LEHIGH UNIVERSITY

Space Frames with Biaxial Loading in Columns

COMPUTER PROGRAM
FOR AN INELASTIC
BEAM-COLUMN PROBLEM

by
Sampath Iyengar
W. F. Chen

May 1970

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A computer program to obtain the response of an eccentrically loaded beam-column under a concentrated load at the midspan is presented.

The relationships, derived in earlier papers, between the concentrated load and curvature at midspan are the bases for this program.

Seven different types of cross sections for which the moment-curvature-thrust relations have been previously approximated in the form of analytical expressions can be handled by the program. The maximum load that the beam-column can carry is computed within close tolerances. The end slope and central deflection at all stages of loading are available at user's option.

Both the elastic and elastic-plastic stages of the beam-column behavior, including unloading, are covered in the solution.
1. INTRODUCTION

The behavior of an eccentrically loaded beam-column under a concentrated load at the midspan, as shown in Fig. 1, is an important technical problem with frequent engineering applications. The obvious example is a compression member in building frames or bridge trusses.

Analytical solutions using the central deflection as a parameter exist, if the beam-column behavior is limited to the elastic range [1]. It has recently been shown that the central curvature is a more suitable parameter for the solution of the problem, if the beam-column behavior extends into the inelastic range [2].

The procedure, using the latter approach, can be stated very briefly as follows: The moment-curvature-thrust relationships (m-ψ-p curves) for various structural sections can be closely approximated by the use of simple analytical expressions [3,4]. Such expressions may be used in a computer program to solve numerically the equations established between external loads (thrust and concentrated load) and the central curvature [3].

2. STATEMENT OF THE PROBLEM

The beam-column shown in Fig. 1 is symmetrically loaded and, hence, only one-half of its length need be considered. The origin is chosen at the left end with the axes directed as shown.

It is assumed that lateral torsional buckling of the beam-column is effectively prevented so that failure is always caused by
excessive bending in the plane of the applied loads. The thrust \( P \) with an eccentricity \( e \) at the ends of the beam-column is assumed to be applied first and maintained at a constant value. The lateral load \( Q \) is then gradually increased from zero to its maximum value and subsequently allowed to drop off steadily.

The beam-column behaves, under loads \( P \) and \( Q \), in one of the six different ways shown in Fig. 1. In the zone marked 'Primary Plastic' the compression fibers only have yielded, whereas in the zone marked 'Secondary Plastic', the tension fibers also have yielded.

The computer program establishes the relationship between \( Q \) and the central curvature. Analytical expressions are needed to relate curvature and the corresponding moment on the one hand and the lateral load \( Q \) and the central curvature (through the central moment) on the other. The former are discussed in the next section and the latter in the succeeding one.

3. **GENERALIZED MOMENT-CURVATURE-THRUST RELATIONSHIPS**

If \( M, P, \) and \( \phi \) are the actual moment, thrust, and curvature, respectively, the corresponding quantities non-dimensionalized with respect to conditions at initial yield for a given cross section may be represented by

\[
m = \frac{M}{M_y}, \quad p = \frac{P}{P_y}, \quad \varphi = \frac{\phi}{\phi_y}
\]  

A general \( m-\varphi-p \) curve of a common structural section with or without the influence of residual stresses usually has the shape shown
in Fig. 2. The curve sketched is for a constant value of \( p \). Similar curves for higher \( p \)-values would be below such a curve, while those for lower \( p \)-values would lie above it.

The curve may be divided into three regimes: Elastic, primary plastic, and secondary plastic. These are separated by the points \((m_1, \varphi_1)\) and \((m_2, \varphi_2)\) in the figure. As \( \varphi \) tends to infinity, the moment \( m \) approaches the value \( m_{pc} \) asymptotically. The value of \( m_{pc} \) is the maximum (non-dimensionalized) moment that the section can be subjected to in the presence of the thrust \( p \).

The \( m-\varphi-p \) curve is assumed to be closely represented by the following expressions, wherein \( a, b, c, \) and \( f \) are constants [4].

**Elastic regime**

\[
m = a \varphi \quad \text{valid for } 0 \leq \varphi \leq \varphi_1 \quad (2)
\]

**Primary Plastic regime**

\[
m = b - \frac{c}{\varphi^{1/2}} \quad \text{valid for } \varphi_1 \leq \varphi \leq \varphi_2 \quad (3)
\]

**Secondary Plastic regime**

\[
m = m_{pc} - \frac{f}{\varphi^2} \quad \text{valid for } \varphi_2 \leq \varphi \leq \infty \quad (4)
\]

Equations can be established for the constants \( a, b, c, \) and \( f \) in terms of \( m_1, m_2, m_{pc}, \varphi_1, \) and \( \varphi_2 \) using these relations. The solutions of these equations lead to the following expressions for the constants

\[
a = \frac{m_1}{\varphi_1} \quad (5)
\]
The process of curve fitting thus involves the parameters $m_1$, $q_1$, $m_2$, $q_2$, $q_{pc}$. The term $m_{pc}$ can be computed directly in terms of $p$ using standard expressions. It is recalled that $m_{pc}$ is independent of the residual stress distribution over the cross section.

The choice of the parameters which are functions of $p$ is explained in detail in an earlier paper [4]. For the following six cross sections their values are listed in Table 1.

<table>
<thead>
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<th>NTYPE</th>
<th>Description</th>
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<td>1</td>
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<td>Square tubular section (without residual stresses)</td>
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If $\text{NTYPE} = 1$ (rectangle), the expressions are simple and exact [3]. The moment $m$ may be expressed directly in terms of curvature $\varphi$ and thrust $p$, since the constants are simple functions of $p$.

**In the elastic range**

\[ m = \varphi \quad \text{valid for } 0 \leq \varphi \leq 1-p \quad (9) \]

**In the primary plastic range**

\[ m = 3 (1-p) - \frac{2}{\varphi^{1/2}} (1-p)^{3/2} \quad \text{valid for } 1-p \leq \varphi \leq \frac{1}{1-p} \quad (10) \]

**In the secondary plastic range**

\[ m = \frac{3}{2} (1-p^2) - \frac{1}{2\varphi^{3/2}} \quad \text{valid for } \frac{1}{1-p} \leq \varphi \leq \infty \quad (11) \]

Hence,

\[ a = 1 \]

\[ b = 3 (1-p) \]

\[ c = 2 (1-p)^{3/2} \quad (12) \]

\[ f = 0.5 \]

\[ m_{pc} = \frac{3}{2} (1-p^2) \]

If $\text{NTYPE} = 2$ [idealized (very thin flange elements) wide-flange shape (without residual stresses) bent about the strong axis], an additional parameter $R$, called the shape variable, is necessary to distinguish between the existence and non-existence of the secondary plastic regime [3].
If \( p \geq \frac{1}{1+R} \),

\[
a = 1 \quad \text{(13)}
\]

\[
b = \frac{3}{1+3R} \frac{(1+R)}{1-p} \quad \text{(14)}
\]

\[
c = \frac{2}{1+3R} (1-p)^{3/2} \quad \text{(15)}
\]

\[
\varphi_1 = 1-p \quad \text{(16)}
\]

\[
\varphi_2 = \infty \quad \text{(17)}
\]

In this case, therefore, there is no secondary plastic regime.

If \( p \leq \frac{1}{1+R} \),

\[
a = 1 \quad \text{(18)}
\]

\[
b = 1-p + \frac{\lambda}{\mu} \quad \text{(19)}
\]

\[
c = (1-p)^{1/2} \frac{\lambda}{\mu} \quad \text{(20)}
\]

where

\[
\lambda = 2 \ p \ [1 + 2R - (1+R)^2 \ p] \quad \text{(21)}
\]

and

\[
\mu = (1+3R) \left\{ 1 - (1-p)^{1/2} \left[ 1 - (1+R) \ p \right]^{1/2} \right\} \quad \text{(22)}
\]

\[
f = \frac{1}{2} \ \frac{1}{(1+3R)} \quad \text{(23)}
\]

\[
m_{pc} = \frac{3}{2} \ \frac{1 + 2R - (1+R)^2 \ p^2}{1 + 3R} \quad \text{(24)}
\]

\[
\varphi_1 = 1-p \quad \text{(25)}
\]

\[
\varphi_2 = \frac{1}{1 - (1+R) \ p} \quad \text{(26)}
\]
For the special case of $p = 0$, since $\frac{\lambda}{\mu}$ assumes the form $0/0$, the values of $b$ and $c$, by the use of L'Hospital's rule, are

$$c = \frac{4}{2 + R} \frac{1 + 2R}{1 + 3R}$$  \hspace{1cm} (27)

$$b = 1 + c$$  \hspace{1cm} (28)

If $R$ is zero (and since $p$ is always less than 1), Eqs. (18) through (28) reduce to those of a solid rectangular section. The value of $R$ for good curve-fitting has been shown to lie between 1.0 and 1.4 [3].

4. LOAD-CURVATURE RELATIONSHIPS

The basic differential equation for the bending of a beam-column is

$$\frac{d^2 m}{dx^2} + k^2 \varphi = 0$$  \hspace{1cm} (29)

where $k^2 = P/EI$ and $EI$ is the flexural rigidity of the section in the plane of bending.

This equation may be combined with Eqs. (2), (3), and (4) for $m$ in terms of $\varphi$ presented in the previous chapter. The corresponding equations in the three zones are:

**Elastic Zone**

$$\frac{d^2 \varphi}{dx^2} + \frac{k^2}{a} \varphi = 0$$  \hspace{1cm} (30)

**Primary Plastic Zone**

$$\varphi \frac{d^2 \varphi}{dx^2} - \frac{3}{2} \left( \frac{d \varphi}{dx} \right)^2 + \frac{2k^2}{c} \varphi \gamma^2 = 0$$  \hspace{1cm} (31)
**Secondary Plastic Zone**

\[
\varphi \frac{d^2 \varphi}{dx^2} - 3 \left( \frac{d \varphi}{dx} \right)^2 + \frac{k_a^2}{2 f} \varphi^5 = 0
\]  

(32)

The general solutions of these equations are, respectively,

\[
\varphi = A \cos \frac{kx}{\sqrt{a}} + B \sin \frac{kx}{\sqrt{a}}
\]  

(33)

\[
x - x_p = - \frac{\sqrt{c}}{\sqrt{2k}} \frac{1}{D} \left[ \frac{D}{\varphi} \right]^{1/2} + \frac{1}{\sqrt{D}} \tanh^{-1} \frac{\left( D - \frac{\varphi}{D} \right)^{1/2}}{D^{1/2}}
\]  

(34)

\[
x - x_s = \frac{2}{3} \frac{\sqrt{f}}{k} \left( G + \frac{1}{\varphi} \right) \left( 2G - \frac{1}{\varphi} \right)
\]  

(35)

where \( A, B, D, G, x_p, \) and \( x_s \) are integration constants.

These constants can be evaluated from known boundary conditions for each zone (elastic, primary plastic, and secondary plastic) as well as the conditions at the end and at the midspan of the beam-column. The reader is referred to an earlier paper for details [3]. The final resulting equations for each case (Fig. 1) are listed here and are the basic building blocks for the computer program. Additional symbols which appear in these equations have been defined under Nomenclature.

For cases 1 through 6,

\[
m_o = \frac{1}{2} p \left( \frac{\sigma_y}{E} \right) \left( \frac{h}{r} \right)
\]  

(36)

\[
kL = \frac{L}{r} \left( \frac{p \sigma_y}{E} \right)^{1/2}
\]  

(37)
Case 1 \((0 \leq \varphi_m \leq \varphi_1)\)

\(q - \varphi_m\) relation

If \(p \neq 0\), \(\varphi_m = \left[ \frac{m_0}{a} + \frac{2aq}{\sqrt{a} \ k \ t} \sin \frac{k \ t}{2/a} \right] \sec \frac{k \ t}{2/a} \) \hspace{1cm} (38)

If \(p = 0\), \(\varphi_m = \frac{aq}{a} \) \hspace{1cm} (39)

\(x - \varphi\) relation \((0 \leq \frac{x}{\ell} \leq 0.5, \varphi_o \leq \varphi \leq \varphi_m)\)

If \(p \neq 0\), \(\varphi = \left[ \frac{m_0}{a} \cos \left\{ k \ell \left( 1 - 2 \frac{x}{\ell} \right) \frac{1}{2/a} \right\} \right. \hspace{1cm} (40)

+ \frac{2aq}{\sqrt{a} \ k \ t} \sin \left( k \ell \cdot \frac{x}{\ell} \cdot \frac{1}{2/a} \right) \sec \frac{k \ t}{2/a} \]

If \(p = 0\), \(\varphi = \frac{2aq}{a} \frac{x}{\ell} \) \hspace{1cm} (41)

Case 2 \((\varphi_1 \leq \varphi_m \leq \varphi_2)\)

\(q - \varphi_m\) relation

\(\varphi_m = \varphi_1 \eta^2 \left[ 1 - \frac{\eta^2}{(k \ell)^2} \right] \) \hspace{1cm} (42)

where

\(\eta = 1 + \frac{aq_1^{3/2}}{2c} \left[ \cot \left( \frac{1}{\sqrt{a}} \cdot k \ell \cdot \frac{\rho_1}{\ell} \right) - \frac{m_0}{a \varphi_1} \csc \left( \frac{1}{\sqrt{a}} \cdot k \ell \cdot \frac{\rho_1}{\ell} \right) \right] \) \hspace{1cm} (43)

\(\xi = \sqrt{2} \frac{aq_1^{1/4}}{\sqrt{c} \varphi_1^{1/2} \eta^{1/2}} \) \hspace{1cm} (44)
and the following equation is satisfied:

$$\frac{k\ell}{2} + \sqrt{\frac{\ell}{2}} \frac{1}{\phi_1^\frac{3}{4}} \eta^{\frac{1}{2}} \left[ \frac{k\ell \cdot \xi}{(k\ell)^2 - \xi^2} + \tanh^{-1} \frac{\xi}{k\ell} \right]$$

$$- k\ell \cdot \frac{\rho_1}{\ell} - \sqrt{\frac{\ell}{2}} \frac{1}{\phi_1^\frac{3}{4}} \eta^{\frac{1}{2}} \left[ \eta^{-\frac{1}{2}} \eta^{-\frac{1}{2}} + \tanh^{-1} \left( 1 - \frac{1}{\eta} \right)^{\frac{1}{2}} \right] = 0$$

(45)

$$\rho_1$$ is the length of the elastic zone ($$0 \leq \frac{\rho_1}{\ell} \leq 0.5$$, Fig. 1)

**x - q relations**

**Elastic zone** ($$0 \leq \frac{x}{\ell} \leq \frac{\rho_1}{\ell}$$, $$\varphi_0 \leq \varphi \leq \varphi_1$$)

$$\varphi = \left[ \frac{m}{\alpha} \sin \left( \frac{1}{\sqrt{\alpha}} k\ell \left( \frac{\rho_1}{\ell} - \frac{x}{\ell} \right) \right) + \varphi_1 \sin \left( \frac{1}{\sqrt{\alpha}} k\ell \cdot \frac{x}{\ell} \right) \right] \csc \left( \frac{1}{\sqrt{\alpha}} \cdot k\ell \cdot \frac{\rho_1}{\ell} \right)$$

(46)

**Primary plastic zone** ($$\frac{\rho_1}{\ell} \leq \frac{x}{\ell} \leq 0.5$$, $$\varphi_1 \leq \varphi \leq \varphi_m$$)

$$\frac{x}{\ell} = \frac{1}{2} - \sqrt{\frac{\ell}{2}} \frac{1}{\phi_1^\frac{3}{4}} \eta^{\frac{1}{2}} \left[ \eta^{-\frac{1}{2}} \left\{ \frac{\varphi_1}{\varphi} \eta^{-\frac{1}{2}} - \frac{\varphi_1}{\varphi} \right\} \right]^{\frac{1}{2}} - \frac{k\ell \cdot \xi}{(k\ell)^2 - \xi^2}$$

$$+ \tanh^{-1} \left\{ 1 - \left( \frac{\varphi}{\varphi_1} \right) \frac{1}{\eta} \right\}^{\frac{1}{2}} - \tanh^{-1} \frac{\xi}{k\ell}$$

(47)

**Case 3** ($$\varphi_1 \leq \varphi_m \leq \varphi_2$$)

$$q - \varphi_m$$ relation
\[
\frac{\sqrt{2c}}{\left(\varphi_m + R\right)^{3/2}} \left[ \left(\varphi_m^{1/2} + R\right)^{1/2} \left(\frac{\varphi_m^{1/2} - \varphi_o^{1/2} + R}{\varphi_o^{1/2} - \varphi_m^{1/2}} \right) - \frac{1}{R} \right]
\]

\[
+ \tanh^{-1} \left( 1 - \frac{\varphi_o^{1/2}}{\varphi_m^{1/2} + R} \right)
\]

\[- \tanh^{-1} \left( \frac{1}{\varphi_m^{1/2} + R} \right) - k\ell = 0 \tag{48}\]

where

\[
R^{1/2} = \frac{\sqrt{2}}{\sqrt{c}} \left( \frac{2q}{k\ell} \right) \tag{49}\]

and

\[
\varphi_o = \text{end curvature} = \frac{c^2}{(b - m_o)^2}, \quad (\varphi_1 \leq \varphi_o \leq \varphi_m) \tag{50}\]

\[
\text{x - } \varphi \text{ relation} \quad (0 \leq \frac{x}{\ell} \leq 0.5, \ \varphi_o \leq \varphi \leq \varphi_m)
\]

\[
\frac{x}{\ell} = \frac{x_p}{\ell} - \frac{\sqrt{c}}{\sqrt{2} k\ell \left(\varphi_m^{1/2} + R\right)^{3/2}} \left[ \left(\varphi_m^{1/2} - \varphi^{1/2} + R\right)^{1/2} \left(\varphi_m^{1/2} + R\right)^{1/2} - \frac{1}{R} \right]
\]

\[- \tanh^{-1} \left( 1 - \frac{1/2}{\varphi_m^{1/2} + R} \right) \tag{51} \]

where

\[
\frac{x_p}{\ell} = 0.5 + \frac{\sqrt{c}}{\sqrt{2} k\ell \left(\varphi_m^{1/2} + R\right)^{3/2}} \left[ \frac{1}{R} \left(\varphi_m^{1/2} + R\right)^{1/2} \right] + \tanh^{-1} \left( \frac{1}{\varphi_m^{1/2} + R} \right) \tag{52} \]
Case 4 \( (\varphi_2 \leq \varphi_m \leq \infty) \)

\[ q - \varphi_m \text{ relation} \]

If \( p \neq 0 \), \( \varphi_m = \frac{\varphi_2}{\zeta + \frac{c}{2f} R \varphi_2} \) \hspace{1cm} (53)

where

\[ \zeta = 1 - \frac{c}{2f} \varphi_2 \left( \varphi_1^{1/2} \eta - \varphi_2^{1/2} \right) \] \hspace{1cm} (54)

\[ R = \frac{2}{c} \left( \frac{\alpha q^2}{k \ell^2} \right) \] \hspace{1cm} (55)

and the following equation is satisfied

\[ (k \ell)^4 - 2\left[ k \ell \cdot \frac{\rho_2}{\ell} + \frac{2}{3} \frac{\sqrt{c}}{\varphi_2^{3/2}} \left( 1 - \zeta \right) \varphi_2^{1/2} \right] (k \ell)^3 \]

\[ + \frac{4\rho_2 \zeta}{\varphi_2} (k \ell)^2 + \frac{4(\alpha q)^3}{3f} = 0 \] \hspace{1cm} (56)

where \( \rho_2 \), the distance to the boundary of the secondary plastic zone from the end (Fig. 1), is given by

\[ k \ell \cdot \frac{\rho_2}{\ell} = k \ell \cdot \frac{\rho_1}{\ell} + \frac{\sqrt{c}}{\sqrt{2}} \frac{1}{\eta^{3/2}} \varphi_1^{3/4} \left[ \eta^{1/2} \left( \eta - 1 \right) \varphi_1^{1/2} \right] \]

\[ - \left( \frac{\eta^2 \varphi_1}{\varphi_2} - \frac{\eta \varphi_1^{1/2}}{\varphi_2^{1/2}} \right) \]

\[ + \tanh^{-1} \left( 1 - \frac{1}{\eta} \right)^{1/2} - \tanh^{-1} \left( 1 - \frac{\varphi_2}{\eta \varphi_1^{1/2}} \right) \] \hspace{1cm} (57)

The expression for \( \eta \) is the same as in Case 2 (Eq. (43))
If \( p = 0 \), \((\varphi_1 = \varphi_2, \rho_1 = \rho_2)\)
\[
\varphi_m = \frac{f}{\alpha (1 - q)} \tag{58}
\]
and
\[
q = \frac{a \varphi_2}{2\alpha} \frac{1}{\frac{\rho_2}{\ell}} \tag{59}
\]

\(x - \varphi\) relations

If \( p \neq 0\)

**Elastic zone**  Same as for Case 2 (Eq. (46))

**Primary plastic zone** \((\frac{\varphi_1}{\ell} \leq \frac{x}{\ell} \leq \frac{\varphi_2}{\ell}, \varphi_1 \leq \varphi \leq \varphi_2)\)

\[
\frac{x}{\ell} = \frac{\rho_1}{\ell} + \frac{\sqrt{2}}{k \ell} \frac{1}{\eta} \frac{1}{\varphi_1 \varphi_2^{1/4}} \left[ \frac{1}{\eta} (\eta - 1)^{1/3} - \left( \frac{\eta}{\varphi} \right)^{1/3} \frac{1}{\eta} \frac{1}{\varphi_1^{1/3}} \right] \\
+ \tanh^{-1} \left( \frac{1}{\eta} \right)^{1/2} - \tanh^{-1} \left( \frac{1}{\eta} \frac{1}{\varphi_1^{1/3}} \right) \tag{60}
\]

**Secondary plastic zone** \((\frac{\varphi_2}{\ell} \leq \frac{x}{\ell} \leq 0.5, \varphi_2 \leq \varphi \leq \varphi_m)\)

\[
\frac{x}{\ell} = 0.5 - \frac{2\sqrt{k}}{3} \frac{1}{k \ell} \left[ \left( \frac{1}{\varphi} - \frac{\ell}{\varphi_2} \right)^{1/2} \left( \frac{1}{\varphi} + \frac{2\varphi}{\varphi_2} \right) \\
- \left( \frac{1}{\varphi_m} - \frac{\ell}{\varphi_2} \right)^{1/2} \left( \frac{1}{\varphi_m} + \frac{2\varphi}{\varphi_2} \right) \right] \tag{61}
\]
If $p = 0$, Elastic zone \[
\left[ 0 \leq \frac{x}{k} \leq \left( \frac{\rho_1}{k} = \frac{\rho_2}{k} \right), \quad 0 \leq \varphi \leq \varphi_1 \right] = \varphi_2 \right] \]

\[
\varphi = \frac{x}{k} \frac{\varphi_1}{\rho_1} \frac{\varphi_2}{\rho_2} \tag{62}
\]

Secondary plastic zone \[
\left[ \frac{\rho_1}{k} = \frac{\rho_2}{k} \right] \leq \frac{x}{k} \leq 0.5, \quad (\varphi = \varphi_2) \leq \varphi \leq \varphi_m \right] \]

\[
\frac{x}{k} = \frac{1}{2q} \left( 1 + \frac{f_1}{\alpha q^2} \right) \tag{63}
\]

Case 5 \((\varphi_2 \leq \varphi_m \leq \infty)\)

\[q - \varphi_m \text{ relation} \]

\[
\frac{2}{3} \sqrt{f} \left[ \frac{1}{\varphi_2} - \frac{1}{\varphi_m} + \frac{1}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \right]^{1/2} \left[ \frac{1}{\varphi_2} + \frac{2}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \right]^{1/2} - \frac{2}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \left[ \frac{1}{\varphi_2} - \frac{2}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \right]^{1/2} + \frac{\sqrt{c}}{\sqrt{2}} \left( \frac{D - \varphi_2^{1/2}}{\varphi_2^{1/2}} \right)^{1/2} + \frac{1}{D^{1/2}} \left\{ \tanh^{-1} \left( 1 - \frac{\varphi_2^{1/2}}{D} \right) \right\}
\]

where \[
D = \varphi_2^{1/2} + \frac{2f_1}{c \varphi_2} \left[ 1 - \frac{\varphi_2}{\varphi_m} + \frac{\varphi_2}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \right] \tag{64}
\]

and $\varphi_o$ is as in Case 3 (Eq. (50)).

