 and 9. In all cases, the columns were set up in a 5,000,000 lb. capacity hydraulic testing machine. The column was placed at the centerline of the machine, and the beams were welded to the column at one end and attached to a supporting tower at the other end. In the single curvature tests, the upper beam was welded to the left flange of the columns and the lower beam to the right flange. In the double curvature tests, both beams were welded to the left flange of the column.

Only one supporting tower was required in the double curvature tests as both beams rested on rollers attached to the same tower. However, in the single curvature tests, two supporting towers were required.
Each supporting tower was fabricated from two braced 8W67 sections of 35 ft. length and was independently attached to the test floor. Beams of length 15 ft., 17 ft., 18 ft., and 20 ft. could be accommodated.

A close-up view of the roller support for the beam is shown in the photograph in Fig. 10. At each joint level, a pinned-end link assembly was provided to restrain the columns from sway. This link assembly, as shown in Fig. 11, consisted of a stiff turnbuckle which was pinned at one end to a fixture attached on the test column and was similarly pinned at the other end to a box beam that was securely bolted to the supporting columns of the testing machine. All pins, holes, and fixtures were machined to very close tolerances in order to remove play in the assembly. The turnbuckle permitted adjustments to be made before the test began, after which time no adjustments were made.

The ends of the columns were carried on cylindrical end fixtures the details of which have been described in Ref. 24. A diagrammatic representation of the end fixtures is shown in Fig. 12. These end fixtures permitted the applied axial load to always pass through two fixed points which were the centers of the cylindrical surfaces. The test columns were designed such that the centers of the surfaces were also the centers of the bases of the columns. In the single curvature tests, one end fixture was bolted to the crosshead of the testing machine, and the other was placed on the test floor. Only the friction between the fixture and floor was necessary to resist the shearing forces. In the double curvature tests, the same two fixtures were used, but the bottom fixture was placed on a four foot high pedestal in order to provide more room for the loading devices for the beams.
Special bracing systems designed based on the Watt's linkage concept were provided for the columns to prevent out-of-plane deformations. (25) (See Fig. 13) For the single curvature subassemblages designed to study lateral-torsional instability, lateral braces were provided for the columns only at the story levels in order to simulate the situations in which lateral restraints were presumably provided only by the floor beams and slab. For the subassemblages designed to study their in-plane behavior, additional braces were provided at the mid-height of each column.

Lateral movement of the beams were prevented by channel-type lateral braces that fitted snugly on both sides as shown in Fig. 14. (26) These braces were bolted to a strong box which was positioned securely above the test beam.

Loads on the columns were applied directly by the testing machine through the end fixtures. Beam loads were applied at the quarter points by the tension jack inside of a gravity load simulator through an adjustable spreader beam (see Fig. 15). A gravity load simulator is a mechanism which always maintains a vertical orientation of load even as a structure sways. (25) It was positioned directly under the beam. In the double curvature tests, two gravity load simulators were required to load the upper beam for the reason that two beams were on the same side and one above the other. Loads on the upper beams were applied through an auxiliary spreader beam which was welded to the top of the adjustable spreader beam and perpendicular to it.
2.3 Design of Test Specimens

The objectives of the tests have been stated previously. The sections and dimensions for the test specimens depend mainly on the objectives of the tests. For the tests design primarily to study the in-plane behavior of the columns, the variables were the slenderness ratio about the strong axis $h/r_x$ and the end moment ratio. Whereas for the study of out-of-plane behavior, the variable was the slenderness ratio about the weak-axis $h/r_y$.

In the tests performed on two single-curvature subassemblies with laterally braced columns, the columns were 8W67 shapes with $h/r_x$ of 35 and 30 respectively; the upper beams were 10B17 shapes of length 20'0" and the lower beams were 8B15 of length 15'0". The $h/r_x$ ratio of 35 and 30 corresponded to a column height $h$ of 10'0" and 9'4" respectively.

For the two single curvature subassemblies with laterally unbraced columns, the columns were 8W35 and 8W24 shapes corresponding to a $h/r_y$ ratio of 60 and 76 respectively. The height of all columns was 10' 2 $\frac{3}{8}$". The upper beam was 10W21 shape for the first subassemblage and 8I18.4 shape for the second. The lower beams were 12B16.5 and 8B15 respectively.