The distance $\rho_2 \left( 0 \leq \frac{\rho_2}{k} \leq 0.5 \right)$ is given by
\[ k \ell \cdot \frac{\rho_a}{\ell} = \frac{k \ell}{2} - \frac{2}{3} \sqrt{f} \left[ \frac{cR}{2f} + \frac{1}{\phi_a} - \frac{1}{\phi_m} \right]^{1/2} \left[ \frac{1}{\phi_a} + \frac{2}{\phi_m} - \frac{cR}{f} \right] + \sqrt{2cR} \left[ \frac{1}{\phi_m} - \frac{cR}{3f} \right] \]

(66)

with

\[ \frac{1}{\phi_m} = \frac{1}{\sqrt{c \cdot \frac{\alpha q}{k \ell}}} \]

(67)

\[ x - \phi \] relations

**Primary plastic zone** \[ 0 \leq \frac{x}{\ell} \leq \frac{\rho_a}{\ell}, \quad \phi_0 \leq \phi \leq \phi_a \]

\[ \frac{x}{\ell} = \frac{\sqrt{c}}{\sqrt{2}} \frac{1_{1/2}}{k \ell D} \left[ \left( D - \frac{\phi_a}{\phi} \right)^{1/2} - \left( D - \frac{\phi_a}{\phi_0} \right)^{1/2} \right] + \frac{1}{\phi^{1/2}} \left\{ \tanh^{-1} \left( 1 - \frac{\phi_a}{D} \right) - \tanh^{-1} \left( 1 - \frac{\phi_a}{D} \right) \right\} \]

(68)

**Secondary plastic zone** \[ \frac{\rho_a}{\ell} \leq \frac{x}{\ell} \leq 0.5, \quad \phi_a \leq \phi \leq \phi_m \]

\[ \frac{x}{\ell} = \frac{1}{2} - \frac{2}{3} \sqrt{f} \left[ \frac{1}{\phi} - \frac{1}{\phi_m} + \frac{1}{f} \frac{\alpha q}{k \ell} \right]^{1/2} \left[ \frac{1}{\phi_m} - \frac{2}{\phi_m} - \frac{2}{f} \left( \frac{\alpha q}{k \ell} \right)^2 \right] + \frac{2\alpha q}{(k \ell)^2} \left[ \frac{1}{\phi_m^2} - \frac{2}{3f} \left( \frac{\alpha q}{k \ell} \right)^2 \right] \]

(69)

**Case 6** \( \phi_a \leq \phi_m \leq \infty \)

\[ q - \phi_m \] relation
\[2 \left( \frac{\alpha q}{k_\ell} \right) \left[ \frac{2}{3} f \left( \frac{\alpha q}{k_\ell} \right)^2 \right] \frac{1}{\varphi_m} \]

\[+ \frac{2}{3} \sqrt{f} \left[ \frac{1}{\varphi_o} - \frac{1}{\varphi_m} + \frac{1}{f} \left( \frac{\alpha q}{k_\ell} \right)^2 \right]^{1/2} \left[ \frac{1}{\varphi_o} + \frac{2}{f} \left( \frac{\alpha q}{k_\ell} \right)^2 \right] - \frac{k_\ell}{2} = 0 \]  

\[\text{where} \]

\[\varphi_o = \left[ \frac{f}{m_{pc} - m_o} \right]^{1/2}, \quad \left[ \varphi_{o_1} = \varphi_o \leq \varphi \leq \varphi_m \right] \]  

\[x - \varphi \text{ relation} \quad \left[ 0 \leq \frac{x}{\ell} \leq 0.5, \varphi_o \leq \varphi \leq \varphi_m \right] \]

Secondary plastic zone

\[\frac{x}{\ell} = \frac{1}{2} + 2 \frac{\alpha q}{(k_\ell)^2} \left[ \frac{1}{\varphi_m} - \frac{2}{3} f \left( \frac{\alpha q}{k_\ell} \right)^2 \right] \]

\[+ \frac{2}{3} \sqrt{f} \left[ \frac{1}{\varphi_o} - \frac{1}{\varphi_m} + \frac{1}{f} \left( \frac{\alpha q}{k_\ell} \right)^2 \right]^{1/2} \left[ \frac{1}{\varphi_o} + \frac{2}{f} \left( \frac{\alpha q}{k_\ell} \right)^2 \right] \]  

5. FURTHER CLASSIFICATION OF BEAM-COLUMN BEHAVIOR

In Fig. 1 the behavior of the beam-column under the loads has been classified into six different cases depending on the nature and extent of yielding.

The behavior may also be classified into three groups based on conditions at the ends of the beam-column due to initial loading \((q = 0)\) to facilitate programming.

One of the assumptions in the problem is that the thrust ratio \((p)\) and the eccentricity ratio \((\frac{e}{r})\) are held constant as the transverse
load ratio $q$ is varied. (Since only ratios, which are non-dimensionalized quantities, will be referenced throughout most of this text, the term ratio itself will be dropped in further discussion). The transverse load does not alter the curvature at the ends of the beam-column since these are simple supports. Hence, the end curvature $\phi_0$ is invariant throughout the solution of the problem. Its magnitude depends only on the value of the end moment $m_o$. [Further, since the beam-column has some finite length (other than zero), $\phi_m$, the curvature at midspan, is never less than $\phi_0$, the curvature at the ends].

The curvature $\phi_0$ may be in one of the following three ranges:

a. $0 \leq \phi_0 \leq \phi_1$  
   Ends are elastic (Group 1)

b. $\phi_1 \leq \phi_0 \leq \phi_2$  
   Ends are in primary plastic zone (Group 2)

c. $\phi_2 \leq \phi_0 \leq \infty$  
   Ends are in secondary plastic zone (Group 3)

If the ends are elastic (Group 1) under the action of $p$ alone, the behavior under the transverse load $q$ is confined to one of the three following cases:

Case 1. Beam-column is elastic throughout. ($0 \leq \phi_m \leq \phi_1$)

Case 2. A central portion of the beam-column is in the primary plastic zone. ($\phi_1 \leq \phi_m \leq \phi_2$)

Case 4. A central portion of the beam-column is in the secondary plastic zone. ($\phi_2 \leq \phi_m \leq \infty$)

Similarly, Group 2 includes Cases 3 and 5 while Group 3 includes Case 6 only.

The order in which the several cases in a specific group may occur with increasing $\phi_m$ is also of importance in programming.
If the beam-column is elastic throughout under thrust p alone, its behavior under the transverse load q is initially governed by Case 1. The addition of the lateral load q will increase $\varphi_m$ and for some value, say $q = q_1$, $\varphi_m$ will attain the value $\varphi_1$. That is, the top fibers begin to yield and the primary plastic zone begins to spread from the center towards the ends. Hence, the behavior is now governed by Case 2. If $\varphi_m$ can further be increased to $\varphi_2$, the bottom fibers also begin to yield and the behavior is finally governed by Case 4.

The initial loading conditions ($p$ and $\frac{p}{r}$) and the geometry of the beam-column ($\frac{L}{r}$) may, however, be such that Case 2 governs at start. That is, with $q = 0$, $\varphi_1 < \varphi_m < \varphi_2$. Subsequent behavior under $q$ is then confined to this case and perhaps Case 4.

By similar reasoning, the following table can be constructed:

**Group 1**

<table>
<thead>
<tr>
<th>Starting Case</th>
<th>Subsequent Cases in Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Cases 1, 2, and 4</td>
</tr>
<tr>
<td>Case 2</td>
<td>Cases 2 and 4</td>
</tr>
<tr>
<td>Case 4</td>
<td>Case 4</td>
</tr>
</tbody>
</table>

**Group 2**

<table>
<thead>
<tr>
<th>Starting Case</th>
<th>Subsequent Cases in Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 3</td>
<td>Cases 3 and 5</td>
</tr>
<tr>
<td>Case 5</td>
<td>Case 5</td>
</tr>
</tbody>
</table>
The starting case is decided by the combination of the three factors $p$, $\frac{e}{r}$, and $\frac{d}{r}$ for a given set of material properties. Among the subsequent cases noted previously all of them may be covered in the order indicated or the beam-column may unload, that is, $q$ becomes negative and only the first few cases will be covered. The exception to this statement will be noted when Case 1 is discussed.

The maximum value of $q$ may be attained in any one of the cases. Again, Case 1 is an exception.

6. **INTRODUCTION TO THE COMPUTER PROGRAM**

A. **Major Details of the Program**

The computer program is actually a set of subprograms. In this set SUBROUTINE BMCOL (INTER) must be CALL-ed by the user's main program. This subprogram acts in a supervisory capacity guiding the path of calculations.

The length (in octal numbers) of each subprogram, using the RUN compiler of the CDC 6400 installation at Lehigh University, is as follows:
Beginning with the next chapter, the algorithm of the solution will be presented covering Cases 1, 2, 4, 3, 5, and 6 in that order. Frequent references to the flow-chart (Fig. 3) are desirable. The subprograms themselves will be discussed independently at a later stage to complete the coverage.

B. Preliminary Steps

The details regarding input to the routine will be explained in the User's Guide. For now, it is assumed that when SUBROUTINE BMCOL (INTER) is entered all the necessary details of the problem are available. [For the sake of brevity, in further references to the subprograms, only the corresponding names will be used in the following form: SR BMCOL instead of SUBROUTINE BMCOL (INTER)].

At the end of the compilation of the user's program along with this set of subprograms, certain arrays (IVAR, JVAR, AND LVAR) which store FORMAT statements and a variable called THIRD will have been initialized by the BLOCK DATA subprogram.
Besides initializing certain other variables, the first task of SR BMCOL is to produce an echo of the input data and print the values of the coefficients a, b, c, f etc. referred to in Chapter 3. These coefficients are functions of \( p \) and the nature of the section (NTYPE). The equations listed in Table 1 and in Chapter 3 are used.

The variable FORMAT arrays are also suitably modified to accommodate the user's request for a printout of the end slope and central deflection of the beam-column for all values of \( q \).

Next, the values of two important quantities \( k \ell \) and \( m_0 \) are checked to determine the existence or otherwise of a solution to the problem defined. The end moment \( m_0 \) has the magnitude \( \frac{1}{2} p \left( \frac{C}{r} \right) \left( \frac{h}{r} \right) \). If this exceeds \( m_{pc} \), there is obviously no solution to the problem. From \( k^2 = \frac{P}{EI} \) the value of \( k \ell \) is \( \frac{\ell}{r} \left( \frac{p\sigma_y}{E} \right)^{1/2} \). If \( k \ell \) exceeds \( \pi \), the implication is that the elastic buckling load of the column is exceeded. Again, there is no solution. Under either of these conditions SR BMCOL returns control to the main program after printing corresponding messages and resetting the variable FORMAT arrays.

C. Use of the Argument INTER

If \( m_0 \geq m_{pc} \) or \( k \ell \geq \pi \) for a specific set of values \( p, \frac{\ell}{r}, \frac{C}{r} \), it is evident that for a subsequent higher set of values for these variables there will be no solution. For example, if \( p = 0.60 \) and there is no solution, the limit of \( p \) has been reached, other parameters remaining the same.
In SR BMCOL, INTER is initially set to the value 1. This value is not modified if no solution can exist for the set of input data. Otherwise, it is reset to 0.

Assume now that the user is generating results for increasing values of, say, \( p \) in a DO-loop in the main program (the discussion is equally applicable if the parameter is \( e/r \) or \( l/r \)). After each CALL to SR BMCOL the value of INTER should be checked. If INTER \( \neq 0 \), further CALL-s in the loop must be INTERRUpted, as in the following program section.

```fortran
DO 1000 NP = 1, 20
  P = NP - 1
  P = 0.05 * P
  CALL BMCOL (INTER)
  IF (INTER .NE. 0) GO TO 2000
1000 CONTINUE
2000 CONTINUE
```

Other cases where such an interruption is desirable will be pointed out when the several cases are discussed.

D. General Approach

In general, solutions will be sought if \( m_o < m_{pc} \) and \( k \ell < \pi \). The basic logic is to decide the group first and the case next by a process of successive eliminations.

From Eqs. (40) and (46) (also for Case 4) with \( x = 0 \) the end curvature \( \phi_0 \) is \( \frac{m_o}{a} \) when Group 1 controls. Conversely, therefore, if \( \frac{m_o}{a} \leq \phi_1 \), the behavior is governed by Cases 1, 2, and/or 4 which belong to Group 1.
If $\frac{m_0}{a} > \varphi_1$, Group 2 or Group 3 must control. If $\varphi_0$, as given by Eq. (50) (also for Case 5) does not exceed $\varphi_2$, Group 2 controls. Else, finally, Group 3 controls and $\varphi_0$ has the value given by Eq. (71).

Similarly, the value of $\varphi_m$ with $q = 0$ indicates the starting case.

If $p = 0$, $m_0 = \varphi_0 = 0$ (Eqs. (41) and (62)) and, hence, $\varphi_0$ is always less than $\varphi_1$. Further, $\varphi_m$ (with $q = 0$) = 0 (Eq. (39)). Hence, $\varphi_m < \varphi_1$. Thus, if $p = 0$, even the starting case (Case 1) is directly known. More simply, if $p = 0$, there is no load and the behavior is elastic initially under load $q$.

7. SOLUTION - CASE 1

A. The Special Case of $p = 0$

Earlier discussion has shown that when $p = 0$, Case 1 is the starting case. The initial values $q = 0$ and $\varphi_m = 0$ are first printed. The end slope and central deflection are also zero. These are printed only if the user has requested a printout of end slope ($\theta_o$) and central deflection ($\delta/h$) through setting INDEX (input parameter) to a non-zero integer value (from 1 to 9).

Equation (39) shows that $\varphi_m$ is a linear function of $q$. Since one terminal point on the $q - \varphi_m$ curve is available ($q = \varphi_m = 0$), the response in Case 1 is completely defined when the other terminal point is known. By setting $\varphi_m = \varphi_1$ the limiting value of Case 1, $q = \frac{a\varphi_1}{\alpha}$, can be evaluated. The values $q$ and $\varphi_m$ are printed next.
If \( \theta_0 \) and \( \delta/h \) have been requested, SR EMCOL CALL-s SR XVSFI to compute these quantities and prints them out. The details of these calculations are presented in the discussion under SR XVSFI.

B. The General Case with \( p \neq 0 \)

It is assumed that the check of \( \varphi_0 \) (Eq. (40), \( x = 0 \)) has indicated that the solution is in Group 1, that is, \( \varphi_0 \leq \varphi_1 \).

Case 1 will be the starting case if \( \varphi_m \) (with \( q = 0 \)) \( \leq \varphi_1 \) (Eq. (38)). In SR EMCOL, therefore, \( \varphi_m \) is evaluated using Eq. (38) and its value checked against \( \varphi_1 \). If \( \varphi_m > \varphi_1 \), the solution will be sought in Case 2.

If \( \varphi_m \leq \varphi_1 \), the initial values \( q = 0 \) and \( \varphi_m \) are printed. If \( \theta_0 \) and \( \delta/h \) are requested, these are made available. Equation (38) also shows that \( \varphi_m \) has a linear relationship with \( q \). The second terminal point on the \( q - \varphi_m \) curve is, hence, easily established by setting \( \varphi_m = \varphi_1 \) in Eq. (38) and computing \( q \).

\[
q = \left( \varphi_1 \cos \left( \frac{kt}{2a} \right) - \frac{m_0}{a} \right) \frac{kt/a}{2\alpha} \csc \frac{kt}{2a}
\]  
(73)

Values \( q \) and \( \varphi_m \) are then printed. The values of \( \theta_0 \) and \( \delta/h \) are made available, if requested.

C. Remarks

By virtue of the linear relationship between \( q \) and \( \varphi_m \) in Case 1, \( \frac{dq}{d\varphi_m} \) is a constant (which is positive). Hence, \( q \) does not attain a maximum value in Case 1. Neither does \( q \) become negative in Case 1.
8. SOLUTION - CASE 2

Of the six cases Case 2 requires the most programming effort. This is due to several factors which will become evident in subsequent discussions. Most operations are done in SR BC.

A. The Special Case of $p = 0$

When there is no thrust on the beam-column, the problem is indeed that of a simple beam. It was shown previously that Case 1 is the starting case when $p = 0$.

For a symmetrical section without residual stresses (say, an annealed section) the stress distribution under load $q$ will be symmetrical about the mid-height of the section considering magnitudes only. If the beam is loaded beyond Case 1, both the top and bottom fibers will yield together to the same length. In other words, $\rho_1 = \rho_2$. That is, Case 2 is non-existent when $p = 0$. This conclusion may also be reached considering that $\varphi_1 = \varphi_2$ if $p = 0$ for all the shapes.

For a $W$ section with residual stresses bent about the strong axis yielding is non-uniform along a line parallel to the neutral axis in the flanges. The flange tips at the top yield in compression first and yielding progresses towards the junction with the web later. A strict definition for $\rho_1$ is, hence, difficult. At the bottom yielding in tension begins at the junction and progresses towards the tips.

A consideration of all such details will lead to excessive complications in the solution. From a macro approach, as long as the
m-φ-p curves are reasonably well approximated, the overall theory is to be considered adequate, although not exact. An improvement even within the bounds of this theory seems possible when bending about the weak axis (with residual stresses) is involved.

For the present the assumption will be made that when p = 0 Case 2 is non-existent in the solution. Hence, Case 4 is the case to be considered after Case 1 when p = 0.

B. Approach to the Solution in Case 2

Equations (42) to (45) are to be used and, fundamentally, Eq. (45) must be satisfied. This equation is quite complicated and, hence, a trial and error approach will be adopted. By its transcendental nature it has many mathematical solutions and only the particular values that represent a practical solution to the problem on hand have to be chosen. Further, to define the q - φ_m curve adequately, enough points have to be obtained in the interval φ_1 ≤ φ_m ≤ φ_2.

In general, assume that a solution has been obtained for the situation when φ_1 (Fig. 1) is the length of the elastic portion, \( \frac{\rho_1}{\ell} \leq 0.5 \). This value of \( \rho_1/\ell \) is termed SMALL. The lateral load \( q \) is then known. The problem is to find a new value of \( q \) when the ratio \( \frac{\rho_1}{\ell} \) is decreased to a new value by a prescribed decrement (input data DRH012). The new ratio \( \frac{\rho_1}{\ell} \) is termed BIG.

The term \( \eta \) may be evaluated from Eq. (43) using the value of BIG. The new value of \( q \) is in the neighborhood of the previous \( q \) value (SMALLQ), which therefore serves as a trial value. Equation (44)
may then be used to obtain $\xi$. Using $\xi$ and $\eta$ so obtained, the left-hand side (LHS) of Eq. (45) may be evaluated. The result, say $FKL_1$ (a signed quantity), will obviously differ from zero.

The next trial value of $q$ is $BIGQ = SMALLQ + DQ$ where $DQ$ in SR BC has been assumed as 0.05. (The incremental value 0.05 has been found to be adequately small. Since Eq. (45) may have multiple solutions for $q$, a continuous $q - q_m$ curve cannot be ensured if $DQ$ has a much larger value. On the other hand, a much smaller value would slow down the entire process. However, if $SMALLQ$ is itself smaller than 0.05 but not zero, the value of $DQ$ is set to the value of $SMALLQ$).

Assume that the LHS of Eq. (45) now yields the result $FKL$. If $FKL_1 (FKL) = 0$, $FKL$ is zero and the solution for $q$ is the trial value $BIGQ$. This happenstance rarely ever occurs. If $FKL_1 (FKL)$ is positive, the LHS has not passed through the value zero in the range $SMALLQ \leq q \leq BIGQ$. Further, if $|FKL| > |FKL_1|$, the solution is diverging. The choice of $BIGQ$ was therefore wrong and the beam-column is unloading. The next trial value should therefore be $BIGQ = SMALLQ - DQ$. The solution must now converge. The inequality $|FKL| < |FKL_1|$ may, hence, be assured one way or the other.

The trial value of $q$ ($BIGQ$) may now be redesignated $SMALLQ$ and further incremented (or decremented, controlled by the sign of $DQ$) if $FKL_1 (FKL)$ was positive. The new trial value is $BIGQ$. The LHS is re-evaluated for this value. The result is again termed $FKL$. At some stage of this process $FKL_1 (FKL)$ will become negative. The LHS has
therefore passed through zero for a value of \( q \) in the range SMALLQ to BIGQ. (It is immaterial which of these is indeed small or big in magnitude).

Once the range is established the value of \( q \) that satisfies Eq. (45) may be found within the prescribed tolerance of TOL1 (input data) by CALL-ing SR HALVE. The process is known as the Method of Interval Halving which will be discussed under SR HALVE. Equation (42) then establishes the value of \( \varphi_m \) corresponding to this set of \( \frac{\rho}{t} \) and \( q \) values. The solution may now be continued by assuming a new value of \( \frac{\rho}{t} \). Evidently, a 'previous' value for \( q \) is a necessary requirement of this iterative process at each step. Once the process has been well started it can feed itself. If a continuation of Case 1 solution is sought, the value of \( q \) at the end of that solution will serve to start the iterative process. If, on the other hand, Case 2 is the starting case, \( q = 0 \) is the starting value.

C. Types of Solution in Case 2

A solution in Case 2 may exist in one of several forms. The background for each of these has already appeared in prior discussions.

1. Continuation of Case 1 Solution

If Case 1 was the starting case and the solution in that case has been covered, Case 2 will govern for further \( q \) values until \( \varphi_m \) exceeds \( \varphi_a \) or until \( q \) becomes negative in which case \( \varphi_m < \varphi_a \). Here the initial value of \( \frac{\rho}{t} \) is 0.5. At start, only for the requirements of the program, SMALL is set at 0.5 + DRH012 and, hence, BIG is 0.5.
As explained in B previously, the value of $q$ is obtained for $\frac{\rho}{\ell} = 0.5$. This provides a check as $q$ is calculated from two different cases (1 and 2) for the same loading condition, $\varphi_m = \varphi_1$. Now BIG is redesignated SMALL and a new BIG value assumed. The value of $q$ is again calculated by the iterative process and this procedure continues.

However, at each stage it is necessary to decide whether the end of Case 2 solution is reached. Two checks are essential. If $\varphi_m$ (Eq. (42)) > $\varphi_2$, the value assumed for BIG was too small. The same conclusion is true if $q$ is negative. The value of BIG in both of these instances is modified to the average of the current values of BIG and SMALL or, in other words, the decrement DRH012 is effectively reduced to half its previous value.

After some more points on the $q - \varphi_m$ curve have thus been established the interval between BIG and SMALL will progressively reduce in magnitude. The process is terminated when this interval is less than another prescribed tolerance $TOL2$ (input data).

2. Starting Case

If the solution is in Group 1 but Case 1 is not the starting case, Case 2 may be the starting case. Here, at start itself, the ratio $\frac{\rho}{\ell}$ has a value less than 0.5, since the top fibers at the center have already yielded. This unknown ratio must first be established with $q = 0$.

The logic is essentially the same as in item 1. The value of $\zeta$ (Eq. (44)) will remain zero. Initially, $\frac{\rho}{\ell}$ is assumed at 0.5.
and this represents BIG. With this value $\eta$ is obtained from Eq. (43) and the LHS (Eq. (45)) is evaluated. The result is FKLI. Redesignating BIG as SMALL, the new trial value of $\frac{\rho}{l}$ is BIG = SMALL-DRHO12. The value of the LHS is now FKL.

The possibilities here are, as before, FKLI (FKL) > 0. Even if FKLI (FKL) > 0, $|FKL|$ will not exceed $|FKLI|$ as the process is necessarily convergent from physical considerations of the problem. At some stage FKLI (FKL) < 0. The range for $\frac{\rho}{l}$ having been established (SMALL to BIG) its value is more precisely obtained by using SR HALVE to a tolerance of TOL2.

From this stage on (START = FALSE, see item D) the process to get q values for several $\frac{\rho}{l}$ values is the same as that in item 1. As pointed out already, the starting value of q is zero.

When this program was being developed, it was observed that after the starting $\frac{\rho}{l}$ value was established the beam-column would not load at all for some prescribed parameters of initial loading. That is, q was negative without every being positive.

The reason is that the transcendental Eq. (45) has several mathematical solutions. Unlike in the case of q where a neighborhood is very well defined (previous q) that for $\frac{\rho}{l}$ is not. There exist, sometimes, two values of $\frac{\rho}{l}$ with corresponding $\varphi_m$ values in the acceptable range $\varphi_1 \leq \varphi_m \leq \varphi_2$. The first value, by virtue of the process, is the larger value. Hence, the following procedure.
First, the larger value is found by the iteration process and saved. The q-value for the subsequent decremented value of \( \frac{p_1}{L} \) is next obtained. All is well if q has a positive value. If not, the first value of \( \frac{p_1}{L} \) is discarded (a message appears on output) and the second value attempted to be found using the first value as a 'lower' bound. If it exists, the q-values found later will be positive.

The program therefore allows for the following possibilities:

i) Case 2 is not the starting case. Case 4 may be. This is indicated by the product FKLI (FKL) remaining positive throughout the range of trial values for \( \frac{p_1}{L} \), the corresponding \( \varphi_m \) values for which lie in the range \( \varphi_1 \leq \varphi_m \leq \varphi_2 \).

ii) The first value found for \( \frac{p_1}{L} \) is acceptable as the beam-column will load (q is positive).

iii) The first value is not acceptable and the second one exists. If the second value exists, it is acceptable as the q-value will be positive for a subsequent smaller \( \frac{p_1}{L} \) value.

iv) The first value is not acceptable, but the second value does not exist. Hence, Case 2 is not the starting case. Case 4 may be.