In the tests on two double curvature subassemblies with laterally braced columns, the columns were 8W67 shapes with $h/r_x$ of 35 and 30 respectively; the upper beams were 10I25.4 and the lower 8W24 shapes. Both the upper and lower beam spanned 20'0".
All the sections and the dimensions for the subassemblages had been designed according to the criterion that the upper and lower joints failed simultaneously at the formation of beam mechanism for the upper and the lower beams. The slenderness ratios of the columns are in the range commonly encountered in multi-story frame design. The selection of the column size had further been influenced by the maximum height (about 35 ft.) of a compression member that could be accommodated in the testing machine.

The beam-to-column connections have been designed on the criterion that no failure should occur in the connection prior to the failure of the subassemblage. Accordingly and for practical reasons, horizontal stiffeners of the same thickness as the flanges of the beam and diagonal stiffeners were provided at each joint. Figure 16 shows a typical connection detail. The detail of a typical load point in a beam is shown in Fig. 17.

Plates of 1 1/4 in. thick were welded to the end of the top and bottom columns and then milled to remove any uneveness. Each plate provided for four bolts to attach the specimen to the end fixtures. Milling was essential for rapid alignment of the columns.
3. MEASUREMENT OF FORCES AND DEFORMATIONS

3.1 Measurement of Forces

As mentioned in Section 1.4 the loading that was adopted was to apply an axial force $P_0$ initially on the columns and then to apply incremental vertical loads on the beams. The resulting bending moment diagrams for the two types of subassemblages are shown qualitatively in Fig. 18.

The force on the column $P_0$ was measured by the testing machine. The beam loads $F_u$ and $F_d$ were measured by the dynamometers as well as from the pressure in the hydraulic system.

Strains in the beams and columns of the braced subassemblages were measured by SR-4 resistance strain gages. In addition to these gages, strain rosettes were also used in the two subassemblages with laterally unbraced columns. Different arrangements of gages were selected for each particular test in order to achieve the test objectives.

For the subassemblies with laterally braced columns, strain gages were placed at each section on all four flange tips. (See Fig. 19a) With gages located in this manner, the axial load and moments at the section at any particular test run could be separately determined. Five sections which remained elastic throughout the loading history were gaged on each beam. In the single curvature tests, thirteen sections were gaged on the column and at fourteen locations in the double curvature tests.
For the subassemblages with laterally unbraced columns, gages were placed at each section on all four flange tips of the beam. Again five sections were gaged on each beam. On the columns, eight sections were gaged, each with 8 SR-4 gages and two rosetts arranged in a pattern as shown in Fig. 19b. One section (at midheight of middle column) was gaged with 12 SR-4 gages and 2 rosetts.

Figure 20 shows a schematic layout of the gage locations at a beam. The quantities $M_1$, $M_2$, $M_3$, $M_4$ and $M_5$ are the computed moments from the recorded strains and the moment at the roller support is known to be zero. The moment at the joint $M_{J(B)}$ was determined by two methods: First, it was obtained by linear extrapolation of the computed moments $M_1$ and $M_2$:

$$M_{J(B)} = \frac{x_2}{x_2 - x_1} (M_1 + M_2) - M_2 \quad (4)$$

Secondly, it was determined by linear extrapolation of a straight line obtained by least square method to pass through moment ordinates of $M_1$, $M_2$ and $M_5$. The Moment $M_5$ was previously determined by linear extrapolation of the computed moments $M_3$ and $M_4$. Figure 21 shows two typical ratios of $M_{J(B)e}/M_{J(B)s}$ vs. joint rotation where $M_{J(B)e}$ is the joint moment obtained by linear extrapolation and $M_{J(B)s}$ the moment obtained by the least square method. The mean difference between the moments obtained by these methods for all the tests was of the order of 1%.

The shear force at the roller support was determined from the recorded moment $M_5$ as follows:

$$R_2 = \frac{M_5}{x_5} \quad (5)$$
The shear force at the joint $R_1$ was then equal to

$$R_1 = F_{u1} + F_{u2} - R_2 \quad (6)$$

and the total load acting on the column was

$$P_B = P_o + \sum R_1 \quad (7)$$

For the middle column, $R_1$ was from the upper beam only. Whereas for the lower column, the extra axial load was due to $R_1$ from the upper and the lower beams.