3. The Special Case NTYPE = 2

If the section is an idealized W shape bent about the strong axis, \( \varphi_2 \) is infinity when \( p \geq \frac{1}{1+R} \) and, hence, Case 2 is the concluding case. (If NTYPE = 2 and \( p \geq \frac{1}{1+R} \), the logical variable TYPE has the value .TRUE. in the program. Else, its value is .FALSE.).
The considerations in items 1 and 2 hold good even here.
However, since $\varphi_2$ cannot be handled by the machine as a number, $\varphi_2$ is
arbitrarily set in SR BMCOL to 100.0. The user must prescribe the
limit of $\varphi_m$ up to which solution is desired. This is done by pre-
scribing RANGE, an input parameter. The upper limit TPHI2 for $\varphi_m$ is
then set by SR BMCOL at RANGE times $\varphi_2$ which, for convenience, will
be termed $t_2$, henceforth. Obviously, for practical situations RANGE
in this case should be a fraction, say 0.5.

If the initial loading and geometry of the beam-column are
such that Case 1 is not the starting case and a value of $\frac{\rho_1}{k}$ (for which
$\varphi_m$ lies in the range $\varphi_1 \leq \varphi_m \leq t_2$) that satisfies Eq. (45) with $q = 0$
cannot be found, the conclusion is that the initial loading is so heavy
that the beam-column is bent beyond practicable use under the action of
$p$ alone.

On return of control from SR BC a suitable message results in
such a case in SR BMCOL which also resets INTER to 1. Finally, control
is returned to the user's main program.

D. Further Program Details

The function of SR BMCOL is initially to prescribe the value of
a logical variable called START. This value is .FALSE. if Case 1 is the
starting case, .TRUE. otherwise. If $p \neq 0$, SR BC is CALL-ed where START
helps distinguish the action required between items C-1 and C-2.

All the possibilities aforementioned are then investigated by
SR BC and solutions as found or the messages are printed out. For each
set of $q$ and $\varphi_m$ values $\theta_0$ and $\frac{\delta}{h}$ are also printed based upon user's
request. Control returns to SR BMCOL when Case 2 solution is exhausted in one of the several ways.

SR BMCOL now decides the further course of computations:

i) If TYPE = .TRUE., the solution is complete and control is returned to the user's main program (Case 2 is the concluding case).

ii) If the beam-column unloaded in Case 2, again control is returned.

iii) If Case 2 also was not the starting case, a solution is investigated in Case 4.

iv) If Case 2 solution was merely exhausted by virtue of \( \varphi_m \) reaching the limit \( \varphi_a \) with \( q \) positive, an extension of the solution is sought in Case 4.

If \( p = 0 \), control is directly transferred to Case 4 computations bypassing the CALL to SR BC.

9. SOLUTION - CASE 4

A. The Special Case of \( p = 0 \)

The solution from Case 1 is here extended. Equations (58) and (59) are to be used.

Initially, \( \frac{\rho}{l} = 0.5 \). In Chapter 8 it was shown that \( \frac{\rho_1}{l} = \frac{\rho}{l} \). Hence, the values of \( q \) and \( \varphi_m \) can be computed from Eqs. (59) and (58),

-34-
respectively. Thereby, a check on the results is available, since the loading condition is the same as at the end of Case 1 solution.

The ratio \( \frac{P_1}{L} \) is next decremented by DRH014, an input parameter, and the new values of \( q \) and \( \varphi_m \) found for the new ratio \( \frac{P_1}{L} \). This process is iterative. Two checks at each step of the process are required. The value of \( q \) cannot exceed 1.0 and the value of \( \varphi_m \) is limited to \( t\varphi_2 \) (RANGE times \( \varphi_2 \), where RANGE, the input parameter, is greater than 1.0). If either condition is violated, the decrement is reduced to half its own value successively. The solution is complete when the final decrement is less than TOL2.

These calculations are handled by SR BMCOL itself. Theoretically, \( q \) attains the value 1.0 (beam problem) when \( \varphi_m \) is infinity. Hence, there is no 'maximum' in the real sense of the word when \( \varphi_m \) can at best be \( t\varphi_2 \). (The value of the logical variable LESS is, hence, assumed to be TRUE. when \( p = 0 \), see Chapter 21).

For each assumed value of \( \frac{P_1}{L} \) the results are printed. In addition, the values of \( \theta_o \) and \( \frac{\delta}{h} \) are also computed and printed based upon user's request. This part of the computations is handled by SR PHIVSX (an ENTRY point in SR XVSPHI) when CALL-ed by SR BMCOL.

B. Starting Case

If the solution is in Group 1 and Cases 1 and 2 are not the starting cases, Case 4 may be the starting case. Equations (43), (53) through (57) are to be used to find the initial values of \( \frac{P_1}{L} \) and \( \frac{P_2}{L} \) with \( q = 0 \).
The last trial value of \( \frac{P_1}{L} \) which failed to yield a solution in Case 2 will be the first trial value for use here. The iteration process is otherwise essentially the same as in Case 2 (Chapter 8, Section C, Item 2). No difficulties such as multiple values of \( \frac{P_1}{L} \) were experienced in program development runs.

For trial values of \( \frac{P_1}{L} \), \( \eta \) is obtained from Eq. (43), \( \zeta \) from Eq. (54), and \( \frac{P_2}{L} \) from Eq. (57). Finally, Eq. (56) is satisfied by the iterative process. Throughout this process \( q \) and, hence, \( R \) (Eq. (55)) have the value zero.

The corresponding \( \varphi_m \) value is then obtained from Eq. (53). If \( \theta_o \) and \( \frac{\delta}{\theta} \) have been requested, these are computed in SR XVSPHI. SR BC handles the problem of starting Case 4.

If \( \varphi_m \) (with \( q = 0 \)) > \( t \varphi \), the beam-column is so heavily loaded initially that it is bent beyond practicable use. In other words, the solution is beyond the region of interest to the user.

There is also no need in such a case to seek a solution for higher load values. The argument INTER therefore has the value 1 (see Chapter 6). SR EBCOL prints out a message and returns control to the user's main program in such a case.

If, however, \( \varphi_m \) (with \( q = 0 \)) < \( t \varphi \), solutions for \( q \) are obtained in SR EBCOL and the procedure is the same as when an extension of Case 2 solution is sought.
C. Extension of Case 2 Solution (p ≠ 0)

Equation (56) is a cubic equation in αq of the form

\[(αq)^3 + s (αq) + t = 0\]

and, hence, the solution for q may be obtained directly (without the necessity of iteration) [5].

If \(x = \frac{t}{2}\) and \(y = \frac{t^2}{4} + \frac{s^3}{27}\), the real roots \(αq\) are given by

\[αq_1 = (\frac{-x + \sqrt{y}}{3}) - (\frac{x + \sqrt{y}}{3})^{1/3}\]  
if \(y > 0\),

\[αq_1 = -2x^{1/3} \text{ and } αq_2 = x^{1/3}\]  
if \(y = 0\),

and

\[αq_1 = m \cos \theta, \quad αq_2 = m \cos \left(\theta + \frac{2\pi}{3}\right), \quad αq_3 = m \cos \left(\theta + \frac{4\pi}{3}\right)\]  
if \(y < 0\)

where

\[m = 2 \left(-\frac{s}{3}\right)^{1/3} \quad \text{and} \quad \cos 3θ = \frac{3t}{s}m\]

Only the values of q, given by \(q_1\), in each case provide the required continuity in the solution. In other words, if \(q = q_1\), both q and \(φ_m\) are positive [3].

In brief, the approach is to assume a value for \(\frac{ρ_3}{c}\), evaluate \(η\) (Eq. (43)), \(ζ\) (Eq. (54)), \(\frac{ρ_2}{c}\) (Eq. (57)), and finally q (Eq. (56)). The value of \(φ_m\) is obtained by the use of Eqs. (55) and (53).

The procedure for assuming values of \(\frac{ρ_3}{c}\) is the same as in A previously with the difference that the initial value is the last one.
tried in Case 2. [If Case 4 is the starting case, the initial value is the value corresponding to the starting situation theoretically. The value of q in this case is known to be zero. However, due to round-off errors in the computer, this result cannot be expected. In fact, such errors lead to computational difficulties such as square roots of (very small) negative numbers being extracted. Hence, the initial value in this case is the starting value less TOL2, a very small quantity].

At each stage checks are applied to see if \( q > 0 \) and \( \varphi_m < t \varphi_2 \). If either condition is violated, \( \frac{p_1}{T} \) is suitably modified. For such modification the decrement of \( \frac{p_1}{T} \) (DRH014, an input parameter) is successively reduced to half of its previous value. The process ends when the final decrement is less than TOL2.

Values of \( \theta_0 \) and \( \frac{\delta}{h} \) are computed and printed, upon request, in SR PHIVSX (an ENTRY point in SR XVSPHI).

D. Program Details

SR BMCOL transfers control directly from the end of Case 1 solution to Case 4 if \( p = 0 \) and the solutions are obtained in SR BMCOL itself.

If neither Case 1 nor Case 2 is the starting case, SR BMCOL CALL-s SR BC to provide the starting conditions in Case 4. If Case 4 also is not the starting case, control is returned to the user's main program after printing out a message and resetting INTER to 1.
If Case 4 is a continuation of Case 2 or if Case 4 has been the starting case, solutions for \( q \) and the corresponding \( \varphi_m \) are obtained in SR BM COL.

Case 4 is the concluding case in Group 1, and, hence, finally control is returned to the user's main program.

The flow chart for handling Case 4 is given as Fig. 4.

10. SOLUTION - CASE 3

If a solution does not exist in Group 1 \( \left( \frac{m}{a} > \varphi_1 \right) \), Group 2 is investigated. The end curvature \( \varphi_o \) in Group 2 is given by Eq. (50) and the acceptable range is \( \varphi_1 \leq \varphi_o \leq \varphi_2 \). If \( \varphi_0 \) (Eq. (50)) exceeds \( \varphi_2 \), the solution will be sought in Group 3.

For a solution in Case 3 the range of \( \varphi_m \) is \( \varphi_1 \leq \varphi_m \leq \varphi_2 \). Since \( \varphi_m > \varphi_o \), this may also be expressed as \( \varphi_1 \leq \varphi_o < \varphi_m \leq \varphi_2 \).

A. Starting Case

If a solution exists in Case 3, the condition at start, \( \varphi_m \) with \( q = 0 \), must be known. For a solution of \( \varphi_m \) (Eq. (48)) the trial values lie in the range \( \varphi_o \leq \varphi_m \leq \varphi_2 \). The process of iteration is the same as before, explained in detail in Case 2. The increment and tolerance for \( \varphi_m \) in Case 3 are the input parameters DPHI3 and TOL3, respectively.

If, in the prescribed range for \( \varphi_m \), no solution exists, the starting case may be Case 5.
If TYPE = \cdot \text{TRUE} \cdot$, $\varphi_2$ is infinity theoretically. As a practical upper limit, $\varphi_2$ is arbitrarily set at 100.0. Further, the region of interest is prescribed as a fraction of $\varphi_2$ ($\text{RANGE times } \varphi_2 = t \varphi_2$). Under these conditions the trial values for $\varphi_m$ lie in the range $\varphi_0 \leq \varphi_m \leq t \varphi_2$.

**B. Solution for $q$**

If the starting conditions have met the above requirements, values of $q$ for various $\varphi_m$ in the range $\varphi_m$ (with $q = 0$) $\leq \varphi_m \leq \varphi_2$ for TYPE = \cdot \text{FALSE} \cdot$, $\varphi_m$ (with $q = 0$) $\leq \varphi_m \leq t \varphi_2$ for TYPE = \cdot \text{TRUE} \cdot$ are found in the usual manner (iterative process) using Eqs. (48) and (49). The values $\theta_0$ and $\delta \over h$ are available for each $q$, upon request.

**C. Program Details**

SR \text{BMCOL} evaluates $\varphi_\circ$ and checks whether a Group 2 solution can exist. If it can, SR BC is CALL-ed to perform the detailed calculations as above. On return of control SR \text{BMCOL} directs further operations in the following manner:

i) If TYPE = \cdot \text{TRUE} \cdot$ and no solution was found in (the concluding case) Case 3 within the region of interest, INTER is set to 1 because no solutions can exist for higher loading values. Control is returned to the user's program.

ii) If TYPE = \cdot \text{TRUE} \cdot$ and a solution was found, such a solution is complete, as Case 3 is the concluding case. INTER is hence allowed to retain the value 0, and control is returned to the user's program.
iii) If TYPE = "FALSE" and no solution was found in Case 3, the solution in Case 5 is investigated.

iv) If TYPE = "FALSE" and a solution was found in Case 3, (a) further solution in Case 5 is sought if the beam-column did not unload ($q > 0$ when $\varphi_m = \varphi_2$) and (b) control is returned to the user's program if the beam-column unloaded ($q < 0$ when $\varphi_m \leq \varphi_2$).

11. SOLUTION - CASE 5

As in Case 3, the end curvature $\varphi_o$ is given by Eq. (50) ($\varphi_1 \leq \varphi_o \leq \varphi_2$). The central curvature $\varphi_m$ has the range $\varphi_2 \leq \varphi_m \leq \infty$. The upper limit for practical problems is, however, $t\varphi_2$ (= RANGE times $\varphi_2$, where RANGE > 1). Equation (64) to Eq. (67) are to be used.

A. Starting Case

Case 5 may be the starting case in Group 2 if Case 3 was not. The value of $\varphi_m$ at start ($q = 0$) is established by the iteration procedure (Eq. (64)) in the range $\varphi_2 \leq \varphi_m \leq t\varphi_2$. The increment and tolerance for $\varphi_m$ in Case 5 are the input parameters DPHI5 and TOL3, respectively.

B. Solution for $q$

The values of $q$ are obtained by solving Eq. (64) using the iteration process for various values of $\varphi_m$ in the following ranges:
\[ \varphi_2 < \varphi_m \leq t \varphi_2 \] if Case 3 was the starting case

\[ \varphi_m \text{ (with } q = 0 \text{)} \leq \varphi_m \leq t \varphi_2 \] if Case 5 is the starting case.

The values \( \theta_o \) and \( \frac{\delta}{h} \) are available, upon request.

C. Program Details

SR BMCOL CALL-s SR BC to perform the required computations.

On return of control from SR BC, SR BMCOL terminates the solution in one of the following ways:

i) If Case 5 is also not the starting case, the initial loading is too heavy and hence INTER is reset to 1 and control returned to the user's program.

ii) Otherwise, since Case 5 is the concluding case, control is returned to the user's program, INTER retaining the value zero.

12. SOLUTION - CASE 6

Group 3 consists of Case 6 only. If a solution did not exist in Groups 1 and 2, Group 3 is investigated. Equations (70) and (71) are to be used.

If \( \varphi_o \) (Eq. (71)) > \( t \varphi_2 \), the solution is of no interest, since \( \varphi_m \) must exceed \( \varphi_o \).

Else, the starting condition is sought by solving Eq. (70) for \( \varphi_m \) in the range \( \varphi_o \leq \varphi_m \) (with \( q = 0 \)) \leq t \varphi_2 \). If no solution can
be found in this range, the actual solution is again beyond the region of interest.

If the starting conditions are suitable, q values for various $\varphi_m$ are found by the iteration process (Eq. (70)) in the range $\varphi_m$ (with $q = 0$) $\leq \varphi_m \leq t \varphi$. The increment and tolerance for $\varphi_m$ in Case 6 are DPHI6 and TOL3, respectively.

The values $\theta_o$ and $\delta h$ for each q are available upon request.

A. Program Details

SR BNCOL initially checks whether $\varphi_o > t \varphi$. If it is, the initial loading is too heavy, INTER is reset to 1 and control returned to the user's program.

Otherwise, SR BC is CALL-ed.

If Case 6 is not the starting case, again the initial loading is too heavy, INTER is reset to 1 and control returned to the user's program.

Otherwise, since Case 6 is the concluding case in Group 3, control is returned to the user's program, INTER retaining the value zero.

13. SUBROUTINE BNCOL (INTER)

The manner in which the solution to a given problem is sought in the different Cases 1 through 6 has been described in previous pages,
and the supervisory role of SR BMCOL is evident. The cases which are also actually solved in SR BMCOL are Case 1 and Case 4 (excepting the starting conditions for Case 4).

Other routines such as SR BC have been merely referred to, although some of their functions have been discussed in detail. In the following chapters these routines will be discussed at greater length.

14. SUBROUTINE BC (FKLX, ARG, DINCR, TOLARG, SIGN, ICASE, UL, IGROUP)

SR BC is CALL-ed by SR BMCOL to provide solutions by the iterative process. Table 2 summarizes the tasks performed by this subprogram and also provides a list of the parameters (for each case) which match the arguments of the subprogram.

FKLX, an EXTERNAL function, is the subprogram which evaluates the LHS of the specific equation to be solved in each case. As an example, FKL3 evaluates the LHS of Eq. (48) when Case 3 is solved. When the starting conditions of Case 2 are to be found, the LHS of Eq. (45) assumes a simpler form since \( q = \xi = 0 \). This modified LHS is evaluated in another subprogram FKL2A which is CALL-ed absolutely (as against variably for EXTERNAL functions) by SR BC.

ARG is the parameter whose value is assumed in solving the relevant equation in a case for the value of \( q \). The value of this same parameter is sought if the starting conditions are to be found (\( q = 0 \)).
**DINCR** is the increment or decrement of ARG.

**TOLARG** is the tolerance of ARG. Towards the conclusion of a solution in a particular case, the assumed final value of ARG differs from the exact mathematical value by less than this amount.

**SIGN** has only one of two values, -1.0 or 1.0. This term is required in order to compare both increasing \(\frac{\beta_1}{\xi}\) and decreasing \(\frac{\beta_1}{\xi}\) ARG-s against their corresponding lower and upper limits, using common expressions in the subprogram.

**ICASE** is the case number. It is used in labelling output as well as selecting the specific FORMAT for output in each case. Another important function it serves is in aiding SR BC to recognize the peculiar programming features involved when dealing with Case 2. If **ICASE** \(\neq 2\), **NCASE2** (read as NOT Case 2) has the value 'TRUE'.

**UL** is the upper limit that ARG can assume in value. Since \(\frac{\beta_1}{\xi}\) is a decreasing parameter in Cases 2 and 4, the upper limit is actually lower in magnitude than the initial value.

The lower limit or the initial value is assigned to ARG before each CALL to SR BC in SR BMCOL. For Cases 2, 4, 3, 5, and 6 the values are 0.5, \(\frac{\beta_1}{\xi}\), \(\varphi_0\), \(\varphi_2\), and \(\varphi_0\), respectively. For Case 4, \(\frac{\beta_1}{\xi}\) is the last trial value which failed to yield a starting condition in Case 2.

**IGROUP** is the Group number. It is used only in labelling output.
Whenever a solution for q is found in SR BC, it CALL-s other subprograms to provide the values of $\theta_o$ and $\delta/h$, if these have been requested by the user.

The technique of finding the maximum value of q is a feature common with that in SR BMCOL (Case 4) and will be discussed under SR QMAX.

Figure 5 is a flowchart for SR BC. It is abbreviated only where printing of results is indicated.

15. SUBROUTINE FKL2A and SUBROUTINE CALC

Most numerical computations involved in the solutions are carried out by SR FKL2A. Several ENTRY statements have been included. The coding between each ENTRY statement and the succeeding RETURN statement is equivalent to a separate subprogram.

In addition to evaluating the LHS-s of the equations to be solved in the several cases, SR FKL2A also handles the x-q relations given in Chapter 4.

Certain calculations in Cases 2 and 4, as well as in Cases 5 and 6, are the same. SR FKL2A hence CALL-s SR CALC (or its other ENTRY points FKL6 and CASE24) in its turn when such calculations are to be made.

Only a few logical decisions are made in SR FKL2A.
Table 3 is a list of the several ENTRY points in SR FKL2A and the corresponding equations [Chapter 4].

SR CALC handles Eqs. (43), (70), and parts of Eqs. (47), (60), (64), and (72). It is mainly CALL-ed by SR FKL2A. SR BC CALL-s SR CALC (ENTRY point FKL6) directly when Case 6 is being solved (Eq. (70)).

16. **SUBROUTINE HALVE (ARG, SMALL, BIG, TOLARG, FKLX, FKLI)**

The iterative process to determine the range of \( q \) for which a solution is being sought (Eq. (45)) was explained in the discussion under Case 2, Chapter 8. This part is handled by SR BC. In addition, SR BC also determines the ranges of \( q, \frac{\rho_1}{l}, \) and \( \varphi_m \) when other equations are being solved. More precise values of these quantities are furnished by SR HALVE.

ARG is hence one of the parameters \( q, \frac{\rho_1}{l}, \) or \( \varphi_m \).

SMALL is the 'lower' limit of ARG.

BIG is the 'upper' limit of ARG. (The precise value of ARG is hence bracketed between SMALL and BIG).

TOLARG is the tolerance or the accuracy within which the solution for ARG is desired (TOL1, TOL2, and TOL3 respectively for the three parameters \( q, \frac{\rho_1}{l}, \varphi_m \)).
FKLX is the EXTERNAL function which represents the LHS of the equation being solved.

FKLI is the signed value of the LHS of the equation when the initial trial value of ARG is used. The sign of FKLI is the same as that of the resulting value of the LHS of the equation when ARG = SMALL. In other words, SMALL is not necessarily the initial trial value of ARG.

The function of SR HALVE is to find a more precise value for ARG (than either SMALL or BIG) within the tolerance TOLARG, given the range of ARG (SMALL to BIG).

The result of plotting the values of LHS for all ARG between SMALL and BIG may be represented by Fig. 6 or Fig. 7.

In each figure, a represents the value of the LHS when ARG = SMALL. Hence, a has the same sign as FKLI. Similarly, b is the value of the LHS when ARG = BIG. The computations in SR BC have assured that a and b are of opposite signs. Hence, there is a value of ARG (between SMALL and BIG) for which the LHS has the value zero.

Strictly speaking, there may be an odd number of ARG values that satisfy this requirement. This would be true if the interval between SMALL and BIG is large. However, it is here presumed that this is not the case, since the prescribed increment (or decrement) for ARG is chosen to be sufficiently small. To ensure this the user is advised to specify the values for the increments as recommended in the User's Guide (or lower values).
Assuming, therefore, the existence of a unique solution for ARG in the interval SMALL to BIG, it is clear that the average value of SMALL and BIG is a better value for ARG than at least one of these. The interval is thus effectively halved. Hence, the name 'Method of Interval Halving' [6].

Let the LHS have the value c corresponding to this new value of ARG. If ac is zero, c is zero and hence the exact value of ARG is fortuitously found. If ac is positive, ARG is on the same side of the actual solution as SMALL is (Fig. 6). Hence, SMALL may now be redefined as the current value of ARG, thereby reducing the range.

If ac is negative, BIG is redefined (Fig. 7).

The process may now be iterated for convergence towards the actual solution. It is terminated when the final interval between the redefined values of SMALL and BIG is less than TOLARG.

The corresponding flowchart is given in Fig. 8.

In SR HALVE, FKL represents the value of LHS after each CALL to FKLX.

17. **SUBROUTINE XVSFI (XFI, XVSFIX, XFIMIN, XFIMAX)**

This routine is active only if the user has requested values of \( \theta_0 \) and \( \frac{\delta}{h} \) for each q value that is computed, through setting the input parameter INDEX to an integer value from 1 to 9 (non-zero).
The quantities $\theta_o$, the end slope, and $\frac{h}{T}$, the non-dimensionalized central deflection, are computed using the curvature diagram of the beam-column, a typical representation of which is given non-dimensionally as Fig. 9.

From conditions of symmetry the slope at midspan is zero. Hence, the end slope is the area of the left half of the diagram suitably modified to account for dimensions.

$$\theta_o = \int_0^{L/2} \varphi \frac{dy}{dx} \, dx = \frac{y}{h} \int_0^{L/2} \varphi \left(\frac{dx}{L}\right)$$

$$= \frac{2\sigma_y L}{E} \frac{y}{h} \left(\frac{1}{h}\right) (\text{area ABCD}) = (\text{slope modifier}) \cdot (\text{area ABCD})$$

The slope modifier is a function of the geometry and material properties of the beam-column and hence a constant.

The central deflection $\delta$ is the tangential deviation of the support from the (horizontal) tangent at midspan and hence the moment of the left half of the area of the curvature diagram about the left support.

$$\frac{\delta}{h} = \frac{1}{h} \int_0^{L/2} \varphi \frac{dy}{dx} \, dx = \frac{y}{h} \int_0^{L/2} \varphi \left(\frac{dx}{L}\right)$$

$$= \frac{2\sigma_y L^2}{E} \frac{y}{h} \left(\frac{1}{h}\right) \left(\frac{1}{h}\right) (\text{moment of area ABCD about AB})$$

$$= (\text{slope modifier}) \frac{L}{r} \frac{1}{h} \left(\frac{1}{h}\right) (\text{moment of area})$$

$$= (\text{deflection modifier}) (\text{moment of area})$$
The deflection modifier is thus a constant similar to the slope modifier.

The area $ABCD$ is approximated in the routine as the summation of areas of strips such as $EFGH$, assuming $FG$ is a straight line.

$$\text{Area } ABeD = \sum_{1}^{N} (\text{cur}_1 + \text{cur}_2) \left( \frac{x_1}{l} - \frac{1}{l} \right) (0.5), \text{ where } N = \text{number of subdivisions of } AD \text{ the semi-span.}$$

In general, the elastic zone is subdivided into $\text{NSUB}$ (input parameter) equal parts along the semi-span of the beam-column. In each of the primary and secondary plastic zones the curvature range is subdivided into $\text{NSUB}$ equal parts and, hence, the subdivisions along the semi-span will not be equal.

The moment of area $ABCD$ about $AB$ is approximated as

$$\frac{N}{1} \sum_{1}^{\text{area EFGH}} \left( \frac{x_2}{l} + \frac{1}{l} \right) (0.5).$$

A. **Arguments of the Routine**

From the $x$-$\xi$ relations listed in Chapter 4 it is evident that $\xi = f \left( \frac{x}{l} \right)$ in the elastic zones and $\frac{x}{l} = f (\xi)$ in the primary and secondary plastic zones. Hence, $XFI$ is either $\frac{x}{l}$ or $\xi$. If $XFI$ represents $\frac{x}{l}$, the length over which the corresponding equation holds is subdivided into $\text{NSUB}$ equal parts. If $XFI$ is $\xi$, the curvature range is subdivided similarly.

$XVSFIX$ is the EXTERNAL function that represents the actual equation to be used. It is any one of items 6 through 15 in Table 3.
XFIMIN is the lower limit of XFI.

XFIMAX is the upper limit of XFI.

B. ENTRY XVSFE

If the beam-column behavior is governed by Cases 1, 3, or 6 one equation will suffice to define the $x-\varphi$ relationship over the semi-span of the beam-column. The integration is hence a one-step procedure. The normal ENTRY point XVSFI which initializes SLOPE and DEFLN to zero is used (SR XVSFI is CALL-ed) in these cases.

If the governing case is 2, 4, or 5, a combination of elastic, primary, and secondary plastic zones exists and in each of the zones the $x-\varphi$ relations are defined by different equations. Considering Case 2 as an example, initially SR XVSFI is CALL-ed with TWOA representing the EXTERNAL function XVSFIX. That is, integration is performed over the elastic zone first. SLOPE and DEFLN have some values at exit from the subroutine and represent parts of the final answers. Hence, they are needed when the integration over the primary plastic zone is performed. To achieve the desired result, the initialization of SLOPE and DEFLN to zero is bypassed by CALL-ing SR XVSFE subsequently with TWOB representing the EXTERNAL function XVSFIX.

Case 5 is handled in a similar manner. When Case 4 governs, the CALL to SR XVSFI is followed by two CALL-s to SR XVSFE, since all the three zones exist in this case, if $p$ is not zero. If $p$ is zero, the primary plastic zone does not exist, and hence one CALL is to SR XVSFI and another to SR XVSFE (see SR XVSPHI, Chapter 20).
C. Logical Variable NIR

The printing of values $\frac{x}{L}$ and $\varphi$ each time these are computed in this routine is governed by NIR. Assuming INDEX is non-zero (else, this routine is not activated), NIR, which is an abbreviation for 'No Intermediate Results' has the value ·TRUE· (SR BMCOL) if the input parameter INDR is zero. The printing of $\frac{x}{L}$ and $\varphi$ values is suppressed in such a case. If (INDEX ≠ 0 and) INDR ≠ 0, NIR = ·FALSE·, and printing results.

The user is cautioned that excessive output may result if both INDEX and INDR are set to non-zero integer values (from 1 to 9).

D. Remarks

The approximations introduced in computing $\theta_o$ and $\frac{b}{h}$ need to be recognized in the interpretation of final results. The integration procedure adopted here is elementary. Further, although the curvature diagram exhibits steeper slopes when passing from elastic to primary to secondary plastic zones, the number of subdivisions is not altered. Refinements are obviously warranted. But they have been omitted considering that $\theta_o$ and $\frac{b}{h}$ are by-products and their values do not affect the solution of the major problem.

Within the bounds of the existing limitations, it is, however, possible to improve the accuracy by specifying NSUB to be sufficiently large. Its value was chosen as 10 in development runs.

The flowchart for this routine is given as Fig. 10.
18. **BLOCK DATA**

This subprogram is active during compilation only and provides an easy way of initializing a few variables in the COMMON block labelled BLA.

Of the four variables that it initializes, THIRD is a simple variable having the value 1.0/3.0 in decimal form 0.333...3. This value is required in the Arithmetic Statement Function CROOT (SR BMCOL) which may be referenced a number of times. If the fractional form were retained in the A. S. Function, the computer would need to convert it to the decimal form each time the function was referenced. Also, the variable THIRD is used elsewhere in SR BMCOL and other subprograms as a multiplier wherever division by 3.0 is required.

The other three variables initialized are subscripted variables or arrays. These store FORMAT statements. Array JVAR is used by SR BMCOL and SR BC and, hence, storing the FORMAT in an array represents some saving in memory. This array is modified in SR BMCOL to suit output of results when Case 5 is handled.

Arrays IVAR and LVAR are needed to store FORMAT statements which may later be modified at execution time by SR BMCOL. The modification is with respect to the manipulation of output based on user's request for the printing of $\theta_o$ and $\frac{\Delta}{h}$. If these quantities are not to be printed, no modification is done.

Since this subprogram is inactive during execution time, SR BMCOL takes care of resetting these arrays to the initialized forms.
before it returns control to the user's program after solving each problem.

19. SUBROUTINE LABELS (IGROUP, ICASE)

The function of this routine is to label the output at appropriate locations. IGROUP and ICASE represent the group and case numbers, respectively.

SR LABELS is CALL-ed by SR BMCOL and SR BC.

It saves a little coding besides affording convenience in programming.

20. SUBROUTINE XVSPHI

This routine handles the calculations regarding $\theta_0$ and $\frac{\delta}{h}$ in Case 4 only.

Two separate situations exist as far as yielded zones are concerned in this case depending on whether $p$ is zero or non-zero.

If $p = 0$, only the elastic and secondary plastic zones exist.

If $p \neq 0$, the primary plastic zone also exists between the elastic and secondary plastic zones.

This routine is CALL-ed mainly by SR BMCOL (assuming user's request for $\theta_0$ and $\frac{\delta}{h}$) whenever a value of $q$ is evaluated for a specific
The decision to use the proper set of \( x - \varphi \) relations in evaluating \( \theta_0 \) and \( \frac{\varphi}{h} \) is made in SR XVSPHI based on the value of \( p \). The ENTRY point is PHIVSX in this case.

If Case 4 is the starting case, \( p \) is not zero. Earlier, it has been pointed out that the start of Case 4 is handled by SR BC. When the starting conditions are found, SR BC CALL-s SR XVSPHI, the normal ENTRY point for this routine since \( p \) need not be tested for the value zero.

If \( p = 0 \), Eqs. (62) and (63) apply. These are represented by EXTERNAL functions ONEA and FOURD, respectively [Table 3].

If \( p \neq 0 \), Eqs. (46), (60), and (61) apply corresponding to TWOA, FOURB, and FOURC, respectively.

21. **SUBROUTINE QMAX (ARG, DARG, ICM2, TOLARG, SIGN)**

The direct method of obtaining \( q_{\text{max}} \), the maximum value of \( q \), is by solving the equation \( \frac{dq}{dq_m} = 0 \) for \( q_m \) and computing the corresponding \( q \). The complexity of the \( q - q_m \) relations, however, renders this approach impractical. A numerical approach which does not require the development and use of additional equations is therefore adopted.

A. **General Approach**

In the ensuing discussion it is helpful but not necessary to assume that Case 3 is the starting case. The procedure is the same for any combination of the cases that may occur in the solution.
Three successive points on the \( q - q_m \) curve are plotted in Fig. 11 to indicate the loading sequence from start. For small \( q_m \)-intervals between successive points the condition \( q_3 > q_2 > q_1 \) holds good.

After redesignating \( q_2 \) as \( q_1 \) and \( q_3 \) as \( q_2, q_3 \), which is a fresh value of \( q \), may be computed. The same general situation, that is, \( q_3 > q_2 > q_1 \), as in Fig. 11 will most probably prevail. If it does, the process can be repeated until the pattern changes to that given by either Fig. 12 or Fig. 13.

In Fig. 12, \( q_3 < q_2 \) and, hence, the violation of the criterion \( q_3 > q_2 > q_1 \) is a clear indication of the fact that \( q_{\text{max}} \) has been passed. In Fig. 13, however, this fact cannot yet be recognized since the criterion holds good. A continuation of the process in the latter case results in the pattern given by Fig. 14 where \( q_3 < q_2 \), thus indicating the existence of \( q_{\text{max}} \) in this region.

In general, therefore, when \( q_3 < q_2 \), the peak has been passed and \( q_3 \) lies to the right of the peak. The location of \( q_2 \) cannot be generalized with respect to the peak but \( q_1 \) is to its left.

Hence, whenever \( q_3 < q_2 \), both the current values of \( q_2 \) and \( q_3 \) are discarded and the \( q_m \)-interval reduced to one half of its current value. Fresh computations for \( q_2 \) and \( q_3 \) will probably show that \( q_3 > q_2 > q_1 \). The process may then be continued until again \( q_3 < q_2 \) and a further reduction of the \( q_m \)-interval becomes necessary. The
process is thus iterative and may be terminated when the $q_m$-interval is finally less than a prescribed tolerance. In such a scheme the set of values $q_2$ and the corresponding $q_m$ will represent the conditions at the actual $q_{\text{max}}$ very closely.

B. Special Cases

The procedure just described covers also certain special situations that may arise.

If the initial $q_m$-interval is large, $q_2$ may have passed the peak of the curve as in Fig. 15. This is likely to occur when the initial loading is such that the starting case is one of the concluding cases 4, 5, or 6. The beam-column has little reserve capacity to resist the lateral load $q$ under these conditions. However, no difficulty is experienced in computing $q_{\text{max}}$ by the prescribed method, since the conditions are similar to those in Fig. 14.

Three other possible situations arise when the solution to the beam-column problem extends over more cases than one, as in Cases 1, 2, and 4 or Cases 3 and 5.

The description of these is combined in what follows with an explanation of the choice of an initial value for $q_1$ when each case is handled in turn.

Until the starting case is established and two successive values of $q$ ($q_2$ and $q_3$) are found such that $q_3 > q_2$ the value of $q_1$ is zero. Its value is later modified in the search process.
If the maximum does not occur in earlier cases (say, Case 3),
the condition $q_3 > q_2 > q_1$ will hold good for the entire range of values
of $q$ in those cases. The maximum may occur in a succeeding case (Case 5).
Here also it would seem at first sight that $q_1 = 0$ may be assumed at start
when seeking $q_{\text{max}}$ in the succeeding case.

If $q_{\text{max}}$ occurs for a value of $q_m$ which is close to the delimiting value between two cases ($q_2$ for Cases 3 and 5), the three possibilities
are as in Figs. 16, 17, and 18. Assume that solutions in the earlier cases
(Case 3) have been covered and those in a succeeding case (Case 5) are
sought. In these figures $q_1$ is shown to be the last computed value of $q$.
The first value of $q$ found in the succeeding case (Case 5) is for the
delimiting value ($q_2$). The $q_m$-interval between $q_1$ and $q_2$ is at best the
tolerance prescribed for $q_m$. Continuing the solution in the succeeding
case, the $q_m$-interval between $q_2$ and $q_3$ is the initial increment of $q_m$
(DPHI5 for Case 5).

If the conditions are as in Fig. 16, there is no special
problem since these are similar to the conditions in Fig. 12. From
Figs. 17 and 18 it is obvious that $q_{\text{max}}$, which actually occurred in
the earlier case, was not found, since the tolerance specified for $q_m$
was too crude.

With the choice of $q_1$ as zero the search process would yield
$q_2$ for $q_{\text{max}}$ in both the latter instances (note that the 'succeeding' case
is being solved). The value $q_2$ is a 'better' maximum than the last com-
puted value in the previous case in Fig. 17 and is actually less than
that quantity in Fig. 18.
In the program a message is output indicating the existence of \( q_{\text{max}} \) when for the first time \( q_3 < q_2 \). Such a message would be misleading if conditions are as in Fig. 18.

Assume now \( q_1 \) is chosen as in Figs. 17 and 18. The search process is satisfactory if conditions of Fig. 17 (similar to Fig. 14) are true as this will yield \( q_2 \) for maximum. Of course, the actual \( q_{\text{max}} \) cannot be approximated to the same degree of accuracy as would have been possible with a more refined value of the tolerance for \( q_m \). Since \( q_2 < q_1 \) in Fig. 18 only, the entire search process can be bypassed in programming after a suitable message is output, when this condition is met. Hence, the initial choice of \( q_1 \) as in Figs. 16, 17, and 18.

C. Remarks

Practically all the details remain the same, if Case 6 is considered. Since Case 6 is the only case in Group 3, it is the starting case and concluding case as well in that group. Hence, the situations of Figs. 16, 17, and 18 will simply not arise.

The value of \( q_{\text{max}} \) is never attained in Case 1. Hence, in SR BMCOL the search for \( q_{\text{max}} \) is omitted as far as this case is concerned.

Further, if \( p = 0 \), the cases to be considered are 1 and 4 in that order and \( q_{\text{max}} = 1 \) when \( q_m = \infty \). Hence, for practical situations where \( q_m \) is limited to \( \tau \phi_2 \) the search for \( q_{\text{max}} \) is bypassed in Case 4 if \( p = 0 \) (by setting LESS = .TRUE., see discussion under 'Other programming details').
In Cases 3, 5, and 6 the solutions for $q$ are obtained assuming incremented values for $\varphi_m$. In Cases 2 and 4 these are obtained for decremented values of $\frac{\varphi_m}{l}$. Equal decrements of $\frac{\varphi_m}{l}$ do not represent equal increments of $\varphi_m$ but this is no inconvenience in the search process since its success does not depend on the $\varphi_m$-interval between $q_1$ and $q_2$ being the same as between $q_2$ and $q_3$.

D. Other Programming Details

Figure 19 represents a typical $q - \varphi_m$ curve which exhibits a value for $q_{\text{max}}$ also.

In the program the logical variables called LESS and MESAGE have the value \texttt{FALSE} until the condition $q_3 > q_2 > q_1$ is violated. When this occurs, a message is output indicating the existence of $q_{\text{max}}$ and MESAGE is given the value \texttt{TRUE}. Further printing of $q - \varphi_m$ values is suspended until $q_{\text{max}}$ is found by the iterative process. This suspension is supervised by the logical variable MESAGE (which now has the value \texttt{TRUE}). When the iteration for $q_{\text{max}}$ is completed, LESS is assigned the value \texttt{TRUE}. From this point on printing of results is resumed and the search process is completely bypassed.

Because of the foregoing choices, it is evident that when $\text{LESS} = \texttt{\text{F}}$, the increment $DQ$ (for $q$ when solving in each case for a given $\varphi_m$ or $\frac{\varphi_m}{l}$, SR BC) should be allowed to be either positive or negative. From the stage when MESAGE = \texttt{\text{TRUE}}, $DQ$ is allowed to assume positive values only (since $q_{\text{max}}$ is sought) until LESS is set to \texttt{\text{TRUE}}.
that is, \( q_{\text{max}} \) is found. Now, \( q \) values must decrease and \( DQ \) is therefore allowed to assume negative values only.

The prescription of a sign for \( DQ \) in SR BC in the previous manner speeds up operations.

The search process is essentially the function of SR QMAX. The CALL to this subroutine is made only when LESS has the value \( .FALSE. \). To bypass the CALL altogether in Case 4 when \( p = 0 \), LESS is set to the value \( .TRUE. \) from the very beginning. In all other cases (with the exception of Case 1) LESS initially has the value \( .FALSE. \) and hence after each computation of \( q \), SR QMAX is CALL-ed. Only after \( q_{\text{max}} \) is established LESS is \( .TRUE. \) and the CALLs to SR QMAX are suspended.

SR QMAX also has an ENTRY point MAXQ to enable only increasing values of \( q \) to be sought once MESSAGE is \( .TRUE. \), as explained in a foregoing paragraph. This is referenced by SR BC only.

SR QMAX has the following arguments:

- \( ARG \) is either \( q_m \) (Case 3, Case 5, or Case 6) or \( \frac{\rho_1}{L} \) (Case 2 or Case 4).

- \( DARG \) is the increment of \( ARG \). If the CALL is from SR BC, it is a signed quantity and hence represents the decrement of \( \frac{\rho_1}{L} \) if \( ARG \) represents \( \frac{\rho_1}{L} \).

- \( ICM2 \) is the case number of the preceding case in which \( q_{\text{max}} \) may have occurred but could not be found since the tolerance specified
was too crude. This is used in output of a message only (ICM2 = 3 if
the CALL is from SR BC, ICM2 = 2 if it is from SR BMCOL).

**TOLARG** is the tolerance of ARG.

**SIGN** is ±1.0 depending on whether the CALL is from SR BMCOL
(-1.0, so that the product DARG*SIGN represents the decrement for \( \frac{\rho_1}{L} \),
Case 4) or from SR BC (1.0, since DARG itself carries a sign).

The flowchart for this routine is presented as Fig. 20.

22. **USER'S GUIDE**

The computer program to solve the problem of an eccentrically
loaded beam-column under transverse load has been written in the form
of subprograms. Of these **SUBROUTINE BMCOL** (INTER) must be **CALL-ed** by
the user's main program.

The use of the argument **INTER** has been explained in Chapter 6
in the documentation.

The input to **SR BMCOL** consists of details pertaining to the
problem as well as the details regarding the solution itself. For
communication with the subprograms the user must declare a **COMMON** block
labelled **BL** in his main program and list the input parameters in the
order indicated on the next page (arranged alphabetically). Further,
the input parameters must be assigned the desired values in some manner
convenient to the user (such as reading in values from data cards).
The input values such as length of the beam-column and thrust must be non-dimensionalized. In trial runs the values noted as recommended values in parentheses were found to be satisfactory for the corresponding parameters:

**DPHI3**: Increment of $\phi_m$ for solution in Case 3.
(Recommended value 0.50)

**DPHI5**: Increment of $\phi_m$ for solution in Case 5.
(Recommended value 0.50)

**DPHI6**: Increment of $\phi_m$ for solution in Case 6.
(Recommended value 0.50)

**DRH012**: Increment of $\frac{p_1}{L}$ for solution in Case 2, hence, a positive quantity. The program takes care to see that $\frac{p_1}{L}$ must be actually decremented.
(Recommended value 0.05)

**DRH014**: Increment of $\frac{p_1}{L}$ for solution in Case 4, a positive quantity.
(Recommended value 0.01)

**E**: Young's modulus, in kips per square inch.

**EOVERR**: The ratio $\frac{e}{r}$, where $e$ is the actual eccentricity of the thrust and $r$ is the radius of gyration about the axis of bending. This value requires to be either hand-computed or computed in the user's main program before CALL-ing SR BMCOL.
HOVERR: The ratio $\frac{h}{r}$, where $h$ is the depth of the section of the beam-column. This value also requires to be hand-computed. If NTYPE = 1 (Rectangular Section), HOVERR need not be specified, because the program assumes the value $\sqrt{12.0}$, irrespective of what has been prescribed.

INDEX: An integer to specify whether the end slope ($\theta_o$) and central deflection ($\frac{\delta}{h}$) of the beam-column for each value of $q$, the lateral load are to be computed and printed or not.

If INDEX is assigned the value zero, no computations for $\theta_o$ and $\frac{\delta}{h}$ are made.

If INDEX is assigned any integer value other than zero (from 1 to 9), the values $\theta_o$ and $\frac{\delta}{h}$ are computed and printed.

INDR: If INDEX is assigned the value zero, the value of INDR is assumed to be zero irrespective of any value prescribed. If INDEX is non-zero but INDR is zero, only the values of $\theta_o$ and $\frac{\delta}{h}$ are printed.

If INDEX and INDR are both non-zero (from 1 to 9), the following output will result for each value of $q$ computed:
a) Curvature $\varphi$ for values of $\frac{x}{L}$ in the range 0.0 to 0.5 (half the span). The elastic zone is divided into $\text{NSUB}$ equal parts and corresponding $\varphi$ values are computed. The primary and secondary plastic zones are divided into $\text{NSUB}$ regions having equal increments of $\varphi$ and corresponding $\frac{x}{L}$-values are computed.

Hence, for example, in Case 1, $\text{NSUB}$ sets of values will be printed and in Case 4 ($p \neq 0$) 3 times $\text{NSUB}$ sets of values will be printed.

b) The values of $\theta_0$ and $\frac{\delta}{h}$.

**IRITE:** The value to be assigned to IRITE depends on the (machine) system being used and refers to the output unit of the computer. To specify printer output on CDC 6400 at Lehigh, any integer 1 through 98 is acceptable, provided it is not the same as the input (say card reader) unit number. The value of 6 for IRITE will probably cover most IBM machines.

(Recommended value 6)
LOVERR: The ratio \( \frac{L}{r} \), where \( L \) is the length of the beam-column. LOVERR should be declared REAL in the user's main program.

NSUB: The number of desired subdivisions when computing the \( \frac{X}{L} \) versus \( \phi \) relationships (see INDR). The value for NSUB must be specified even if INDR is zero, since \( \theta_o \) and \( \frac{\delta}{h} \) are computed following the computations for \( \frac{X}{L} \) and \( \phi \) when INDEX is non-zero.
(Recommended value 10)

NTYPE: An integer 1 through 7 to describe the nature of the section of the beam-column (see Table 1 and Chapter 3).

P: The longitudinal thrust \( p \) on the beam-column.
It is the non-dimensionalized quantity \( \frac{P}{P_Y} \)
where \( P \) is the actual thrust and \( P_Y \) is the thrust corresponding to yield (no moment).
Hence, \( p \) must always be less than 1.0.

R: The shape variable (Chapter 3).
(Recommended value 1.40, recommended range 1.0 to 1.40)
This value is pertinent to the program only if NTYPE = 2. For all other values of
NTYPE, R is assumed to be zero, irrespective of the value specified.

**RANGE:** A multiplier of \( q_2 \) to describe the user's region of interest in the solution of the problem. The recommended value is 10.0 if NTYPE is different from 2.

If NTYPE = 2, \( q_2 \) is infinity whenever

\[
p \geq \frac{1}{1 + R}.
\]

In the program, for practical reasons, \( q_2 \) is arbitrarily set at 100.0 under these conditions. A recommended value for RANGE here is, therefore, 0.5.

For NTYPE = 2 and \( R = 1.40 \) the value of RANGE in trial runs was prescribed in terms of \( p \) as follows:

\[
\text{RANGE} = 10.0 + 3.6 \, p - 63.36 \, p^2
\]

with a minimum value of 0.5 (see Fig. 21).

**SIGMAY:** The yield stress \( \sigma_y \) in kips per sq. in. [\( E \) and SIGMAY will be echoed on output in ksi, and hence the restriction. Other units, same for both, may be chosen as only the ratio \( E/\text{SIGMAY} \) affects the solution, if the user is prepared to overlook the units (ksi) on printout].

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TOL1: The prescribed tolerance on \( q \), the lateral load. The value of \( q \) computed by the iteration technique differs from the actual solution by less than TOL1.

(Recommended value 1.0E-9)

TOL2: The prescribed tolerance on \( \frac{p_1}{\ell} \).

(Recommended value 1.0E-6)

TOL3: The prescribed tolerance on \( \varphi_m \).

(Recommended value 1.0E-3)

None of the input parameters will change in value when SR BMCOL is CALL-ed.

A. Details of Output

All the input parameters except IRITE are echoed on the first sheet of output with suitable labels. In addition, the values of the following are also printed.

ALPHA (shape factor), MZERO (\( m_o \), end moment), A,B,C,F,MPC, PHI1,PHI2 (the parameters of the generalized stress-strain relationship \( a,b,c,f,m_{pc}, \varphi_1 \), and \( \varphi_2 \)), and KL (the value of \( k\ell \)).

Next, if no solution can exist either because \( k\ell > \pi \) or \( m_o > m_{pc} \), a corresponding message is output and execution terminated in the subprograms.