For the section with gages shown in Fig. 19a, the axial load $P_s$ and the major axis bending moment $M_{bx}$ acting on the column were computed from the recorded strains $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ as follows:

$$P_s = \frac{AE}{4} \sum_{i=1}^{4} \varepsilon_i \quad (8)$$

$$M_{bx} = -\frac{EI_x}{2d} (\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \quad (9)$$

where $A = \text{cross-sectional area of the column}$,

$E = \text{elastic modulus} = 29,600 \text{ ksi}$,

$I_x = \text{moment of inertia about the strong axis}$, and

$d = \text{depth of the beam}$.

For the section with gages shown in Fig. 19b, the recorded strain at each strain location in the flanges was the resultant of the following four types of strain:

1. normal strain due to axial force $\varepsilon_a$,
2. normal strain due to strong axis bending $\varepsilon_{bx}$,
3. normal strain due to weak axis bending $\varepsilon_{by}$, and

4. normal warping strain $\varepsilon_{w}$.

The strain recorded by rosette mounted on the web surface was the resultant of the normal strain due to axial force and St. Venant shearing strain $\varepsilon_{s}$. The decomposition of the recorded strains could be accomplished in the following manner:

$$
\varepsilon_a = \frac{1}{8} \sum_{i=1}^{8} \varepsilon_i \quad (10)
$$

$$
\varepsilon_{bx} = \frac{d}{8t} (\varepsilon_1 + \varepsilon_2 + \varepsilon_5 + \varepsilon_6 - \varepsilon_3 - \varepsilon_4 - \varepsilon_7 - \varepsilon_8) \quad (11)
$$

$$
\varepsilon_{by} = \frac{1}{8} (\varepsilon_1 + \varepsilon_3 + \varepsilon_5 + \varepsilon_7 - \varepsilon_2 - \varepsilon_4 - \varepsilon_6 - \varepsilon_8) \quad (12)
$$

$$
\varepsilon_{w} = \frac{1}{8} (\varepsilon_1 + \varepsilon_3 - \varepsilon_5 - \varepsilon_7 - \varepsilon_2 - \varepsilon_4 + \varepsilon_6 + \varepsilon_8) \quad (13)
$$

$$
\varepsilon_{s} = \frac{1}{2} (\varepsilon_9 + \varepsilon_{14} - \varepsilon_{11} - \varepsilon_{12}) \quad (14)
$$

In Eq. 11, $t$ is the thickness of the flange.

The axial force and the various moments were calculated from the decomposed strains by using the following formula:

$$
P_s = AE \varepsilon_a \quad (15)
$$

$$
M_{bx} = -\frac{2EI_x}{d} \varepsilon_{bx} \quad (16)
$$

$$
M_{by} = -\frac{EI_y}{b_1} \varepsilon_{by} \quad (17)
$$

$$
M_{z} = -\frac{EI_f}{b_1} \varepsilon_{w} \quad (18)
$$

$$
M_{z} = -\frac{GK_T}{w} \varepsilon_{s} \quad (19)
$$
where \( M_{by} \) = Bending moment about the y axis,
\( M_w^T \) = warping torsional moment,
\( M_T \) = St. Venant torsional moment,
\( M_z \) = total torsional moment,
\( K_T \) = torsional constant,
\( I_f \) = moment of inertia of a flange about the y-axis,
\( I_y \) = moment of inertia about y axis,
\( b_l \) = location of strain gage from the y axis as shown in Fig. 19b, and
\( w \) = web thickness.

A cross-check for the axial load \( P_B \) on the column as calculated from Eq. 7 was made by comparing it to the value \( P_S \) calculated with the aid of Eq. 8 for the laterally supported columns and of Eqs. 10 and 15 for the laterally unsupported columns. Figure 22 shows a typical comparison between \( P_B \) and \( P_S \) for subassemblage S-2 (with laterally unsupported columns). Within the elastic range of steel, the discrepancy of measuring \( P \) by the two methods was only about 2%.