If a solution exists, the starting group and case are labelled. The values of \( q, \varphi_m, \frac{p_1}{\ell}, \) and \( \frac{p_2}{\ell} \) are listed for each step in the
computations. (The values $\frac{p}{l}$ and $\frac{p}{l}$, which are pertinent to some cases, are listed only if the solution is in these cases).

If INDEX $\neq 0$ and INDR $\neq 0$, the values of $X$ and corresponding $q$ are printed for each $q$ computed. This is followed by the printing of $\theta_o$ and $\frac{\delta}{h}$. The output in this case will hence normally run into a few pages.

If INDEX $\neq 0$ but INDR $= 0$, the values of $\theta_o$ and $\frac{\delta}{h}$ are printed on the same line as the $q$-value.

If INDEX $= 0$, the values of $\theta_o$ and $\frac{\delta}{h}$ are neither computed nor printed. In general, therefore, if INDR $= 0$, the total output per problem will not exceed 2 or 3 sheets of paper.

After the solution in the starting case is completed the group and case numbers of the succeeding case are labelled and the output continued in a similar manner.

If a maximum value of $q$ exists in the range prescribed for the solution, the printing of such a value is preceded by a message to the same effect.

Other messages may appear depending on the nature of the problem, particularly when the initial loading is so heavy that a solution cannot exist within the region of interest.

In the interpretation of the output the following definitions will be useful:
\[ q = \frac{Q}{Q_Y} \]

where \( Q \) is the actual transverse load and \( Q_Y \) is the transverse load corresponding to initial yield of the beam-column treated as a beam (no thrust). \( Q_Y = \frac{4M}{L} \), where \( M = \) yield moment = \( \sigma_Y S \) with \( S = \) section modulus, \( L = \) length.

\[ \varphi_m = \frac{\bar{\varphi}_m}{\bar{\varphi}_Y} \]

where \( \bar{\varphi}_m \) is the actual midspan curvature and \( \bar{\varphi}_y \) is the curvature at initial yield.

\[ \bar{\varphi}_y = \frac{2}{h} = \frac{2}{h} \frac{\sigma_Y}{E} \]

where \( h = \) depth of section.

\[ \frac{\rho_1}{\ell} = \text{ratio of length of elastic zone from end to span} \]

\( \ell \). (Fig. 1)

\[ \frac{\rho_2}{\ell} = \text{ratio of (length to beginning of secondary plastic zone from end) to span} \]

\( \ell \). (Fig. 1)

\[ \frac{x}{\ell} = \text{ratio of length} \ x \ \text{from end to span} \]

\( \ell \).

\[ \varphi = \frac{\bar{\varphi}}{\bar{\varphi}_y} \]

where \( \bar{\varphi} = \) actual curvature.

\[ \theta_0 = \text{actual slope in radians at the ends of the beam-column.} \]

\[ \frac{\delta}{h} = \text{ratio of midspan deflection to} \ h. \]

B. Sample Program

In Appendix 1 the control card deck (down to the first 7/8/9 card, multiple punches in Column 1) suitable for use on the
CDC 6400 at Lehigh University is followed by an example of a user's main program. Assuming that source cards (FORTRAN) of the subprograms described in this documentation are to be used, these would be placed after the END card of the main program. The deck ends with the (orange) 6/7/8/9 card. No data requires to be read in for the example program.

The output follows on the pages following the program.

The complete listing of the subprograms appears in Appendix 2.

C. Points to ponder when using machines other than CDC 6400

The authors believe that this group of subprograms written in FORTRAN IV language is compatible for use in most machines with minor modifications, if any. Some of the features of the CDC 6400 which may not be common with those of other machines are noted below.

1. It is permissible to print in 136 columns excluding the first. Some FORMAT statements and arrays may need to be revised when using other machines.

2. An ENTRY statement does not have an argument list, since the list is assumed to be the same as the one associated with the name of the subprogram in which the ENTRY statement appears. However, references to the ENTRY statement must include an argument list if it exists.
3. Names of variables or subprograms may be 7 characters long. (In this group of subprograms, however, the length has been limited to 6 characters).

4. When the carriage control character is +, spacing is suppressed and printing occurs on the same line as the last printed line.

5. The logical constants ·TRUE· and ·FALSE· may be replaced by ·T· and ·F·, respectively. However, in this set the longer forms have been used.

23. SUMMARY

The computer program described in this report utilizes the theory developed in earlier reports [2,3,4]. The problem solved here, that of an eccentrically loaded beam-column carrying a concentrated load at midspan, occurs frequently in practice and therefore the program is expected to be of benefit to civil engineers. The input to the program is not complicated, and yet provides a good deal of flexibility in specifying the degree of accuracy required in the solution as well as the magnitude of the output.

Research workers investigating beam-column behavior will probably find this a handy tool in their studies. It may also serve as a starting point for new programs that need to be written when the theory is extended to cover additional variations in loading.
Table 4 is intended to be of use to programmers attempting such modifications.

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25. TABLES
**TABLE 1 -- PARAMETER FUNCTIONS FOR THE APPROXIMATE m-φ-p EXPRESSIONS**

**Solid Rectangular Section**

For all \( p \)

\[
\begin{align*}
  m_1 &= 1 - p \\
  \phi_1 &= 1 - p \\
  m_2 &= 1 + p - 2p^2 \\
  \phi_2 &= \frac{1}{1 - p} \\
  m_{pc} &= \frac{3}{2} (1 - p^2)
\end{align*}
\]

**Strong Axis Bending No Residual Stress**

For all \( p \)

\[
\begin{align*}
  m_1 &= 1 - p \\
  \phi_1 &= 1 - p \\
  m_2 &= 1 + 0.778p - 4.78p^2 \\
  \phi_2 &= \frac{1}{1 - 3.7p + 8.4p^2} \\
  m_{pc} &= 1.238 - 1.143p - 0.95p^2 \\
  m_2 &= 1.20 (1-p) \\
  \phi_2 &= 2.20 (1-p)
\end{align*}
\]
TABLE 1 -- CONTINUED

(4)

Weak Axis Bending
No Residual Stress

For all $p$

$$m_1 = 1 - p$$
$$\varphi_1 = 1 - p$$

For

$$0 \leq p \leq 0.4$$

$$m_2 = 1 + 1.5p - 2.5p^2$$
$$\varphi_2 = \frac{1}{1 - 1.57p + 0.725p^2}$$

$$0.4 \leq p \leq 1$$

$$m_2 = 0.85 + 2.03p - 2.88p^2$$
$$\varphi_2 = \frac{1}{0.368 + 0.645p - 0.862p^2}$$

For

$$0 \leq p \leq 0.252$$

$$m_{pc} = 1.51 (1 - 0.185p^2)$$

$$0.252 \leq p \leq 1$$

$$m_{pc} = 2.58 (0.52 + p) (1 - p)$$

(5)

Strong Axis Bending
with Residual Stress

For

$$0 \leq p \leq 0.8$$

$$m_1 = 0.9 - p$$
$$\varphi_1 = 0.9 - p$$

$$0.8 \leq p \leq 1$$

$$m_1 = -1.1 + 3.1p - 2p^2$$
$$\varphi_1 = 3.3 - 8p + 5p^2$$

For

$$0 \leq p \leq 0.225$$

$$m_2 = 1.11 - 2.64p^2$$
$$m_a = 0.9 + 1.94p - 9.4p^2$$
$$\varphi_2 = \frac{1}{1.11 - 7.35p + 29.2p^2}$$

$$0.225 \leq p \leq 1$$

$$m_2 = 1.238 - 1.143p - 0.095p^2$$
$$m_a = 1.1 (1-p)$$
$$\varphi_2 = 1.3 - p$$
<table>
<thead>
<tr>
<th>Weak Axis Bending with Residual Stress</th>
<th>Square Tubular Section No Residual Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6)</td>
<td>(7)</td>
</tr>
</tbody>
</table>
| \begin{align*} 
  \varphi_1 & = 0.9 - p \\
  m_1 & = 0.9 - p \\
  m_2 & = 0.9 + p - 2.5p^2 \\
  \varphi_2 & = \frac{1}{1.11 - 2.11p + 2.81p^2} \\
 0.4 \leq p \leq 1 \end{align*} | \begin{align*} 
  \varphi_1 & = 1 - p \\
  m_1 & = 1 - p \\
  \varphi_2 & = \frac{1}{1.51 - 1.31p - 0.2p^2} \\
 0.467 \leq p \leq 1 \end{align*} |
| For \( 0 \leq p \leq 0.4 \) | For all \( p \) |
| \( m_{PC} = 1.51 - 0.28p^2 \) | \( m_{PC} = 1.20 - 1.60p^2 \) |
| \( 0.252 \leq p \leq 1 \) | \( m_2 = 1 + 0.9p - 3.25p^2 \) |
| \( m_{PC} = 2.58 (0.52 + p)(1 - p) \) | \( \varphi_2 = 2.50 (1 - p) \) |
TABLE 2
ARGUMENTS OF SUBROUTINE BC

<table>
<thead>
<tr>
<th>Case</th>
<th>Purpose</th>
<th>FKLX</th>
<th>ARG</th>
<th>DINCR</th>
<th>TOLARG</th>
<th>SIGN</th>
<th>ICASE</th>
<th>UL</th>
<th>IGROUP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Start and solve or solve (extend Case 1)</td>
<td>FKL2, Eq. (45)</td>
<td>$p_1$</td>
<td>0.0</td>
<td>DRH012</td>
<td>TOL2</td>
<td>-1.0</td>
<td>2</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>Start and solve</td>
<td>FKL3, Eq. (48)</td>
<td>$p_1$</td>
<td>-0.0</td>
<td>DRH014</td>
<td>TOL2</td>
<td>-1.0</td>
<td>4</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>Start only</td>
<td>FKL4, Eq. (56) (q=0)</td>
<td>$p_1$</td>
<td>-0.0</td>
<td>DRH014</td>
<td>TOL2</td>
<td>-1.0</td>
<td>4</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>Start and solve or solve (extend Case 3)</td>
<td>FKL5, Eq. (64)</td>
<td>$p_1$</td>
<td>-0.0</td>
<td>DRH014</td>
<td>TOL2</td>
<td>-1.0</td>
<td>5</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>Start and solve</td>
<td>FKL6, Eq. (70)</td>
<td>$p_1$</td>
<td>-0.0</td>
<td>DRH014</td>
<td>TOL2</td>
<td>-1.0</td>
<td>6</td>
<td>0.0</td>
</tr>
</tbody>
</table>
### TABLE 3

**SUBROUTINE FKL2A**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>ENTRY point</th>
<th>Corresponding Equation/s (Chap. 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FKL2A (CALL-s CALC)</td>
<td>42 ($\xi=0$), 43 (CALC), 45 ($\xi=0$)</td>
</tr>
<tr>
<td>2</td>
<td>FKL2</td>
<td>42, 44, 45 (Note: FKL2 is a part of FKL2A, hence, 43 is not re-evaluated)</td>
</tr>
<tr>
<td>3</td>
<td>FKL4 (CALL-s CALC)</td>
<td>43 (CALC), 53 (q=0, hence, R=0), 54, 56 (q=0), 57</td>
</tr>
<tr>
<td>4</td>
<td>FKL3</td>
<td>48, 49, 52</td>
</tr>
<tr>
<td>5</td>
<td>FKL5 (CALL-s FKL6)</td>
<td>64 (Part in FKL6), 65, 66, 67</td>
</tr>
<tr>
<td>6</td>
<td>ONEA</td>
<td>41, 62</td>
</tr>
<tr>
<td>7</td>
<td>ONE</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>TWOA</td>
<td>46</td>
</tr>
<tr>
<td>9</td>
<td>TWOB (CALL-s CASE24)</td>
<td>47 (Part in CASE24)</td>
</tr>
<tr>
<td>10</td>
<td>FOURB (CALL-s CASE24)</td>
<td>60 (Part in CASE24)</td>
</tr>
<tr>
<td>11</td>
<td>FOURC</td>
<td>61</td>
</tr>
<tr>
<td>12</td>
<td>FOURD</td>
<td>63</td>
</tr>
<tr>
<td>13</td>
<td>THREE</td>
<td>51</td>
</tr>
<tr>
<td>14</td>
<td>FIVEA</td>
<td>68</td>
</tr>
<tr>
<td>15</td>
<td>FIVEB</td>
<td>69, 72</td>
</tr>
</tbody>
</table>
### TABLE 4

**STRUCTURE OF THE COMPUTER PROGRAM**

<table>
<thead>
<tr>
<th>No.</th>
<th>ROUTINE</th>
<th>ENTRY points</th>
<th>EXTERNALs</th>
<th>CALL-s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>User's program</td>
<td>User's program</td>
<td></td>
<td>BMCOL</td>
</tr>
<tr>
<td>2</td>
<td>BMCOL</td>
<td>BMCOL</td>
<td>FKL2,FKL4, FKL3,FKL5, FKL6,ONEA, ONE</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>BC</td>
<td>BC</td>
<td>TWOA,TWOB, THREE, FIVEA, FIVEB, FKL2A</td>
<td>FKL2A,LABELS, XVSFI,FKL6,FKL4,FKL3,FKL5,FKL6,ONEA,ONE,HALVE,QMAX,PHIVSX</td>
</tr>
<tr>
<td>4</td>
<td>FKL2A</td>
<td>See Table 3 (15 ENTRY points)</td>
<td></td>
<td>CALC,FKL6,CASE24</td>
</tr>
<tr>
<td>5</td>
<td>CALC</td>
<td>CALC,CASE24, FKL6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>HALVE</td>
<td>HALVE</td>
<td></td>
<td>FKLX(= FKL2A,FKL2, FKL4,FKL3, FKL5, or FKL6)</td>
</tr>
<tr>
<td>7</td>
<td>XVSFI</td>
<td>XVSFI,XVSFE</td>
<td></td>
<td>XVSFIX (= any one of items 6 thru 15 of Table 3)</td>
</tr>
<tr>
<td>8</td>
<td>BLOCK DATA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>LABELS</td>
<td>LABELS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>XVSPHI</td>
<td>XVSPHI,PHIVSX</td>
<td>TWOA, FOURB, FOURC, ONEA , FOURD</td>
<td>XVSFI,XVSFE</td>
</tr>
<tr>
<td>11</td>
<td>QMAX</td>
<td>QMAX,MAXQ</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
26. FIGURES
Case 1
(a) Elastic

Case 2
(b) One Side Plastic

Case 3

Fig. 1 Eccentrically Loaded Beam-Column
Fig. 1 Eccentrically Loaded Beam-Column (Concluded)
Fig. 2 Moment-Curvature-Thrust Relationship for a Common Structural Section
Fig. 3 General Flowchart
Fig. 3 General Flowchart (Concluded)
Solve Case 1

Investigate Case 1, Case 2 solutions.
If Case 4 is the starting case, find starting conditions.
Evaluate program constants TB and TC.

\[
\text{QM}(1) = \text{TEMPQ} \\
I = 2 \\
T = \text{RHO1L}
\]

\[
\text{DRHO1L} = \text{DRHO14} \\
UL = 0.0
\]

\[
\alpha \\
\text{p} \neq 0 \\
\text{F}
\]

\[
\text{RH02L} = \text{RHO1L} \\
Q = \text{TEMPY}/\text{RH02L}
\]

\[
\text{Q} \geq 1.0 \\
\text{T}
\]

\[
\text{Computes PHIM}
\]

\[
\text{PHIM} > \text{TPHI2} \\
\text{F}
\]

\[
\text{LESS} \\
\text{F}
\]

\[
\text{CALL}
\]

\[
\text{QMAX}
\]

\[
\beta
\]

\[
\gamma
\]

\[
\delta
\]

\[
\text{RETURN}
\]

Fig. 4 Flowchart - Case 4
Fig. 5. Flowchart - SUBROUTINE BC

-89-
Fig. 5 Flowchart - SUBROUTINE BC (Contd.)

-90-
CALL MAXQ

DQ = -DQ

Q =/small/

HALVE (Q,...,FKLX)

NCASE2

PHIM > TEMP

Q < 0

LESS

CALL QMAX

MESSAGE

Fig. 5 Flowchart - SUBROUTINE BC (Concluded)

-91-
Figs. 6, 7 Plot of LHS vs. ARG - SUBROUTINE HALVE
Fig. 8 Flowchart - SUBROUTINE HALVE
Fig. 9 Non-Dimensionalized Curvature vs. Length Diagram
Fig. 10 Flowchart - SUBROUTINE XVSFI

-95-
Figs. 11, 12, 13, 14  Plot of q vs. $\phi_m$ - SUBROUTINE QMAX

-96-
Figs. 15, 16, 17, 18 Plot of $q$ vs. $q_m$ - SUBROUTINE QMAX (Concluded)
Fig. 19 Values of LESS and MESAGE - SUBROUTINE QMAX
Fig. 20 Flowchart - SUBROUTINE QMAX

-99-
Fig. 21 Plot of RANGE vs. p (NTYPE = 2, R = 1.40)
27. APPENDICES
APPENDIX 1

IYENGAR, A2023, CM60000, T20, *IYENGAR.
RUN(S)
LGO.
7/8/9 MULTIPLE PUNCHES IN COLUMN 1, END-OF-RECORD CARD.

PROGRAM USER (OUTPUT, TAPE5=OUTPUT)
REAL LOVERR
COMMON /BL/ DPHI3, DPHI5, DPHI6, DRHO12, DRHO14, E, EOVERR, HOVERR, INDEX,
1 INDR, IRITE, LOVERR, NSUB, NTYPE, P, R, RANGE, SIGMAY, TOL1, TOL2, TOL3
DATA DPHI3, DPHI5, DPHI6, DRHO12, DRHO14, E/3*0.5, 0.05, 0.01, 30000.0/
DATA INDEX, INDR, IRITE, NSUB/1, 0, 6, 10/
DATA SIGMAY, TOL1, TOL2, TOL3/34.0, 1.0E-9, 1.0E-3/
NTYPE=5
EOVERR=8.0/3.47
LOVERR=20.0
EOVERR=1.0
RANGE=10.0
DO 1000 NP=40, 95, 5
P=NP
P=P*0.01
CALL BMCOL (INTER)
IF (INTER .NE. 0) GO TO 1010
1000 CONTINUE
1010 CALL EXIT
END

SOURCE CARDS (FORTRAN) OF SUBROUTINE BMCOL, ETC (SEE APPENDIX 2).
6/7/8/9 MULTIPLE PUNCHES IN COLUMN 1, END-OF-FILE (ORANGE) CARD.
GENERAL SOLUTION OF AN INELASTIC BEAM-COLUMN PROBLEM

1. MATERIAL PROPERTIES
   \( E = \text{MODULUS OF ELASTICITY} = 30000.0 \text{ KSI} \)
   \( \sigma_Y = \text{YIELD STRESS} = 34.00 \text{ KSI} \)

2. SECTION AND GEOMETRY
   \( \text{NYTYPE} = \text{NATURE OF SECTION} = 5 \) (WF SHAPE, STRONG AXIS BENDING WITH RESIDUAL STRESSES)
   \( \alpha = \text{SHAPE FACTOR} = 1.11 \)
   \( \text{LOVERR} = \text{SLENDERNESS RATIO} = \frac{L}{R} = 20.0 \)
   \( \text{DOVERR} = \text{DEPTH RATIO} = \frac{H}{R} = 2.31 \)
   \( \text{R} = \text{SHAPE VARIABLE} = 0.00 \)

3. LOADING CONDITIONS
   \( \text{ECCENTRICITY RATIO} = \frac{E}{R} = 1.000 \)
   \( \text{P} = \text{ECCENTRIC LOAD (FRACTION OF YIELD LOAD)} = 0.400 \)
   \( \text{MZERO} = \text{END MOMENT} = 0.461095 \)

4. GENERALIZED STRESS-STRAIN RELATIONSHIP
   \( A = 1.000000 \)
   \( B = 1.128328 \)
   \( C = 0.444295 \)
   \( F = 0.985936 \)
   \( M_P = 0.765600 \)
   \( \phi_I = 0.500000 \)
   \( \phi_2 = 0.900000 \)

5. PRESCRIBED INCREMENTS, ACCURACIES AND RANGE
   INCREMENTS
   \( \text{DRO1} = \text{INCREMENT OF RO1/L IN CASE 2} = 0.050000 \)
   \( \text{DRO14} = \text{INCREMENT OF RO1/L IN CASE 4} = 0.010000 \)
   \( \text{DPHA3} = \text{INCREMENT OF PHM IN CASE 3} = 0.500000 \)
   \( \text{DPHA5} = \text{INCREMENT OF PHM IN CASE 5} = 0.500000 \)
   \( \text{DPHA6} = \text{INCREMENT OF PHM IN CASE 6} = 0.500000 \)

   ACCURACIES
   \( \text{TOL1} = \text{TOLERANCE ON Q} = 1.00E-09 \)
   \( \text{TOL2} = \text{TOLERANCE ON RO1/L} = 1.00E-06 \)
   \( \text{TOL3} = \text{TOLERANCE ON PHM} = 1.00E-03 \)

   RANGE
   \( \text{RANGE} = \text{LIMIT OF PHM} = 10.00 \text{ TIMES PHI2} \)

6. END SLOPE AND CENTRAL DEFLECTION
   \( \text{INDEX} = 1 \) (RESULTS PRINTED)
   \( \text{INDR} = 0 \) (CURVATURE VS. LENGTH RESULTS SUPPRESSED)
   \( \text{NSUN} = \text{NUMBER OF SUBDIVISIONS} = 10 \)

7. \( \text{KL} = 4254.793 \)
GROUP 1, CASE 1

<table>
<thead>
<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
<th>END SLOPE</th>
<th>CENTRAL DEFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000</td>
<td>0.471748</td>
<td>0.500000</td>
<td>0.500000</td>
<td>4.6029E-03</td>
<td>1.0020E-02</td>
</tr>
<tr>
<td>0.025967</td>
<td>0.471748</td>
<td>0.500000</td>
<td>0.500000</td>
<td>4.7423E-03</td>
<td>1.0422E-02</td>
</tr>
</tbody>
</table>

GROUP 1, CASE 2

<table>
<thead>
<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
<th>END SLOPE</th>
<th>CENTRAL DEFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025967</td>
<td>0.471748</td>
<td>0.500000</td>
<td>0.500000</td>
<td>4.7423E-03</td>
<td>1.0422E-02</td>
</tr>
<tr>
<td>0.050136</td>
<td>0.471748</td>
<td>0.500000</td>
<td>0.500000</td>
<td>4.5764E-03</td>
<td>1.0162E-02</td>
</tr>
</tbody>
</table>

GROUP 1, CASE 4

<table>
<thead>
<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
<th>END SLOPE</th>
<th>CENTRAL DEFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.146449</td>
<td>0.000000</td>
<td>0.02749</td>
<td>0.99999</td>
<td>6.2840E-03</td>
<td>1.5183E-02</td>
</tr>
<tr>
<td>0.145782</td>
<td>1.028193</td>
<td>0.02749</td>
<td>0.99999</td>
<td>6.1467E-03</td>
<td>1.5183E-02</td>
</tr>
</tbody>
</table>

THE NEXT SET OF VALUES CORRESPONDS TO A VALUE OF Q WHICH IS VERY CLOSE TO THE ACTUAL MAXIMUM VALUE OF Q.
GENERAL SOLUTION OF AN INELASTIC BEAM-COLUMN PROBLEM

1. MATERIAL PROPERTIES
   F = MODULUS OF ELASTICITY = 30000.0 KSI  SIGMA Y = YIELD STRESS = 34.00 KSI

2. SECTION AND GEOMETRY
   NTYPF = NATURE OF SECTION = 5 (WF SHAPE, STRONG AXIS BENDING WITH RESIDUAL STRESSES)
   ALPHA = SHAPE FACTOR = 1.11
   LOVERR = SLENDERNESS RATIO = L/R = 20.0
   HOVERR = DEPTH RATIO = H/R = 2.31  R = SHAPE VARIABLE = 0.00

3. LOADING CONDITIONS
   EOVERR = ECCENTRICITY RATIO = E/R = 1.000  P = ECCENTRIC LOAD (FRACTION OF YIELD LOAD) = .450
   MZERO = END MOMENT = .518732

4. GENERALIZED STRESS-STRAIN RELATIONSHIP
   A = 1.000000  R = 1.019031  C = .381717  F = .071826
   HPC = .704412  PHI1 = .450000  PHI2 = .050000

5. PRESCRIBED INCREMENTS, ACCURACIES AND RANGE
   INCREMENTS
   DRHO12 = INCREMENT OF RHO1/L IN CASE 2 = .050000
   DRHO14 = INCREMENT OF RHO1/L IN CASE 4 = .010000
   DPHI3 = INCREMENT OF PHIM IN CASE 3 = .500000
   DPHI5 = INCREMENT OF PHIM IN CASE 5 = .500000
   DPHI6 = INCREMENT OF PHIM IN CASE 6 = .500000
   ACCURACIES
   TOL1 = TOLERANCE ON Q = 1.00E-09
   TOL2 = TOLERANCE ON RHO1/L = 1.00E-06
   TOL3 = TOLERANCE ON PHIM = 1.00E-03
   RANGE
   RANGE = LIMIT OF PHIM = 10.00 TIMES PHI2

6. END SLOPE AND CENTRAL DEFLECTION
   INDX = 1 (RESULTS PRINTED )
   IND2 = 5 (CURVATURE VS. LENGTH RESULTS SUPPRESSED)
   NSUB = NUMBER OF SUBDIVISIONS = 10

7. KL = .451644
### GROUP 2, CASE 3

<table>
<thead>
<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
<th>END SLOPE</th>
<th>CENTRAL DEFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>620223</td>
<td></td>
<td></td>
<td>5.9595E-03</td>
<td>1.3074E-02</td>
</tr>
<tr>
<td>0.03643</td>
<td>745223</td>
<td></td>
<td></td>
<td>6.5543E-03</td>
<td>1.4794E-02</td>
</tr>
<tr>
<td>0.05125</td>
<td>807723</td>
<td></td>
<td></td>
<td>6.8291E-03</td>
<td>1.5611E-02</td>
</tr>
<tr>
<td>0.05801</td>
<td>939973</td>
<td></td>
<td></td>
<td>6.9528E-03</td>
<td>1.6011E-02</td>
</tr>
<tr>
<td>0.05964</td>
<td>846795</td>
<td></td>
<td></td>
<td>6.9959E-03</td>
<td>1.6110E-02</td>
</tr>
<tr>
<td>0.06048</td>
<td>848739</td>
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The next set of values corresponds to a value of Q which is very close to the actual maximum value of Q.