The joint moments as computed from Eq. 4 were cross-checked by moments developed at the column ends. The top joint of a single-curvature subassemblage was chosen to illustrate the technique used to compute the joint resisting moment. If the column deflections \( \Delta_1, \Delta_2 \)
and $\Delta_3$ at the level of gage locations 1, 2 and 3 respectively are known (see Fig. 23), the column end moments $M_C(a)$ and $M_C(b)$ are given by:

$$M_C(a) = \frac{h}{h_1} (M_{m1} - P \Delta_1)$$

$$M_C(b) = (1 - \frac{h_2}{h_3}) (M_{m3} - P \Delta_3) + \frac{h_2}{h_3} (M_{m2} - P \Delta_2)$$

The subscripts $a$ and $b$ refer to moments above and below the joint. The moments $M_{m1}$ and $M_{m2}$ are the strong axis moments calculated from Eqs. 9 and 16 depending on the particular test. The resisting joint moment is then

$$M_{J(C)} = M_C(a) + M_C(b)$$

For equilibrium of the joint,

$$M_{J(B)} = M_{J(C)}$$

The relationship between the two predictions of joint moments for test S-1 within elastic range of steel is shown in Fig. 24. It is seen that except for the first couple of readings, there is a variation of only 5% to -2% for the relevant region of the test. This variation is within the limits of experimental accuracy. The results shown in Figs. 22 and 24 serve to confirm the adequacy and the validity of the measuring technique.

Figure 25 shows typical twisting moment vs. joint rotation relationships as predicted by Eqs. 18, 19 and 20. The section considered was the midheight of the upper column of subassemblage S-2. The warping torsional moment increased with increasing joint rotation while the
recorded St. Venant torsion remained fairly constant through the whole range of rotation.

3.2 Measurement of Deformations

Altogether five instruments were used to measure deformations: mechanical level bar and electrical rotation gage for rotation measurements (see Fig. 26); level, micrometer depth gage and linearly varying potentiometer for deflection measurements (see Fig. 27).

The use and the accuracy of mechanical level bar and electrical rotation gage for rotation measurements have been described in detail elsewhere. (16, 27)

The vertical beam deflections were measured through a kern level which were sighted on a scale glued to the beams at selected locations. Displacements were thus measured directly from the scales.

The in-plane displacements of the columns of subassemblies with laterally braced columns were measured by the linearly varying potentiometers which were independently supported in the testing machine. A fine piece of wire exactly two feet long connected every potentiometer to the center of the column flange. When the column deflected horizontally, the wire changed the pointer on the potentiometer, thus changing the voltage which was calibrated directly into inches. Deflection measurements were taken at all levels of the columns where strain gages were glued, at the levels of the joints, at the top of the upper column and the bottom of the lower column.
In addition to the potentiometer, micrometer depth gages were used to measure the in-plane as well as out-of-plane deformations of the columns of subassemblies with laterally unbraced columns. The depth gage measured the distance of the tip of the two flanges of the column from a steel bar drilled with a horizontal slot in the middle. This slot was specially designed so as to enable the depth gage to move horizontally with the column and at the same time not too loose to allow inaccurate measurement. (see Fig. 27a) A scale was glued to the top of the slotted bar for the transverse reading of the depth-gage. Two slotted bars were used at each section. They were clamped securely to a steel frame and parallel to the web of the column. The steel frame supporting the slotted bars was in turn bolted tightly to the strong columns of the testing machine. Typical out-of-plane deformations are shown in Fig. 28. The twist of the section was then computed from:

\[ \beta = \frac{(u_1 - u_2)}{(d-t)} \]  

which is shown in Fig. 29. The quantity \( u_1 \) is the lateral displacement of the compression flange and \( u_2 \) the lateral displacement of the tension flange.

A comparison between the in-plane displacement of the columns measured by the potentiometer and by the depth gage is made in Fig. 30. The large discrepancy in the initial readings can be attributed to two reasons. First, the displacement readings were actually very small for small joint rotation and therefore any slight error in recording the displacement was largely amplified. Secondly, it was difficult to take depth-gage readings from the exact point in the flange of the column where the first reading was taken because the hole initially indented
in the flange had moved away as the result of axial deformation of the column. However, the discrepancy converged rapidly to within 10%, which is quite tolerable under the difficult recording situation. It is believed that the potentiometer gave more reliable readings since it was calibrated with consistent results before each test.
4. SECONDARY EFFECTS INTRODUCED BY THE TESTING ARRANGEMENT

4.1 Erection Stresses

Each subassemblage was shipped in three pieces: column and two beams. Thus, field welding was required after the specimen was fully assembled in the testing machine. After all members had been positioned, all bracing attached and the members clamped together, the strain gages were wired and connected to an automatic data acquisition system, the B & F unit. Readings were taken immediately before welding, at intervals during welding and after all welds had cooled. These readings permitted an evaluation of the welding stresses present in the testing specimens, which were then converted into bending moments. Some of the moment was due to the dead weight of the two beams. Typical residual moment distributions are shown in Fig. 31. The maximum moment of each subassemblage represented only about 4% of the beam plastic moment $M_p$ or the reduced plastic moment $M_{pc}$ of the column.

4.2 Alignment

After welding was completed and before the test, it was necessary to align the column. The procedure used was similar to the one used in testing centrally loaded columns. Column strain gage readings were taken first at zero load and then at a load approximately equal to $P_y/3$. Initially the resulting differences showed a load that was being applied eccentrically. Adjustments were made, and the process continued until the strain gages showed that an almost concentric load was being applied. The criterion for acceptance was 5% difference among
the four readings of strain gages mounted on the outside surfaces of the two flanges at every gage level.

4.3 Effect of Vertical Deformation of Columns

Under the combined action of axial load and applied moments, the columns of each subassemblage deformed vertically as well as transversely. The vertical deformations can be attributed to two reasons:

1. settlement of the end fixture, and
2. axial and bending deformations.

In Table 1 are listed the theoretical and measured vertical deformations at the upper and the lower joints of the six subassemblages at two loading stages: (1) prior to the application of beam loads and (2) when the subassemblages exhibited maximum resisting strength. It should be emphasized that only axial shortening was considered in the theoretical calculation. The difference in readings between columns (8) and (9) gives the magnitude of the settlement of the end fixture. A comparison shows that, in most cases, fixture settlement was small, the biggest being 0.07". Subassemblage DC-1 exhibited most vertical displacements at the maximum load stage. Assuming that the upper column shortened as much as the lower column, the maximum vertical displacement (at the top of the subassemblage) was about 0.76" for subassemblage DC-1.

Vertical deformations of the columns produced two minor effects:

1. They induced reversed moments at the joints;
2. They changed what would be the true horizontal column displacement readings as recorded by the linear potentiometers which were set in a horizontal position relative to the originally undeformed shape of the subassemblage.
The reversed moments at the joints had no detrimental effects on the behavior of the subassemblages. The applied joint moment computed by Eq. 4 or Eq. 23 included this secondary effect.

Vertical deformations had little effect on the potentiometer readings. As stated earlier, the maximum vertical displacement recorded by the six subassemblages was 0.76" of subassemblage DC-1. The maximum change in horizontal reading $\Delta_h$ was:

$$\Delta_h \approx l \cdot (1 - \cos \alpha) \tag{28}$$

where $l$ is the original length of the wire and $\alpha$ is the inclination of the wire to the horizontal. For wire length $l = 24''$ and $\alpha = \frac{0.76''}{2.4''}$, the change is:

$$\Delta_h = 24 \cdot (1 - \cos \frac{0.76}{24}) = 0.003 \text{ in.}$$

This maximum change of 0.003 in. was too small to be of significance in influencing the recorded readings of horizontal displacement of column.
5. SUMMARY

The techniques described in this report allow full-scale three-story high structural subassemblages with laterally braced or unbraced columns to be tested under no-sway conditions. The most important aspect of these techniques is the unique arrangement of the strain gages and rosettes on the unbraced columns. The arrangement enables various types of strains to be analyzed separately.