### GROUP 2, CASE 5

<table>
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<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
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<th>CENTRAL DEFLECTION</th>
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1. MATERIAL PROPERTIES
   \( E = \text{MODULUS OF ELASTICITY} = 30000.0 \, \text{ksi} \)
   \( \text{SIGMAY} = \text{YIELD STRESS} = 34.00 \, \text{ksi} \)

2. SECTION AND GEOMETRY
   \( \text{NTYPE} = \text{NATURE OF SECTION} = 5 \) (WF SHAPE, STRONG AXIS BENDING WITH RESIDUAL STRESSES)
   \( \text{ALPHA} = \text{SHAPE FACTOR} = 1.11 \)
   \( \text{LOWERR} = \text{SLENDERNESS RATIO} = L/R = 20.0 \)
   \( \text{HOVERR} = \text{DEPTH RATIO} = H/R = 2.31 \)
   \( \text{R} = \text{SHAPE VARIABLE} = 0.00 \)

3. LOADING CONDITIONS
   \( \text{EOVERR} = \text{ECCENTRICITY RATIO} = E/R = 1.000 \)
   \( \text{P} = \text{ECCENTRIC LOAD (FRACTION OF YIELD LOAD)} = .500 \)
   \( \text{HZERO} = \text{END MOMENT} = .976369 \)

4. GENERALIZED STRESS-STRAIN RELATIONSHIP
   \( A = 1.000000 \)
   \( R = .912132 \)
   \( C = .323901 \)
   \( F = .059960 \)
   \( \text{MPC} = .642750 \)
   \( \text{PHI1} = .400000 \)
   \( \text{PHI2} = .800000 \)

5. PRESCRIBED INCREMENTS, ACCURACIES AND RANGE
   \( \text{DRHO12} = \text{INCREMENT OF RH01/L IN CASE 2} = .050000 \)
   \( \text{DRHO14} = \text{INCREMENT OF RH01/L IN CASE 4} = .010000 \)
   \( \text{DPHI3} = \text{INCREMENT OF PHIM IN CASE 3} = .500000 \)
   \( \text{DPHI5} = \text{INCREMENT OF PHIM IN CASE 5} = .500000 \)
   \( \text{DPHI6} = \text{INCREMENT OF PHIM IN CASE 6} = .500000 \)
   \( \text{ACCUACIES} \)
   \( \text{TOL1} = \text{TOLERANCE ON G} = 1.00E-09 \)
   \( \text{TOL2} = \text{TOLERANCE ON RH01/L} = 1.00E-05 \)
   \( \text{TOL3} = \text{TOLERANCE ON PHIM} = 1.00E-03 \)
   \( \text{RANGE} \)
   \( \text{RANGE} = \text{LIMIT OF PHIM} = 10.00 \text{ TIMES PHI2} \)

6. END SLOPE AND CENTRAL DEFLECTION
   \( \text{INDEX} = 1 \) (RESULTS PRINTED )
   \( \text{INDR} = 0 \) (CURVATURE VS. LENGTH RESULTS SUPPRESSED)
   \( \text{NSUM} = \text{NUMBER OF SUBDIVISIONS} = 10 \)

7. KL = .674695
GROUP 3, CASE 6

<table>
<thead>
<tr>
<th>Q</th>
<th>PHIM</th>
<th>RH01/L</th>
<th>RH02/L</th>
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<th>CENTRAL DEFLECTION</th>
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THE NEXT SET OF VALUES CORRESPONDS TO A VALUE OF Q WHICH IS VERY CLOSE TO THE ACTUAL MAXIMUM VALUE OF Q

<table>
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<th>CENTRAL DEFLECTION</th>
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</table>
GENERAL SOLUTION OF AN INELASTIC BEAM-COLUMN PROBLEM

1. MATERIAL PROPERTIES
   \( E = \text{MODULUS OF ELASTICITY} = 30000.0 \text{ KSI} \)
   \( \text{SIGMA} = \text{YIELD STRESS} = 34.00 \text{ KSI} \)

2. SECTION AND GEOMETRY
   \( \text{NTYPE} = \text{NATURE OF SECTION} = 5 \)  
   (WF SHAPE, STRONG AXIS BENDING WITH RESIDUAL STRESSES)
   \( \text{ALPHA} = \text{SHAPE FACTOR} = 1.11 \)
   \( \text{LOVERR = SLENDERNES RATIO} = L/R = 20.0 \)
   \( \text{HOVERR = DEPTH RATIO} = H/R = 2.31 \)
   \( R = \text{SHAPE VARIABLE} = 0.00 \)

3. LOADING CONDITIONS
   \( \text{EOVERR = ECCENTRICITY RATIO} = E/R = 1.000 \)
   \( P = \text{ECCENTRIC LOAD (FRACTION OF YIELD LOAD)} = 0.550 \)
   \( \text{MZERO = END MOMENT} = 0.634086 \)

4. GENERALIZED STRESS-STRAIN RELATIONSHIP
   \( A = 1.000000 \)
   \( B = 0.807601 \)
   \( C = 0.270720 \)
   \( F = 0.048157 \)
   \( \text{MPC} = 0.580612 \)  \( \text{PHI1} = 0.390000 \)
   \( \text{PHI2} = 0.750000 \)

5. PRESCRIBED INCREMENTS, ACCURACIES AND RANGE
   \( \text{ORHO12 = INCREMENT OF RH01/L IN CASE 2} = 0.050000 \)
   \( \text{ORHO14 = INCREMENT OF RH01/L IN CASE 4} = 0.100000 \)
   \( \text{DPHI3 = INCREMENT OF PHIM IN CASE 3} = 0.500000 \)
   \( \text{DPH15 = INCREMENT OF PHIM IN CASE 5} = 0.500000 \)
   \( \text{DPH16 = INCREMENT OF PHIM IN CASE 6} = 0.500000 \)
   \( \text{ACCURACIES} \)
   \( \text{TOL1 = TOLERANCE ON G} = 1.00E-09 \)
   \( \text{TOL2 = TOLERANCE ON RH01/L} = 1.00E-06 \)
   \( \text{TOL7 = TOLERANCE ON PHIM} = 1.00E-03 \)
   \( \text{RANGE} \)
   \( \text{RANGE = LIMIT OF PHIM} = 10.00 \text{ TIMES PHI2} \)

6. END SLOPE AND CENTRAL DEFLECTION
   \( \text{INDEX} = 1 \) (RESULTS PRINTED )
   \( \text{INDR} = 0 \) (CURVATURE VS LENGTH RESULTS SUPPRESSED)
   \( \text{NSUB} = \text{NUMBER OF SUBDIVISIONS} = 10 \)

7. \( KL = 1.499773 \)
MZERO EXCEEDS MPC, HENCE NO SOLUTION.
SUBROUTINE BMCOL (INTER)

AUTHOR-SAMPATH IYENGAR, FRITZ ENGINEERING LABORATORY,
LEHIGH UNIVERSITY, BETHLEHEM, PA. 18015.

THIS GROUP OF SUBPROGRAMS IS WRITTEN TO SOLVE THE PROBLEM OF A
BEAM-COLUMN IN BOTH THE ELASTIC AND THE INELASTIC RANGES.
ASSUMPTIONS - THRUST IS APPLIED FIRST AT THE ENDS OF THE
BEAM-COLUMN, WITH OR WITHOUT ECCENTRICITY. BOTH THRUST AND
ECCENTRICITY REMAIN CONSTANT.
THE BEAM-COLUMN IS NEXT SUBJECT TO A VARIABLE CONCENTRATED LATERAL
LOAD AT M IDSPAN.
THE PROGRAM PREDICTS THE CENTRAL CURVATURE FOR EACH VALUE OF THE
CONCENTRATED LOAD.

ALL QUANTITIES CONSIDERED ARE NON-DIMENSIONALIZED WITH RESPECT TO
QUANTITIES AT INITIAL YIELD.

FOR DETAILED DOCUMENTATION, SEE FRITZ ENGINEERING LABORATORY
REPORT NO. 331.,.

INPUT TO THE ROUTINE IS THE LIST IN THE COMMON BLOCK LABELLED BL.

LOGICAL LESS,MESAGE,NCVSL,NIR,PZERO,START,TYPE
COMMON /BL/ DPHI3,DPHI5,DPHI6,DRH012,DRH014,E,E0VRR,HOVRR,INDEX,
1INDR,IRITE,LOVRR,NSUP,NTYPE,P,R,RANGE,SIGMAY,TOL1,TOL2,TOL3
COMMON /BLA/ AKL,ARSHO,BIGQ,B,CKL2SA,DEFLN,DM,DQ,F,ALPHA,FKL,IV
1AR(15),JVAR(5),KL,KLS,KL5A,KL2,KL2SA,KRH01,LESS,LVART(16),MESAGE,MZ
ZEROA,NCVSL,NIR,PHI,PHIM,PHIZ,PHI1,PHI2,PZERO,0,RT(3),RHO41,RH02L,R
3PHIM,RPHIZ,SA,SETA,SHC,SINA,SLOPE,SM,SMALLQ,SPHIZ,SPH1,SPHI2,SSET
4A,START,SUB,S2G,T4,TE,TEMPA,TEMPB,TEMPK,TEMPM,TEMPO,TEMPX,TF
5,TG,TH,THIRD,TPHI2,TYPE,T1,T3,T4,T8,XOVELZ,ZETA
DIMENSION TYPE1(2), TYPE2(2)
REAL KL,KLS,KLSA,KLSA2,KL2,KL2SA,LOVRR,M,MPC,MZERO,MZEROA,M1,M2
EXTERNAL FKL2,FKL4,FKL3,FKL5,FKL6
EXTERNAL ONEA,ONE
CROOT(ARG)=SIGN(1.0,ARG)*ABS(ARG)**THIRD

INITIALIZE VARIABLE INTER TO 1 (SEE CHAPTER 6).
START COMPUTATIONS OF GENERALIZED STRESS-STRAIN RELATIONS.

INTER=1
KL=LOVRR*SORT(P*S IGMAY/E)
WRITE (IRITE,1610) E,SIGMAY,NTYPE
PS0=P*P
TYPE=FALSE.

PZERO=P*FO.0,0
NCVSL=INDEX,EQ.0
NIR=INDR,EQ.0
HOVRR=HOVRR
RR=R
A=1.0
PHI1=1.0-P
SPHI1=SQR RT(PHI1)
M1=PHI1
APPENDIX 2

SEVEN TYPES OF CROSS-SECTION CAN BE HANDLED BY THIS PROGRAM.

GO TO (1000, 1010, 1040, 1090, 1140, 1180, 1200), NTYPE

NTYPE = 1, RECTANGULAR CROSS-SECTION.

1000 WRITE (IRITE, 1620)
F=0.5
B=3.0*PHI1
MPC=1.5*(1.0-PSQ)
PHI2=1.0/PHI1
SPHI2=SQRT(PHI2)
H0VRR=3.464101615137754587054092683
ALPHA=1.50
C=2.0*PHI1*SPHI1
GO TO 1250

NTYPE = 2, IDEALIZED WIDE FLANGE SHAPE BENT ABOUT THE STRONG AXIS.

1010 WRITE (IRITE, 1630)
RP1=1.0+R
RP2=RP1+R
RP3=RP2+R
TEMPZ=RP2/RP3
ALPHA=1.5*TEMPZ

IF PGE. 1/(1+R), THE LOGICAL VARIABLE TYPE HAS THE VALUE .TRUE.,
AND THE SECONDARY PLASTIC ZONE DOES NOT EXIST.

TYPE=PGE.1.0/RP1
IF (TYPE) GO TO 1030
C=4.0*TEMPZ/(2.0+R)
B=1.0+C
TEMPZ=RP1*P
IF (PZERO) GO TO 1020
DOWN=RP3*(1.0-SORT(PHI1*(1.0-TEMPZ)))
UP=2.0*P*(RP2-RP1*TEMPZ)
B=PHI1+UP/DOWN
C=SPHI1*UP/DOWN

1020 F=0.5/RP3
MPC=1.5*(RP2-TEMPZ*TEMPZ)/RP3
PHI2=1.0/(1.0-TEMPZ)
SPHI2=SQRT(PHI2)
GO TO 1260

1030 B=3.0*RP1*PHI1/RP3
C=2.0*PHI1*SPHI1/RP3

ARBITRARY VALUES - VALUES OF MPC AND F ARE NOT USED IN
CALCULATIONS, VALUE OF PHI2 IS USED ONLY IN ESTABLISHING THE
RANGE OF THE SOLUTION.

PHI2=100.0
MPC=B
F=0.5
GO TO 1260
C
C NTYPE = 3, WIDE FLANGE SHAPE WITHOUT RESIDUAL STRESSES BENT ABOUT
C THE STRONG AXIS.
C
1040 WRITE (IRITE,1640)
  IF (P.GT.0.225) GO TO 1050
  M2=1.0+0.778*P-4.78*P*P
  PHI2=1.0-3.7*P+8.4*P*P
  PHI2=1.0/PHI2
1050 MPC=1.11-2.64*P*P
  GO TO 1080
1060 M2=1.20*PHI1
  PHI2=2.20*PHI1
1070 MPC=1.238-1.143*P-0.095*P*P
1080 ALPHA=1.11
  GO TO 1230
C
C NTYPE = 4, WIDE FLANGE SHAPE WITHOUT RESIDUAL STRESSES BENT ABOUT
C THE WEAK AXIS.
C
1090 WRITE (IRITE,1650)
  IF (P.GT.0.4) GO TO 1110
  M2=1.0+1.5*P-2.5*P*P
  PHI2=1.0-1.57*P+0.725*P*P
  PHI2=1.0/PHI2
1100 IF (P.GT.0.252) GO TO 1120
  MPC=1.51*(1.0-0.185*P*P)
  GO TO 1130
1110 M2=0.85+2.03*P-2.88*P*P
  PHI2=0.356+0.645*P-0.862*P*P
  PHI2=1.0/PHI2
1120 MPC=2.58*(0.52+P)*(1.0-P)
1130 ALPHA=1.51
  GO TO 1230
C
C NTYPE = 5, WIDE FLANGE SHAPE WITH RESIDUAL STRESSES BENT ABOUT THE
C STRONG AXIS.
C
1140 WRITE (IRITE,1660)
  IF (P.GT.0.8) GO TO 1150
  PHI1=0.9-P
  SPHI1=SORT(PHI1)
  M1=PHI1
  GO TO 1170
1150 M1=-1.1+3.1*P-2.0*P*P
  PHI1=3.3-4.0*P+5.0*P*P
  A=M1/PHI1
  SPHI1=SORT(PHI1)
1160 M2=1.1*(1.0-P)
  PHI2=1.3-P
  GO TO 1070
1170 IF (P.GT.0.225) GO TO 1160
APPENDIX 2

\[ M_2 = 0.9 + 1.94\times P - 9.4\times P^2 \]
\[ \Phi_2 = 1.0 \times \Phi_1 - 7.35\times P + 29.2\times P^2 \]
\[ \Phi_2 = 1.0 \times \Phi_1 \]
GO TO 1050

C

\[ \text{NTYPE} = 6, \text{ WIDE FLANGE SHAPE WITH RESIDUAL STRESSES BENT ABOUT THE WEAK AXIS.} \]

C

1180 WRITE (IRITE,1670)
IF (P.GT.0.4) GO TO 1190
PHI1 = 0.9-P
SPHI1 = SQRT(PHI1)
M1 = PHI1
M2 = 0.9 + 2.5\times P
\[ \Phi_2 = 1.0 \times \Phi_1 - 2.11\times P + 2.81\times P^2 \]
\[ \Phi_2 = 1.0 \times \Phi_1 \]
GO TO 1100

1190 M1 = 0.567 + 0.1\times P - 0.667\times P^2
PHI1 = 0.5
SPHI1 = 0.7071067811865475244008443621
\[ A = M1/\Phi_1 \]
M2 = 1.0 + 0.25\times P - 1.25\times P^2
\[ \Phi_2 = 1.3 - 2.45\times P + 2.45\times P^2 \]
\[ \Phi_2 = 1.0 \times \Phi_1 \]
GO TO 1120

C

\[ \text{NTYPE} = 7, \text{ SQUARE TUBE WITHOUT RESIDUAL STRESSES.} \]

C

1200 WRITE (IRITE,1680)
IF (P.GT.0.467) GO TO 1210
MPC = 1.20 - 1.596\times P
M2 = 1.0 + 0.9\times P - 3.25\times P^2
\[ \Phi_2 = 1.0 - 2.5\times P + 4.17\times P^2 \]
\[ \Phi_2 = 1.0 \times \Phi_1 \]
GO TO 1220

1210 MPC = 1.51 - 1.31\times P - 0.2\times P^2
\[ M2 = 1.40 \times \Phi_1 \]
\[ \Phi_2 = 2.50 \times \Phi_1 \]
\[ \text{ALPHA} = 1.20 \]
1230 SPHI2 = SQRT(PHI2)

C

\[ \text{ARBITRARY VALUES FOR B AND C IF P = 0.0, VALUES NOT USED IN CALCULATIONS.} \]

C

B = 0.0
C = 0.0
IF (P.ZERO) GO TO 1240
T = SPHI2 - SPHI1
\[ B = (M2 \times \Phi_2 - M1 \times \Phi_1)/T \]
\[ C = \Phi_1 \times \Phi_2 \times (M2 - M1)/T \]

1240 F = (MPC - M2) \times \Phi_2 \times \Phi_1
1250 \[ RR = 0.0 \]
1260 MZERO = 0.5\times P*EOVERR*HOVRR

C
APPENDIX 2

PAGE NO. 5

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C ECHO OF INPUT DATA AND PRINTOUT OF PARAMETERS CONCERNING THE
C GENERALIZED STRESS-STRAIN RELATIONSHIP.
C
C WRITE (IRITE,1690) ALPHA,LOVERR,HOVRR,RR,EOVERR,P,MZERO,A,B,C,F,MP
C,PHI1,PHI2,DRH012,DRH014,DPHI3,DPHI5,DPHI6,TOL1,TOL2,TOL3,RANGE
C MODIFY FORMAT ARRAYS TO SUIT PRINTOUT OF END SLOPE AND CENTRAL
C DEFLECTION, IF THESE HAVE BEEN REQUESTED.
C
C TYPE1(1)=5H SUPPR
C TYPE1(2)=5H ESSED
C TYPE2(1)=5H SUPPR
C TYPE2(2)=5H ESSED
C IND=0
C IF (NCVSL) GO TO 1270
C TYPE1(1)=5H PRIN
C TYPE1(2)=5H TDED
C IF (NIR) GO TO 1270
C TYPE2(1)=5H PRIN
C TYPE2(2)=5H TDED
C IND=INDP
C IVAR(12)=6H3HX/L,
C IVAR(13)=6H17X,
C IVAR(14)=6H3HPHI/
C LVAR(1)=6H4(H0,4
C 1270 WRITE (IRITE,1700) INDEX,TYPE1,IND,TYPE2,NSUB,KL
C
C PRELIMINARIES CONCLUDED.
C
C RETURN CONTROL TO USER'S PROGRAM IF EITHER MZERO EXCEEDS MPC OR
C KL EXCEEDS PI, SINCE NO SOLUTION CAN EXIST IN THESE INSTANCES.
C INTER RETAINS THE VALUE 1.
C
C IF (MZERO.GE.MPC) GO TO 1580
C IF (KL.GE.3.1415926535897932384626433832) GO TO 1590
C
C A SOLUTION IS LIKELY. RESET INTER TO ZERO.
C
C INTER=0
C
C EVALUATE CERTAIN PROGRAM CONSTANTS.
C
C DM=LOVERR/HOVRR
C SM=2.0*SIGMAY/E*DM
C SUB=NSUB
C TF=2.0*SORT(F)*THIRD
C TG=TF/KL
C IF (PZERO) GO TO 1280
C TA=2.0/C*F/PHI2
C S2C=SORT(2.0*C)
C SHC=0.5*S2C
C ARSCH=ALPHA/SHC
C 1280 TPHI2=PHI2*RANGE
MZEROA = MZERO / A
KL2 = 0.5 * KL

C
C INITIALIZE VARIABLES AS UNDER.
C
Q = 0.0
START = .TRUE.
LESS = .FALSE.
MESSAGE = .FALSE.
RHO1L = 0.5

C
C ELIMINATION BY GROUPS.
C IF MZERO / A .LE. PHI1, GROUP 1 CONTROLS. ELSE, INVESTIGATE GROUP 2.
C
IF (MZEROA, GT, PHI1) GO TO 1530

C
C ELIMINATION BY CASES.
C IF P IS ZERO, SOLVE CASE 1.
C
IF (.NOT.PZERO) GO TO 1310
AALPHA = V / ALPHA
PHIM = 0.0
CALL LABELS (1, 1)
WRITE (IRITEx, JVAR) Q, PHIM
IF (NCVSL) GO TO 1290
SLOPE = 0.0
DEFLN = 0.0
WRITE (IRITEx, LVAR) SLOPE, DEFLN

1290
Q = AALPHA * PHI1
PHIM = PHI1

C
C PRINT RESULTS.
C
WRITE (IRITEx, JVAR) Q, PHIM

C
C COMPUTE AND PRINT END SLOPE AND CENTRAL DEFLECTION IF THESE HAVE BEEN REQUESTED.
C
IF (NCVSL) GO TO 1300
TEMPA = PHI1 + PHI1
CALL XVSFI (XOVERL, ONEA, 0.0, 0.5)
WRITE (IRITEx, LVAR) SLOPE, DEFLN

C
C CASE 1 SOLUTION ENDS (P = 0.0).
C EVALUATE PARAMETERS FOR CASE 4 SOLUTION.
C
1300
FALPHA = F / ALPHA
TEMPY = AALPHA * PHI2 * 0.5
CALL LABELS (1, 4)

C
C NO #MAXIMUM# FOR Q IF P = 0.0, HENCE LESS = .TRUE..
C
LESS = .TRUE.
APPENDIX 2

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C PROCEED TO CASE 4 DIRECTLY, AS THERE IS NO CASE 2 SOLUTION WHEN P = 0.0.
C
C GO TO 1400
C
C MORE PROGRAM CONSTANTS (P IS NOT ZERO).
C
1310 SA=SORT(A)
  KLS=KL*KL
  TE=TF/PHI2/SPHI2
  TB=MZEROA/PHI1
  KL2SA=KL2/SA
  KLSA2=KL2*SA
  KLSA=KL/SA
  CKL2SA=COS(KL2SA)
  TEMPX=0.5*PHI1*SPHI1*A/C

C STARTING CASE IS CASE 1 IF PHIM (Q = 0.0) .LE. PHI1.
C
PHIM=MZEROA/CKL2SA
IF (PHIM.GT.PHI1) GO TO 1340
C
SOLVE CASE 1.
C
CALL LABELS (1,1)
WRITE (IRITE,JVAR) Q,PHIM
IF (NCVSL) GO TO 1320
TEMPA=0.0
CALL XVSFI (XOVERL,ONE,0.0,0.5)
WRITE (IRITE,LVAR) SLOPE,DEFNL
1320 Q=KLSA2/ALPHA*TEMPA

C RESET START TO .FALSE., AS CASE 1 IS THE STARTING CASE.
C
START=.FALSE.
PHIM=PHI1
WRITE (IRITE,JVAR) Q,PHIM
IF (NCVSL) GO TO 1330
CALL XVSFI (XOVERL,ONE,0.0,0.5)
WRITE (IRITE,LVAR) SLOPE,DEFNL