The actual test results of these six subassemblages have been reported in Refs. 27 and 28. Figure 32 is provided primarily to illustrate a typical moment-rotation curve obtained from the testing arrangement presented in this report.
6. ACKNOWLEDGMENTS

The work described in this report forms part of a general investigation "Design of Laterally Unsupported Columns" currently being carried out at Fritz Engineering Laboratory, Lehigh University. This study is sponsored by the American Iron and Steel Institute, Naval Ship Engineering Center, Naval Facilities Engineering Command, and the Welding Research Council. Technical guidance is provided by Task Group 10 of the Column Research Council of which T. V. Galambos is Chairman.

The authors express their thanks to K. Harpel and his staff for setting up the tests. The work of R. N. Sopko for his photography, J. Gera and Mrs. S. Balogh for preparing the drawings are acknowledged. Jack Irabin helped in the data reduction and Miss Karen Philbin very carefully typed this report.
7. NOMENCLATURE

The following symbols are used in this report:

- $A$ = cross-sectional area of column,
- $b_i$ = location of strain gages from $y$ axis,
- $C_m$ = end-moment correction factor,
- $d$ = depth of beam,
- $E$ = elastic modulus,
- $F_{u1}, F_{u2}$ = beam load, $u$ refers to upper beam, $\ell$ refers to lower beam,
- $h_1, h_2$ = length of column segment,
- $h_3, h_5$ = length of beam segment,
- $I_f$ = moment of inertia of a flange about $y$ axis,
- $I_x$ = moment of inertia about $x$ axis,
- $I_y$ = moment of inertia about $y$ axis,
- $K_t$ = St. Venant constant,
- $L_u, L_\ell$ = span length of beam,
- $\ell$ = length of potentiometer wire,
- $\ell_1, \ell_2, \ell_3$ = length of beam segment,
- $M_J(B)$ = joint moment,
- $M_J(B)_e$ = joint moment obtained by linear extrapolation,
- $M_J(B)_s$ = joint moment obtained by the least square method,
- $M_J(C)$ = resisting joint moment,
- $M_{bx}$ = strong axis bending moment,
- $M_{by}$ = weak axis bending moment,
\( M_C(a) \) = column end-moment above the joint,
\( M_C(b) \) = column end-moment below the joint,
\( M_j \) = joint moment,
\( M_m \) = member end-moment,
\( M_{m1}, M_{m2} \) = bending moment computed from measured strains
\( M_{m3} \) =
\( M_o \) = maximum moment which the column can sustain if no axial force is present,
\( M_z \) = total torsional moment,
\( M_z^T \) = St. Venant torsional moment,
\( M_z^W \) = warping moment,
\( P \) = axial load on column,
\( P_B \) = axial load on column,
\( P_e \) = elastic buckling load in the plane of bending,
\( P_o \) = maximum axial load which the column can support if no bending moments is present,
\( P_s \) = axial load on column,
\( R_1, R_2 \) = end reaction of beam,
\( t \) = flange thickness,
\( u \) = lateral displacement of flange (subscripts 1 and 2 denote compression and tension respectively).
\( w \) = web thickness,
\( \alpha \) = inclination of potentiometer wire to the horizontal,
\( \beta \) = twisting angle,
\( \Delta_h \) = change in transverse column displacement,
\( \Delta_1, \Delta_2, \Delta_3 \) = transverse displacement of column,
\( \varepsilon_a \) = normal strain due to axial force,
\( \varepsilon_{bx} = \) normal strain due to strong axis bending,
\( \varepsilon_{by} = \) normal strain due to weak axis bending,
\( \varepsilon_i = \) recorded strain \( (i = 1 \text{ to } 14) \),
\( \varepsilon_s = \) St. Venant shearing strain,
\( \varepsilon_w = \) normal warping strain, and
\( \theta_j = \) joint rotation.
Table 1: Summary of Vertical Displacements of Columns

<table>
<thead>
<tr>
<th>Subassemblage</th>
<th>Column Shape</th>
<th>$P_y (kips)$</th>
<th>$P_y$ (ins.)</th>
<th>Upper Joint</th>
<th>Lower Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Prior to Beam Loads</td>
<td>Measured at Max. Strength</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>SC-1</td>
<td>8(\frac{1}{4})/67</td>
<td>0.81</td>
<td>574</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>SC-2</td>
<td>8(\frac{1}{4})/67</td>
<td>0.82</td>
<td>581</td>
<td>0.23</td>
<td>0.11</td>
</tr>
<tr>
<td>DC-1</td>
<td>8(\frac{1}{4})/67</td>
<td>0.80</td>
<td>567</td>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>DC-2</td>
<td>8(\frac{1}{4})/67</td>
<td>0.80</td>
<td>567</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>S-1</td>
<td>8(\frac{1}{4})/35</td>
<td>0.54</td>
<td>329</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>S-2</td>
<td>8(\frac{1}{4})/21</td>
<td>0.55</td>
<td>249</td>
<td>0.16</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Fig. 1 Subassemblages in a Multi-Story Frame
Fig. 2 Braced Subassemblage
Fig. 3 Column Under Axial Load and Bending Moments
(a) Types of Beam-Columns Reported in Ref. 15

(b) Type of Restrained Columns Reported in Ref. 16

Fig. 4 Tests of Beam-Columns and Restrained Columns
Fig. 5 Test Subassemblages
Fig. 6 Test Setup for Single Curvature Subassemblages

Fig. 7 Single Curvature Test in Progress
Fig. 8 Test Setup for Double Curvature Subassemblages

Fig. 9 Double Curvature Test in Progress
Fig. 10  Roller Support

Fig. 11  Link Assembly
Fig. 12 Connection and End Fixture
Fig. 13 Lateral Bracing System

Fig. 14 Knife Edge Guide
Fig. 15 Loading Arrangement for Beam
Fig. 16 A Typical Connection Detail

Fig. 17 Load-Point Detail
Fig. 18 Bending Moment Diagrams

(a) Single Curvature Subassemblage

(b) Double Curvature Subassemblage
Fig. 19 Arrangements of Electrical Resistance Gages on Columns

- SR-4 Gage
- Rosette
Fig. 20 Measurement of Beam Moments
Fig. 21 Joint Moment Ratio
Fig. 22 Load Ratios

\( P_s = \text{Axial Load Computed by Eq. 8 or Eq. 15} \)

\( P_B = \text{Axial Load Computed by Eq. 7} \)
\[ M_i = M_{m1} - P_0 \cdot \Delta_i \]
\[ M_{C(a)} = R h = \left( \frac{M_{m1} - P_0 \cdot \Delta_i}{h_1} \right) h \]
\[ M_2 = M_{m2} - P \cdot \Delta_2 \]
\[ M_3 = M_{m3} - P \cdot \Delta_3 \]
\[ M_{C(b)} = M_3 + \frac{h_2}{h_3} \left( M_2 - M_3 \right) \]
\[ = \left( 1 - \frac{h_2}{h_3} \right) M_3 + \frac{h_2}{h_3} M_2 \]
\[ = \left( 1 - \frac{h_2}{h_3} \right) \left( M_{m3} - P \Delta_3 \right) + \frac{h_2}{h_3} \left( M_{m2} - P \Delta_2 \right) \]

Fig. 23 Joint Moments
Subassemblage S-2

Fig. 24 Joint Moment Ratio

\[ \frac{M_J(c)}{M_J(b)} \]

- Lower Joint
- Upper Joint

Joint Rotation, \( \theta \) (Radian)

\[ M_J(c) = M_C(a) + M_C(b) \]
Fig. 25 Typical Recorded Torsional Moments
(a) Mechanical Level Bar

(b) Electrical Rotation Gage

Fig. 26 Rotation Measuring Devices
(a) Depth-Gage

(b) Displacement Potentiometer

Fig. 27 Deflection Measuring Devices
Fig. 28 Lateral Displacement of Column
Mid-height of Middle Column of Subassemblage S-2

Fig. 29 Twist of Column
Fig. 30 Horizontal Column Displacement Ratios

\[ \frac{\Delta_s}{\Delta_p} \]

\[ \Delta_p = \text{Horizontal Displacement Measured by Potentiometer} \]

\[ \Delta_s = \text{Horizontal Displacement Measured by Micrometer Depth Gorge} \]
Moments Plotted On Tension Side of Members

(a) Subassemblage SC-2

(b) Subassemblage DC-2

Fig. 31 Erection Moments
Fig. 32 Typical Moment-Rotation Relationships
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