C CASE 1 SOLUTION ENDS (P IS NOT ZERO).
C
1330 Q=Q-TOL1
1340 DINCR=-DRHO12
C
INVESTIGATE CASE 2 SOLUTION IN SUBROUTINE BC.
C
CALL BC (FKL2,RHO1L,DINCR,TOL2,-1.0,2,0.0,1)
C
IF TYPE = .TRUE., THE SOLUTION IS COMPLETE, RETURN CONTROL TO
C USER'S PROGRAM.
IF (TYPE) GO TO 1350

C
C IF CASE 2 ALSO IS NOT THE STARTING CASE, CASE 4 MAY BE.
C IF THE BEAM-COLUMN UNLOADED IN CASE 2, RETURN CONTROL.
C OTHERWISE, SEEK AN EXTENSION OF THE SOLUTION IN CASE 4.
C
C IF (START) GO TO 1360
IF (PHIM.LT.PHI2) GO TO 1600
GO TO 1380

1350 IF (START) GO TO 1370
GO TO 1600

1350 WRITE (IRITE, 1740)
DINCR=-DRHO14

C CALL SUBROUTINE BC TO PROVIDE THE STARTING SITUATION IN CASE 4.
C
CALL BC (FKL4, RHO1L, DINCR, TOL2, -1.0, 4, 0.0, 1)
RHO1L=RHO1L-TOL2

C IF CASE 4 ALSO IS NOT THE STARTING CASE, RETURN CONTROL AFTER
C resetting INTER TO 1.
C
IF (.NOT. START) GO TO 1390

1370 WRITE (IRITE, 1750) RANGE
INTER=1
GO TO 1600

1380 CALL LABELS (1,4)

C MORE PROGRAM CONSTANTS FOR CASE 4 (P IS NOT ZERO).
C
1390 TB=KLS*3.0*F/PHI2
TC=0.375*F*KL*KLS
QM(1)=TEMPQ
I=2

C #LOWER# LIMIT OF RHO1/L.
C
T=RHO1L
C
DECREMENT AND #UPPER# LIMIT OF RHO1/L IN CASE 4.
C
1400 DRIHO1L=DRHO14
UL=0.0
C
C CASE 4 SOLUTION.
C
1410 IF (.NOT.PZERO) GO TO 1420
C
P IS ZERO.
C
RH02L=RHO1L
Q=TEMPY/RH02L
C
RESET UPPER LIMIT IF Q EXCEEDS 1.0.
APPENDIX 2

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C
IF (Q.GE.1.0) GO TO 1520
PHIM=SQRTR(FALPHA/(1.0-Q))
GO TO 1470
C
P IS NOT ZERO.
C
TEST RH01/L AGAINST LOWER LIMIT.
C
1420 IF (RH01L.GT.T) RH01L=T
C
SOLUTION OF CUBIC EQUATION.
C
CALL FKL4
IF (TEMPK.LT.1.0) GO TO 1520
AA=ZETA*TB
BB=TC*FKL
DISCR=BB*BR+AA*3/2.7
IF (DISCR) 1430,1440,1450
M=2.0*SORT(-AA*THIRD)
AQ=M*COS(THIRD*ACOS(6.0*R9/AA/M))
GO TO 1460
1440 AQ=-2.0*CROOT(BB)
GO TO 1460
1450 DISCR=SORT(DISCR)
AQ=CROOT(DISCR-BB)-CROOT(DISCR+BB)
1460 Q=AQ/ALPHA
C
RESET UPPER LIMIT IF Q IS NEGATIVE.
C
IF (Q.LT.0.0) GO TO 1520
PHIM=PHI2/(ZETA+AQ*AQ*3.0/TB)
C
RESET UPPER LIMIT IF PHIM EXCEEDS TPHI2.
C
1470 IF (PHIM.GT.TPHI2) GO TO 1520
C
BYPASS SEARCH FOR MAXIMUM VALUE OF Q IF LESS = .TRUE..
C
IF (LESS) GO TO 1480
CALL OMAX (RH01L,DRH01L,2,TOL2,-1.0)
C
SUPPRESS PRINTING UNTIL THE MAXIMUM VALUE OF Q IS ESTABLISHED, IF
IT IS KNOWN TO EXIST IN THIS REGION.
C
IF (MESSAGE) GO TO 1490
1480 WRITE (TRITE,JVAR) Q,PHIM,RH01L,RH02L
IF (NCVSL) GO TO 1490
C
COMPUTE AND PRINT END SLOPE AND CENTRAL DEFLECTION AS THESE HAVE
BEEN REQUESTED.
C
CALL PHIVSX
APPENDIX 2

C DECREASE RHO1/L FOR A NEW SET OF VALUES.
C 1490 RHO1L=RHO1L-DRHO1L
C TEST RHO1/L AGAINST UPPER LIMIT.
C 1500 IF (RHO1L.GT.UL) GO TO 1410
C HALVE THE DECREMENT IF IT EXCEEDS THE TOLERANCE FOR RHO1/L.
C ELSE, TERMINATE THE SOLUTION.
C 1510 IF (DRHO1L.LT.TOL2) GO TO 1500
C DRHO1L=0.5*DRHO1L
C RHO1L=RHO1L+DRHO1L
C GO TO 1500
C RESETTING UPPER LIMIT.
C 1520 UL=RHO1L
C GO TO 1510
C GROUP 1 SOLUTION ENDS.
C GROUP 2 SOLUTION BEGINS.
C MORE PROGRAM CONSTANTS.
C 1530 AKL=ALPHA/KL
C TEMPK=ARSHC/KL
C TEMPL=AKL*AKL/F
C IF END CURVATURE EXCEEDS PHI2, SEEK THE SOLUTION IN GROUP 3. ELSE, IN GROUP 2.
C SPHI2=C/(B-MZERO)
C PHIZ=SPHI2*SPHI2
C IF (PHIZ.GE.PHI2) GO TO 1560
C IF TYPE = .TRUE., CASE 3 IS CONCLUDING CASE IN GROUP 2.
C UL=PHI2
C IF (TYPE) UL=TPHI2
C PHIM=PHI2
C INVESTIGATE CASE 3 SOLUTION IN SUBROUTINE BC.
C CALL BC (FKL3,PHIM,DPHI3,TOL3,1,0,3,UL,2)
C IF TYPE = .TRUE., TERMINATE THE SOLUTION.
C IF (TYPE) GO TO 1350
C IF CASE 3 IS NOT THE STARTING CASE, CASE 5 MAY BE.
C IF THE REAM-COLUMN UNLOADED IN CASE 3, RETURN CONTROL.
APPENDIX 2

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C OTHERWISE, SEEK AN EXTENSION OF THE SOLUTION IN CASE 5. 541
C IF (START) GO TO 1540 542
IF (PHIM.LT.PHI2) GO TO 1600 543
GO TO 1550 544
1540 WRITE (IRITE,1740) 545
C INVESTIGATE CASE 5 SOLUTION IN SUBROUTINE BC. 546
C 1550 RPHIZ=1.0/PHI2 547
Q=TEMPQ 548
PHIM=PHI2 549
JVAR(4)=6H19.6,F 550
CALL BC (FKL5,PHIM,DPHI5,TOL3,1.0,5,TPHI2,2) 551
JVAR(4)=6H 9.6,F 552
GO TO 1350 553
C GROUP 2 SOLUTION ENDS. 554
C GROUP 3 SOLUTION BEGINS. 555
C IF END CURVATURE EXCEEDS TPHI2, THE SOLUTION IS BEYOND USER'S 556
REGION OF INTEREST. RETURN CONTROL, AFTER RESETTING INTER TO 1. 557
C 1560 PHIZ=SQRT(F/(MPC-MZERO)) 558
IF (PHI7.LT.TPHI2) GO TO 1570 559
WRITE (IRITE,1760) RANGE 560
INTER=1 561
GO TO 1600 562
C INVESTIGATE CASE 6 SOLUTION IN SUBROUTINE BC. 563
C 1570 RPHIZ=1.0/PHI2 564
PHIM=PHI7 565
CALL BC (FKL6,PHIM,DPHI6,TOL3,1.0,6,TPHI2,3) 566
GO TO 1350 567
C GROUP 3 SOLUTION ENDS. 568
C MESSAGES. 569
1580 WRITE (TRITE,1720) 570
GO TO 1600 571
1590 WRITE (TRITE,1730) 572
C RESET FORMAT ARRAYS BEFORE CONTROL IS RETURNED TO USER'S PROGRAM. 573
C 1600 IVAR(12)=6H 574
IVAR(13)=6H 575
IVAR(14)=6H / 576
LVAR(1)=6H (1H+,4 577
RETURN 578
C
APPENDIX 2

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1610 FORMAT (1H1,30X,52H GENERAL SOLUTION OF AN INELASTIC BEAM-COLUMN PR 595
18BLEM/1H0,5X,22H. MATERIAL PROPERTIES/1H0,12X,28HE = MODULUS OF E 596
2LASTICITY = ,F10.1,4H KSI,5X,24H SIGMA Y = YIELD STRESS = ,F9.2,4H K 597
3ST/1H0,5X,23H. SECTION AND GEOMETRY/1H0,12X,28H NTYPE = NATURE OF 598
4SECTION = ,I1) 599

1620 FORMAT (1H+,45X,11H(RECTANGLE))

1630 FORMAT (1H+,45X,67H(IDEALIZED WF SHAPE, STRONG AXIS BENDING WITHOUT 601
1 T RESIDUAL STRESSES))

1640 FORMAT (1H+,45X,57H(WF SHAPE, STRONG AXIS BENDING WITHOUT RESIDUAL S 603
1TRESSES))

1650 FORMAT (1H+,45X,55H(WF SHAPE, WEAK AXIS BENDING WITHOUT RESIDUAL STRESS 605
1RESSES))

1660 FORMAT (1H+,45X,54HSWF SHAPE, STRONG AXIS BENDING WITH RESIDUAL S 607
TRESSES)

1670 FORMAT (1H+,45X,52H(WF SHAPE, WEAK AXIS BENDING WITH RESIDUAL STRESS 609
1RESSES))

1680 FORMAT (1H+,45X,39H(SQUARE TUBE WITHOUT RESIDUAL STRESSES))

1690 FORMAT (1H0,12X,23HALPHA = SHAPE FACTOR = ,F4.2,5X,35HLOVERR = SLE 612
1NDERNESS RATIO = L/R = ,F5.1/1H0,12X,29H HOVERR = DEPTH RATIO = H/R 613
2 = ,F5.2,5X,24HR = SHAPE VARIABLE = ,F4.2/1H0,5X,21H. LOADING CO 614
3DITIONS/1H0,12X,36HEHOVERR = ECCENTRICITY RATIO = E/R = ,F7.3,5X,46 615
HP = ECCENTRIC LOAD (FRACTION OF YIELD LOAD) = ,F5.3/1H0,12X,21H. MZ 616
5ERO = END MOMENT = ,F8.6/1H0,5X,41H. GENERALIZED STRESS-STRAIN REL 617
6ATIONSHIP/1H0,12X,4HA = ,F8.6,5X,4HB = ,F8.6,5X,4HC = ,F8.6,5X,4H 618
7F = ,F8.6/1H0,12X,6MPC = ,F8.6,5X,7PHI1 = ,F8.6,5X,7PHI2 = ,F10 619
8.6/1H0,5X,45H5. PRESCRIBED INCREMENTS,ACCURACIES AND RANGE/1H0,12X 620
9,10HINCREMENTS/1H0,14X,41HDRHO12 = INCREMENT OF RHO1/L IN CASE 2 = 621
$,F8.6/1H0,14X,41HDRHO14 = INCREMENT OF RHO1/L IN CASE 4 = ,F8.6/1H0 622
$H0,14X,41HDRPHI15 = INCREMENT OF PHIM IN CASE 5 = ,F8.6/1H0,14X,41H 623
$1HDPHI5 = INCREMENT OF PHIM IN CASE 5 = ,F8.6/1H0,14X,41HDPHI6 624
$ = INCREMENT OF PHIM IN CASE 6 = ,F8.6/1H0,12X,10HACCURACIES/1H0 625
$14X,29HTOL1 = TOLERANCE ON Q = ,E9.2/1H0,14X,29HTOL2 = TOLER 626
$ANCE ON RHO1/L = ,E9.2/1H0,14X,29HTOL3 = TOLERANCE ON PHIM = ,E9 627
$2/1H0,12X,5H RANGE/1H0,14X,24H RANGE = LIMIT OF PHIM = ,F6.2,11H 628
$MES PHII) 629

1700 FORMAT (1H0,5X,35H6. END SLOPE AND CENTRAL DEFLECTION/1H0,12X,8HTIN 630
1DEX = ,I1,10H (RESULTS ,2A5,1H)/1H0,12X,8HINOR = ,I1,31H (CURVATU 631
2RE VS. LENGTH RESULTS ,2A5,1H)/1H0,12X,33HNSUB = NUMBER OF SUBDIV 632
3SIONS = ,I3/1H0,5X,8H. KL = ,F8.6) 633

1710 FORMAT (1H1) 634

1720-FORMAT (1H0,10X,37HMZERO EXCEEDS MPC, HENCE NO SOLUTION.) 635
1730-FORMAT (1H0,10X,33HMPC EXCEEDS PT, HENCE NO SOLUTION.) 636
1740-FORMAT (1H+,10X,24HNO SOLUTION IN THIS CASE/1H) 637
1750 FORMAT (1H+,10X,12HMPC PHIM EXCEEDS,F7.2,5H TIMES PHI2 UNDER NO LOAD, 638
1 Q = 0.0, HENCE NO SOLUTION.) 639
1760 FORMAT (1H0,10X,21HEND CURVATURE EXCEEDS,F7.2,3H TIMES PHI2, HENCE 640
1E NO SOLUTION.) 641

END 642
SUBROUTINE BC (FKLX, ARG, DINCR, TOLARG, SIGN, ICASE, UL, IGROUP).

AUTHOR - SAMPATH IYENGAR.

THIS SUBROUTINE HANDLES CASES 2, 3, 4 (START ONLY), 5 AND 6.

LOGICAL ITER, LESS, MESAGE, NCASE2, NCASE4, NCVSL, START, TYPE
COMMON /RL/ DPHI3, DPHI5, DPHI6, DRH012, DRH014, E, EOVERR, HOVERR, INDEX,
1INDR, IRITE, LOVERR, NSUB, NTYPE, P, R, RANGE, SIGMA, TOL1, TOL2, TOL3
COMMON /RLA/ AKL, ARSHC, BIGQ, C, CKL2SA, DEFLN, DM, DO, F, FALPHA, FKL, I, IV
1AR(15), JVAR(5), KL, KLS, KLSA, KL2, KL3SA, KRH01, LESS, LVAR(16), MESAGE, NZ
ZEROA, NCVSL, NIR, PHI, PHIM, PHI7, PHI1, PHI2, PZERO, Q, QM(3), RH01L, RH02L, R
3PHIM, RPHITZ, SA, SETA, SHC, SINA, SLOPE, SM, SMALLQ, SPHI2, SPHI1, SPHI2, SSE
4A, START, SUB, S2C, TA, TE, TEMPA, TEMPB, TEMPK, TEMPL, TEMPO, TEMPX, TF
5, T0, TH, THIRD, TPHI2, TYPE, T1, T3, T4, T8, XOVERL, ZETA
EXTERNAL TWA, TWHB, THREE, FIVEA, FIVEB
EXTERNAL FKL2A

PROGRAM CONSTANTS AND INITIALIZATION OF VARIABLES.

NCASE2=ICASE .NE. 2
NCASE4=ICASE .NE. 4
TEMP=PHI2
IF (TYPE) TEMP=TPHI2
QM(1)=Q
I=2
TEMPQ=Q
1000 CALL LABELS (IGROUP, ICASE)
ULS=UL
DARG=DINCR
T=ARG
TSIGN=T*SIGN
ARG=ARG-DARG

ITERATION BEGINS HERE.

1010 ITER=.FALSE.
DO=0.05
IF (Q.NF.0.0.AND.Q.LE.0.05) DO=Q

IF LESS = .TRUE. , BEAM-COLUMN UNLOADS.

IF (LESS) DO=-DO

CHOOSE A NEW VALUE FOR ARG AND TEST WHETHER IT LIES IN THE PROPER
RANGE.

1020 SMALL=ARG
ARG=ARG+DARG
ARGSIGN=ARG*SIGN
IF (ARGSIGN.LT.TSIGN) ARG=T
BIG=ARG
IF (ARGSIGN.LT.ULS) GO TO 1040
HALVE THE INCREMENT IF IT EXCEEDS THE TOLERANCE. ELSE, TERMINATE
THE SOLUTION (SEE COMMENTS BEFORE STATEMENT 1270).

1030 IF (ARG(DARG).LT.TOLARG) GO TO 1270
   ARG=SMALL
   DARG=DARG*0.5
   IF (START) GO TO 1020
   Q=TEMPQ
   GO TO 1010

IF ICASE = 2, CALL FKL2A FOR EACH NEW VALUE OF ARG. ELSE, CALL
FKLX. VALUE RETURNED IS FKL.

1040 IF (NCASE2) GO TO 1060
   CALL FKL2A
   IF (.NOT.START) GO TO 1070
   IF (PHIM.GT.TEMP) GO TO 1050
   GO TO 1070

RESET UPPER LIMIT IF Q IS NEGATIVE OR PHIM EXCEEDS ALLOWABLE VALUE
IN CASES 2 AND 4 WHEN STARTING CONDITIONS ARE BEING FOUND.

1050 ULS=ARGSGN
   GO TO 1030
1060 CALL FKLX
   IF (NCASE4) GO TO 1070
   IF (TEMPK.LT.1.0) GO TO 1050
   IF (PHIM.GT.TPHI2) GO TO 1050
1070 IF (ITER) GO TO 1090

STORE THE VALUE OF FKL IN FKL1 AND ITS ABSOLUTE VALUE IN AFKL1.

FKL1=FKL
ITER=.TRUE.
IF (START) GO TO 1020
   AFKL1=ABS(FKL)

INCREMENT Q.

1080 SMALLQ=Q
   Q=Q+DQ
   BIGQ=Q
   GO TO 1060

TEST THE PRODUCT OF FKL1 AND THE NEW VALUE OF FKL.

1090 IF (FKL*FKL1) 1220,1100,1200

PRODUCT = 0, PRINT THE RESULTS.

1100 IF (.NOT.START) GO TO 1230
1110 DARG=DINCR
   START=.FALSE.
APPENDIX 2

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C SELECT THE FORMAT FOR EACH CASE, COMPUTE AND PRINT VALUES OF END
C SLOPE AND CENTRAL DEFLECTION, IF THESE HAVE BEEN REQUESTED.
C
1120 GO TO (1130,1140,1150,1160,1170), ICASE

1130 WRITE (IRITE,JVAR) Q,PHIM,RHO1L
IF (NCVSL) GO TO 1190
CALL XVSI (XOVERL,TWOA,O,O,RHO1L)
CALL XVSE (PHI,TWOB,PHI1,PHIM)
GO TO 1180

1140 WRITE (IRITE,JVAR) Q,PHIM
IF (NCVSL) GO TO 1190
CALL XVSI (PHI,THREE,PHI2,PHIM)
GO TO 1180

1150 WRITE (IRITE,JVAR) Q,PHIM,RHO1L,RHO2L

C ICASE = 4, STARTING CONDITIONS ARE FOUND. RETURN TO SUBROUTINE
C BMCOL.

C IF (.NOT.NCVSL) CALL XVSPHI
RETURN

1160 WRITE (IRITE,JVAR) Q,PHIM,RHO2L
IF (NCVSL) GO TO 1190
CALL XVSI (PHI,FIVEA,PHI2,PHIM)
CALL XVSE (PHI,FIVER,PHI2,PHIM)
GO TO 1180

1170 WRITE (IRITE,JVAR) Q,PHIM
IF (NCVSL) GO TO 1190
CALL XVSI (PHI,FIVER,PHI2,PHIM)

1180 WRITE (IRITE,JVAR) SLOPE,DEFLN

1190 TEMPQ=Q

C BEGIN A NEW CYCLE.
C GO TO 1010

C PRODUCT POSITIVE, RANGE NOT ESTABLISHED YET.

1200 IF (START) GO TO 1020
IF (DO.LT.0.0) GO TO 1280

C CHECK IF SOLUTION IS DIVERGING.
C IF (ABS(FKL).LE.AFKL1) GO TO 1080

C SEEK ONLY INCREASING VALUES OF Q NEAR THE REGION OF MAXIMUM Q.
C IF (MESSAGE) GO TO 1210

C BEAM-COLUMN HAS BEGUN TO UNLOAD.
C DO=-DO
Q=SMALLO
GO TO 1080

1210 CALL MAXQ (ARG,DARG,3,TOLARG,1.0)
GO TO 1010

PRODUCT NEGATIVE, RANGE ESTABLISHED. SEEK MORE PRECISE VALUES, CALL SUBROUTINE HALVE.

1220 IF (START) GO TO 1250
CALL HALVE (Q,SMALL,Q,BIG,TOL1,FKLX,FKL1)

1230 IF (NCASE2) GO TO 1240
IF (PHIM.GT.TEMP) GO TO 1030

1240 IF (Q.LT.0.0) GO TO 1050
ACCEPTABLE VALUE OF Q IS FOUND. IF LESS = .FALSE., TEST HOW THIS VALUE OF Q FITS IN THE SCHEME FOR ESTABLISHING MAXIMUM VALUE OF Q, CALL QMAX.

1250 IF (NCASE2) GO TO 1260
CALL HALVE (ARG,SMALL,BIG,TOLARG,FKL2A,FKL1)
TARG=ARG
GO TO 1110

1260 CALL HALVE (APG,SMALL,BIG,TOLARG,FKLX,FKL1)
GO TO 1110

TERMINATE THE SOLUTION IF ICASE IS NOT 2 OR A POSITIVE VALUE OF Q WAS PRINTED. IF A FICTITIOUS VALUE WAS FOUND FOR THE STARTING CONDITION IN CASE 2, SEEK THE OTHER PROPER VALUE, ASSUMING IT EXISTS.

1270 IF (NCASE2) RETURN
IF (TEMPO.NE.0.0) RETURN

THE OTHER VALUE DOES NOT EXIST, IF START = .TRUE., AFTER THE SECOND TRIAL, RETURN CONTROL.

IF (START) RETURN
START=.TRUE.
ARG=TARG-TOLARG
Q=0.0
WRITE (IRITE,1300)
GO TO 1000

C BEAM-COLUMN IS UNLOADING AND THE RANGE FOR Q IS NOT ESTABLISHED YET. IF, ALREADY, PHIM EXCEEDS THE ALLOWABLE VALUE IN CASE 2, OR THE TRIAL VALUE OF Q IS NEGATIVE, THE ACTUAL VALUE OF Q IS OF NO INTEREST. PROCEED TO MODIFY THE VALUE OF ARG IN EITHER CASE.
C IF Q IS NEGATIVE, ALSO RESET UPPER LIMIT.
C CONTINUE ITERATION OTHERWISE.

1280 IF (NCASE2) GO TO 1290
IF (PHIM.GT.TEMP) GO TO 1030
1290 IF (Q.LT.0.0) GO TO 1050
GO TO 1080

C

1300 FORMAT (1H0,5X,32HTHE SOLUTION ABOVE IS FICTITIOUS,/) END

SUBROUTINE FKL2A
C
C AUTHOR - SAMPATH IYENGAR.
C
THIS SUBROUTINE PERFORMS NUMERICAL COMPUTATIONS IN THE SEVERAL CASES.
C
REAL KL, KLS, KLSA, KL2, KL2SA, KRH01, KRH02, MZERO
LOGICAL START
COMMON /BLA/ AKL, ARSHC, DIGO, C, CKL2SA, DEFLN, DM, DQ, F, FALPHA, FKL, I, IV
1AR(15), JVAR(5), KL, KLS, KLSA, KL2, KL2SA, KRH01, LESS, LVAR(15), MESAEG, MZ
2EROA, NCVSL, NIR, PHI, PHIM, PHIZ, PHI2, PZERO, Q, QM(3), RH01L, RH02L, R3PHM, RPH17, SA, SETA, SHC, SINA, SLOPE, SM, SMALLO, SPH1Z, SPH1I, SPHI2, SSET
4A, START, SUB, S2C, TA, TE, TEMP, TEMP2, TEMPK, TEMPI, TEMPS, TEMPX, TF
5, T6, TH, THIRD, TPHI2, TYPE, T1, T3, T4, T8, XOVEL, ZETA
ATANH(XX)=0.5*Aalog((1.0+XX)/(1.0-XX))
ASF1(XX, YY)=SORT(XX-YY)*(XX+YY+YY)
ASF3(XX, YY)=ATANH(SORT(1.0-XX/YY))
ASF4(XX)=SORT(XX-YY)/XX
C
C CASF 2 - FKL2A IS CALL-ED WHENEVER A NEW VALUE OF RH01/L IS ASSUMED IN ITERATIONS.
C
CALL CALC
T2=0.0
T5=0.0
AS2CS=ARSHC/SSET
PHIM=SETA*SETA
TPHIM=PHTM
FKL=KL2-KRH01-T1*(T4+T3)
TFKL=FKL
IF (START) RETURN

C

CASE 2 - FKL2 IS CALL-ED WHEN Q IS SOUGHT DURING ITERATIONS.

ENTRY FKL2

1000 XI=Q*AS2CS
XIS=XI*KL
IF (ABS(T2) .GE. 1.0) GO TO 1010
T2=ATANH(T2)
T5=KL*XIXI/KLS-XIS
PHIM=TFKL*T1*(T5+T2)
RETURN

1010 Q=(SMALLQ+BIGO)*0.5
BIGQ=Q
DQ=0.5*DQ
GO TO 1000

C

CASE 4 - Q IS ASSUMED TO BE ZERO.

ENTRY FKL4

CALL CALC
TEMPK=SETA/SPHI2
IF (TEMPK .LT. 1.0) RETURN
T2=SQRT(TEMPK*TEMPK-TEMPK)
T5=ASF3(SPHI2,SETA)
KRH02=KPH01+T1*(T4-T2+T3-T5)
RH02L=KRH02/KL
ZETA=1.0-(SETA-SPHI2)/TA
PHIM=PHI2/ZETA
FKL=KL-2.0*(KRH02+TE*ASF1(1.0,ZETA))
RETURN

C

CASE 3.

ENTRY FKL3

SPHI=SQRT(PHI)
SR=Q*TEMPK
SPHIRM=SPHI+SR*SR
T2=SQRT(SPHIRM)
T1=SQRT(T2/SPHIRM)
T9=SPHIRM-SPHI2
T3=SQRT(T9/SPHI2)
T5=ATANH(SQRT(T9/SPHIRM))
T4=SR/SPHI
T6=ATANH(SR/T2)
FKL=T1*(T2*(T3-T4)+T5-T6)-KL
TEMPC=0.5*T1/KL
TEMPB=(SR*T2/SPHI+T6)*TEMPC+0.5
RETURN

C

CASE 5.
ENTRY FKL5
CALL FKL6
SR=O*TEMPK
CRF=C*SR*SR/F
KRHO2=KL2-TF*ASF1(RPHIZ,RPHIM-0.5*CRF)
KRHO2=KRHO2+S2C*SR*(RPHIM-CRF*THIRD)
RH02L=KRHO2/KL
D=SPHI2+(1.0-PHI2*TEMPM)*TA
T3=SHC/D
T4=ASF4(SPHIZ)
T5=ASF4(SPHI2)
T6=ASF3(SPHIZ,D)
T7=ASF3(SPHI2,D)
SD=Sqrt(D)
FKL=FKL+T3*(T4-T5+(T6-T7)/SD)
RETURN

ENTRY X/L VS. PHI RELATIONS.

CASES 1 AND 4, P = 0.0, ELASTIC ZONE.

ENTRY ONEA
PHI=TEMPA*XOVERL
RETURN

CASE 1, P .NE. 0.0, ELASTIC ZONE.

ENTRY ONE
PHI=MZEROA*COS(KL2SA*(1.0-2.0*XOVERL))
PHI=(PHI+TEMPE*SIN(KL2SA*XOVERL))/CKL2SA
RETURN

CASES 2 AND 4, P .NE. 0.0, ELASTIC ZONE.

ENTRY TWOA
PHI=MZEROA*SIN(KL2SA*(RH01L-XOVERL))
PHI=(PHI+PHI1*SIN(KL2SA*XOVERL))/SINA
RETURN

CASE 2, PRIMARY PLASTIC ZONE.

ENTRY TWOB
CALL CASE24
XOVERL=0.5-T1/KL*(XOVERL-T5-T2)
RETURN

CASE 4, PRIMARY PLASTIC ZONE.

ENTRY FOURB
CALL CASE24
XOVERL=RHO1L+T1/KL*(T4-XOVERL+T3)
RETURN
APPENDIX 2

C CASE 4, SECONDARY PLASTIC ZONE.
C ENTRY FOURC
RPHI=1.0/PHI
XOVERL=TEMPB-ASF1(RPHI,TEMPA)*TG
RETURN

C CASE 4, P = 0.0, SECONDARY PLASTIC ZONE.
C ENTRY FOURD
XOVERL=(1.0-FALPHA/PHI/PHI)*0.5/Q
RETURN

C CASE 3, PRIMARY PLASTIC ZONE.
C ENTRY THREE
SPHI=SQRT(PHI)
T4=SQRT((SPHIMR-SPHI)*SPHIMR/PHI)
T5=ASF3(SPHI,SPHIMR)
XOVERL=TEMPB-TEMPA*(T4+T5)
RETURN

C CASE 5, PRIMARY PLASTIC ZONE.
C ENTRY FIVEA
SPHI=SQRT(PHI)
T5=ASF4(SPHI)
T7=ASF3(SPHI,0)
XOVERL=T3/KL*(T4-T5+(T6-T7)/50)
RETURN

C CASES 5 AND 6, SECONDARY PLASTIC ZONE.
C ENTRY FIVEB
RPHI=1.0/PHI
XOVERL=TH-TG*ASF1(RPHI,TEMPM)
RETURN
END

SUBROUTINE CALC
C
C AUTHOR - SAMPATH IYENGAR.
C
C THIS SUBROUTINE IS SUBSIDIARY TO SUBROUTINE FKL2A. IT IS CALL-ED
DIRECTLY (ENTRY FKL6) BY SUBROUTINE BC, ONLY WHEN CASE 6 IS BEING
SOLVED.
C
REAL KL,KL2,KRH01
COMMON /RLA/AKL,ARSHC,BIGQ,C,CKL2SA,DEFLN,DM,DQ,F,FALPHA,FKL,I,IV
1AKR(15),JVAR(5),KL,KLS,KLSA,KL2,KL2SA,KRH01,LESS,LVAR(16),MESSAGE,MZ
2EROA,NOVSL,NIR,PHI,PHIM,PHIZ,PHI1,PHI2,PZERO,Q,QM(3),RH01L,RH02L,R
3PHIM,RPHIZ,SA,SETRA,SHC,SINA,SLOPE,SM,SMALLQ,SPHIZ,SPHI1,SPHI2,SSET

1 2 3 4 5 6 7 8 9 10 11 12 13
CALCULATIONS FOR ETA AND OTHER VARIABLES COMMON TO CASES 2 AND 4.

\[ \eta = 1.0 + \text{TEMPX} \left( \frac{(\cos(\text{ANGLE}) - T8)}{\sin(\text{ANGLE})} \right) \]

\[ \text{SETA} = \text{SPHI} / \eta \]

\[ \text{SSETA} = \sqrt{\text{SETA}} \]

\[ T1 = \frac{\text{SHC}}{\text{SETA} / \text{SSETA}} \]

\[ T3 = \text{ASF2} (\eta) \]

\[ \text{RETURN} \]

CALCULATIONS COMMON TO CASES 2 AND 4 WHEN HANDLING X/L VS. PHI RELATIONS.

ENTRY CASE 24

\[ \text{SPHI} = \sqrt{\text{PHI}} \]

\[ \text{XOVERL} = \text{SETA} / \text{SPHI} \]

\[ \text{XOVERL} = \text{ASF2} (\text{XOVERL}) + \text{ASF3} (\text{SPHI}, \text{SETA}) \]

RETURN

CASE 6 CALCULATIONS, ALSO USED BY CASE 5.

ENTRY FKL6

\[ \text{RPHIM} = 1.0 / \text{PHI} \]

\[ \text{TEMP} = \text{TEMPL} * Q \]

\[ T2 = 2.0 * \text{AKL} * Q * (\text{RPHIM} - 2.0 * \text{TEMP} * \text{THIRD}) \]

\[ \text{TEMPM} = \text{RPHIM} - \text{TEMP} \]

\[ \text{FKL} = - T2 + \text{TF} * \sqrt{\text{RPHI2} - \text{TEMPM}} * (\text{RPHI2} + \text{TEMPM} + \text{TEMPM}) - \text{KL2} \]

\[ T = T2 / KL + 0.5 \]

RETURN

END

SUBROUTINE HALVE (ARG, SMALL, BIG, TOLARG, FKLX, FKL1)

AUTHOR - SAMPATH IYENGAR.

THIS SUBROUTINE FINDS PRECISE VALUES OF ARG WITHIN THE TOLERANCE TOLARG, GIVEN THE RANGE OF ARG AS SMALL TO BIG.
SUBROUTINE XVSFI (XFI,XVFIX,XFINX,XFIMAX)

C
C AUTHOR - SAMPATH IYENGAR.
C
C THIS SUBROUTINE COMPUTES END SLOPE AND CENTRAL DEFLECTION OF THE
C BEAM-COLUMN.
C
C LOGICAL NUI
COMMON /BL/ DPHI3,DPHIS,DPHIS5,DRH012,DRH014,E,EQUIV,HOVRR,INDEX,
1INDR,IRITE,LOVRR,NSUB,NIYPE,P,RANGE,SIGMA,Y,TOL1,TOL2,TOL3
COMMON /BLA/ AKL,ARSHC,BIGQ,G,CLK2SA,DEFLN,DM,DQ,F,FALPHA,FKL1,I,IV
1AR(15),JVAR(5),KL,KS,KS1A,KL2,KL2SA,KRH01,LESS,LVAR(16),MSSAGE,MZ
ZEROA,NCVSL,NIR,BHIM,PHIM,PHIZ,PHI1,PHI2,ZERO,0,QM(3),RHO111,RHO211,R
3PHIM,PH1Z,SA,SETA,SHC,SIMA,SL0PE,SM,SMALLQ,SPHIS,SPH1,SPHI2,SSST
4A,START,SUB,S2C,TA,TE,TEPA,TEMPB,TEMPK,TEMPM,TEMPQ,TEMPX,TF
5,TG,TH,THIRD,TPHI2,TYPE,T1,T3,T4,T8,XOVERL,ZETA
C
C INITIALIZE SLOPE AND DEFLN TO ZERO.
C
SLOPE=0.0
DEFLN=0.0
C
C BYPASS INITIALIZATION IF MORE THAN ONE EQUATION Requires TO BE
C USED IN THE INTEGRATION PROCEDURE.
C
ENTRY XVSFE
C
C ESTABLISH INCREMENT.
APPENDIX 2

--------------------------------- PAGE NO. 23 ---------------------------------

C
C
DXFI=(XFIMAX-XFIMIN)/SUB
IF (DXFI.LE.0.0) RETURN
C
BEGIN INTEGRATION.
C
XFI=XFIMIN
CALL XVSFIX
C
PRINT INTERMEDIATE RESULTS X/L VS. PHI IF THESE HAVE BEEN
REQUESTED.
C
IF (.NOT.NIR) WRITE (IRITE,1010) XOVERL,PHI
1000 X1=XOVERL
CUR1=PHI
XFI=XFI+DXFI
IF (XFI.GT.XFIMAX) RETURN
CALL XVSFIX
IF (.NOT.NIR) WRITE (IRITE,1010) XOVERL,PHI
TEMP=0.5*(CUR1+PHI)*(XOVERL-X1)*SM
SLOPE=SLOPE+TEMP
DEFLN=DEFLN+TEMP*0.5*(XOVERL+X1)*DM
GO TO 1000
C
C
1010 FORMAT (1H,76X,2E20.6)
END

--------------------------------- PAGE NO. 23 ---------------------------------

BLOCK DATA

C
C
AUTHOR - SAMPATH IYENGAR.
C
C
THIS SUBPROGRAM INITIALIZES CERTAIN FORMAT ARRAYS AND A VARIABLE.
C
C
COMMON /BLA/ AKL,ARSHC,BIGQ,C,CKL2SA,DEFLN,DM,DQ,F,FALPHA,FKL,I,IV
                                      1AR(15),JVAR(5),KL,KL5,KLSA,CL2,CL2SA,KRHO1,LESS,LVAR(16),MESAGE,M2
                                      EROA,NCVSL,NIR,PHI,PHIM,PHIZ,PHII,PHII,PZERO,Q,QM(3),RHO1,LRHO2,R3
                                      PHPIM,PPHI2,SA,SETA,SHC,SIGQ,C,CKL2SA,SA,KR2,KL2,L2,EHT,
                                      4A,START,SUB,S2C,TA,TE,TEMPS,TEMPS,TEMPL,TEMPM,TEMPQ,TEMPX,TF
                                      5,6,7,8,9,10,11,12,13,14,15,16,17
                                      DATA IVAR/6H(1HO,6,8HX,1HO,5H8X,6H4HPIH,M,6H5X,6H4HRHO1,6H
                                      1,2H/L,6H4X,6H4HRHO2,6H,2/L,6H48X,6H ,6H ,6H,6H,6H )
                                      18
                                      2 /,6H )
                                      DATA JVAR/6H(1H,F,6H10.6,F,6H11.6,F,6H9.6,F,6H10.6 )/
                                      19
                                      DATA LVAR/6H(1H,F,6H10.6,F,6H9.6,F,6H10.6 )/
                                      20
                                      DATA E1,E1,F,6H5X,6H3HEND,6A3HSL,6A3H0PE,6F3H=,6H
                                      21
                                      DATA THIRD/0.333333333333333333333/
                                      END

--------------------------------- PAGE NO. 23 ---------------------------------
SUBROUTINE LABELS (IGROUP, ICASE)

AUTHOR - SAMPATH IYENGAR.

THIS SUBROUTINE LABELS OUTPUT.

COMMON /BL/ DPHI3, DPHI5, DPHI6, DRH012, DRH014, E, EOVERR, HOVERR, INDEX, 1INDR, IRITE, LOVERR, NSUB, NTYPE, P, R, RANGE, SIGMAE, TOL1, TOL2, TOL3
COMMON /BLA/ A, ALPH, BIGQ, C, CKL2SA, DEFLN, DM, DQ, F, FALPHA, FKL, I, IV 1AR(15), JVAR(5), KL, KLS, KLSA, KL2, KL2SA, KRH01, LESS, LVAR(16), MEGAGE, MZ 2ZEROA, NCVSL, NR, NR, PHIM, PHIM1, PH1, PH12, PZERO, Q, QM(3), RH01L, RH02L, R 3PHIM, RPHI2, SA, SETA, SHC, SINA, SLOPE, SM, SMALLQ, SPHIZ, SPIHI1, SPIHI2, SSET 4A, START, SUB, S2C, TA, TE, TEMPA, TEMPB, TEMPK, TEMPL, TEMPO, TEMPX, TF 5, TG, TH, THRD, TPHI2, TYPE, T1, T3, T4, T8, XOVERL, ZETA
WRITE (IRITE, 1000) IGROUP, ICASE
WRITE (IRITE, IVAR) SLOPE, OEFLN
RETURN

ENTRY PHIVXSPH

SUBROUTINE XVSPHI

AUTHOR - SAMPATH IYENGAR.

THIS SUBROUTINE COMPUTES AND PRINTS THE VALUES OF END SLOPE AND CENTRAL DEFLECTION OF THE BEAM-COLUMN WHEN CASE 4 GOVERNS.

REAL KL
LOGICAL PZERO
COMMON /BL/ DPHI3, DPHI5, DPHI6, DRH012, DRH014, E, EOVERR, HOVERR, INDEX, 1INDR, IRITE, LOVERR, NSUB, NTYPE, P, R, RANGE, SIGMAE, TOL1, TOL2, TOL3
COMMON /BLA/ A, ALPH, BIGQ, C, CKL2SA, DEFLN, DM, DQ, F, FALPHA, FKL, I, IV 1AR(15), JVAR(5), KL, KLS, KLSA, KL2, KL2SA, KRH01, LESS, LVAR(16), MEGAGE, MZ 2ZEROA, NCVSL, NR, NR, PHIM, PHIM1, PH1, PH12, PZERO, Q, QM(3), RH01L, RH02L, R 3PHIM, RPHI2, SA, SETA, SHC, SINA, SLOPE, SM, SMALLQ, SPHIZ, SPIHI1, SPIHI2, SSET 4A, START, SUB, S2C, TA, TE, TEMPA, TEMPB, TEMPK, TEMPL, TEMPO, TEMPX, TF 5, TG, TH, THRD, TPHI2, TYPE, T1, T3, T4, T8, XOVERL, ZETA
EXTERNAL TWOA, FOURS, FOURC, ONEA, FOURD

P IS NOT ZERO. ELASTIC, PRIMARY AND SECONDARY PLASTIC ZONES EXIST.

CALL XVSPFI (XOVERL, TWOA, 0.0, RH01L)
CALL XVSPFE (PHI, FOURB, PHI1, PHI2)
TEMPA=ZETA/PHI2
RPHIM=1.0/PHIM
TEMPB=SQRTP(RPHIM-TEMPA)*RPHIM+2.0*TEMPA)*TG0.5
CALL XVSPFE (PHI, FOURC, PHI2, PHIM)

WRITE (IRITE, IVAR) SLOPE, DEFLN
RETURN

ENTRY PHIVXSPH
IF (.NOT.PZERO) GO TO 1000
C
P = 0.0. PRIMARY PLASTIC ZONE DOES NOT EXIST.
C
TEMPA=PHI1/RH01L
CALL XSVFI (XOVERL,ONEA,0,0,RH01L)
CALL XSVFE (PHI,FOURD,PHI1,PHIM)
GO TO 1010
END

SUBROUTINE QMAX (ARG,DARG,ICM2,TOLARG,SIGN)
C
AUTHOR - SAMPATH IYENGAR.
C
THIS SUBROUTINE ESTABLISHES THE MAXIMUM VALUE OF Q, IF IT EXISTS
THROUGH AN ITERATIVE SEARCH PROCEDURE.
C
LOGICAL LESS,MESAGE
COMMON /BL/, DPHI3,DPHI5,DPHI6,DRH012,DRH014,E,EQVERR,HOVERR,INDEX,
1INDP,IRITE,LOVERR,NSUB,NTYPE,P,R,RANGE,SIGMAY,TOL1,TOL2,TOL3
COMMON /BLA/, AKL,ARSHC,BIGQ,CKL2SA,DFLN,DM,DQ,F,FALPHA,FKL,I,IV
1AR(15),JVAR(5),KL,KLS,KLSA,KL2,KL2SA,KRH01,LESS,LVAR(16),MESAGE,MZ
2EROA,NCVSL,NIP,PHI1,PHIM,PHIZ,PHI11,PHI2,PZERO,Q,QM(3),RH01L,RH02L,R
3PHIM,RPHIZ,SA,SETA,SHG,SINA,SLOPE,SM,SMALLQ,SPHIZ,SPHI1,SPHI2,SSSET
4A,START,SUB,S2C,TA,TE,TEPPA,TEPPB,TEMP,TMPL,TEMPM,TEMPQ,TEMPX,TF
5,TG,TH,THIRD,TPHI2,TYPE,T1,T3,T4,T8,XOVERL,ZETA
QM(I)=Q
C
IF SUCCESSIVE VALUES OF Q ARE INCREASING, REDESIGNATE QM(1) AND
QM(2) WHEN I=3, AND FIND QM(3).
C
IF (Q.LT.QM(I-1)) GO TO 1010
ARGM=ARG-DARG*SIGN
DARGM=DARG
IF (.EQ.3) GO TO 1000
I=3
RETURN
1000 QM(1)=QM(2)
QM(2)=QM(3)
RETURN
C
PRINT MESSAGES TO INDICATE THE EXISTENCE OF THE MAXIMUM VALUE OF Q
C
1010 IF (MESAGE) GO TO 1020
MESAGE=.TRUE.
IF (.EQ.2) GO TO 1030
WRITE (IRITE,1060)
DARG=DARGM
TDARG=DARGM
C
END THE SEARCH PROCESS WHEN DARG AFTER SUCCESSIVE MODIFICATIONS IS
C
LESS THAN TOLARG.
ENTRY MAXQ

1020 IF (ABS(DARG),LT,TOLARG) GO TO 1040

RETURN TO CONDITIONS AT QM(1), FIND A NEW VALUE FOR QM(2) AFTER
HALVING DARG.

ARG=ARGM
DARG=DARG*.50
Q=QM(1)
I=2
RETURN

IF I=2, AND BEAM-COLUMN UNLOADS, CONCLUDE THAT THE MAXIMUM VALUE
OF Q WAS MISSED IN A PREVIOUS CASE, 2 OR 3 CORRESPONDING TO CASE 4
OR 5 BEING SOLVED.

WRITE (IRITE,1070) ICM2,ICM2
GO TO 1050

PREPARE TO BYPASS CALL-S TO THIS SUBROUTINE BY SETTING LESS=.TRUE.
AS THE MAXIMUM VALUE OF Q HAS BEEN INVESTIGATED.

ARG=ARG-DARG*SIGN
DARG=TARG
LESS=.TRUE.
Q=QM(2)+TOL1
RETURN

FORMAT (1HO, 2X,103HTHE NEXT SET OF VALUES CORRESPONDS TO A VALUE
OF Q WHICH IS VERY CLOSE TO THE ACTUAL MAXIMUM VALUE OF Q/1H )
1070 FORMAT (1HO,10X,46HTHE MAXIMUM VALUE OF Q HAS BEEN MISSED IN CASE,
112,11H, SINCE TOL,11,24H SPECIFIED WAS TOO CRUDE/1H )
END
28. NOMENCLATURE
(see also User's Guide)

A
integration constant

B
integration constant

D
integration constant, also used in Eq. (65)

E
Young's modulus in kips per sq. inch

G
integration constant

I
moment of inertia of the cross section about the axis of bending

M
moment on the cross section

M_y
moment at initial yield (no thrust)

NTYPE
integer to specify the nature of cross section (1 through 7)

P
thrust on the cross section

P_y
axial thrust at yield (no moment)

Q
lateral concentrated load at midspan

Q_y
the value of Q causing initial yield (no thrust)

R
shape variable, also used in Eqs. (49), (55), and (67)

START
logical variable used in program to check whether the starting conditions are found or to be found

TYPE
logical variable to differentiate the case

\[ NTYPE = 2 \text{ and } p \geq \frac{1}{1 + \frac{1}{R}} \]  (TYPE = .TRUE.) from all others

a
constant, generalized stress-strain relationship
constant, generalized stress-strain relationship
constant, generalized stress-strain relationship
eccentricity of the thrust on the beam-column
constant, generalized stress-strain relationship
depth of cross section
equals \left( \frac{P}{EI} \right)^{1/2}
length or span of the beam-column
ratio of \( M \) to \( M_y \)
ratio of \( M_{pc} \) to \( M_y \), where \( M_{pc} \) is maximum moment that can be resisted by the cross section in the presence of axial thrust
ratio of moment at end to \( M_y \)
ratio of moment causing compression fibers to yield in the presence of thrust, to \( M_y \)
ratio of moment causing tensile fibers to yield in the presence of thrust, to \( M_y \)
ratio of \( P \) to \( P_y \)
ratio of \( Q \) to \( Q_y \)
radius of gyration about the axis of bending
distance to a given cross section from the left end of the beam-column
integration constant, also used in Eq. (52)
integration constant
curvature at a given cross section
\( \delta_y \) curvature at a cross section corresponding to \( M_y \)
\( \alpha \) shape factor
\( \delta \) deflection at midspan
\( \varepsilon_y \) strain at yield
\( \zeta \) an expression, Eq. (54)
\( \eta \) an expression, Eq. (43)
\( \theta_0 \) slope at ends of the beam-column (radians)
\( \xi \) an expression, Eq. (44)
\( \rho_1 \) length to beginning of primary plastic zone from left end
\( \rho_2 \) length to beginning of secondary plastic zone from left end
\( \sigma_y \) yield stress in kips per sq. inch
\( \varphi \) ratio of \( \delta \) to \( \delta_y \)
\( \varphi_m \) value of \( \varphi \) when \( \delta \) represents curvature at midspan
\( \varphi_0 \) value of \( \varphi \) corresponding to \( m_0 \)
\( \varphi_1 \) value of \( \varphi \) corresponding to \( m_1 \)
\( \varphi_2 \) value of \( \varphi \) corresponding to \( m_2 \)
29. FLOWCHART LEGEND

- Start
- Return
- Logical IF
- Arithmetic IF
- WRITE Statement
- ENTRY Point
- Substitution Statement
  or an operation described briefly
- CALL Statement
- Link/Continuation
1. Timoshenko, S. P. and Gere, J. M.

2. Chen, W. F. and Santathadaporn, S.
   CURVATURE AND THE SOLUTION OF ECCENTRICALLY LOADED COLUMNS,

3. Chen, W. F.

4. Chen, W. F.
   FURTHER STUDIES OF AN INELASTIC BEAM-COLUMN PROBLEM, Fritz Engineering Laboratory Report No. 331.6, January 1970.

5. Selby, S. M. (Editor)

6. Fenves, S. J.