Beam-to-Column Connections

STRENGTH AND STABILITY OF COLUMN WEB IN WELDED BEAM-TO-COLUMN CONNECTIONS

by

D. E. Newlin
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STRENGTH AND STABILITY OF COLUMN WEB IN WELDED BEAM-TO-COLUMN CONNECTIONS

by

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and

W. F. Chen ²

ABSTRACT

In the design of an interior beam-to-column connection, consideration must be given to column web stiffening. The present AISC Specifications require stiffening of the compression region of column web on the basis of two formulas. The first formula compares the strength of the compression region as a function of web and flange thickness to the applied load from the beam flanges. The second formula precludes instability on the basis of the web depth-to-thickness ratio. If stability is the more critical, web stiffening is required regardless of the magnitude of the applied load. Both formulas are conservative.

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This report is a further examination of the criteria for stiffening the web opposite the beam compression flange(s). This compression region is simulated in a manner allowing rapid and easy testing of specimens. A simple formula is developed for predicting the load carrying capacity of the compression region for sections in the range of instability. Moreover, the effects of strength and stability are combined into a single formula. Simulation tests are also made to investigate the effect of column flange thickness and less common loading conditions on the strength and stability of the compression region.
1. **INTRODUCTION**

In the present AISC Specification [1] there are two formulas governing the requirements for stiffening the compression region of an interior beam-to-column moment connection as illustrated in Figure 1. Formula (1.15-1) [1] or (ASCE Manual No. 41, Eq. 8.21 [2]) gives the strength a column web will develop in resisting the compression forces delivered by beam flanges when expressed in the form

\[ P_{\text{max}} = (t_b + 5k) \tau_{\text{y}} \]  

(1)

This formula was developed from the concept that the column flange acts as a bearing plate as illustrated in Figure 2. It distributes the load caused by the beam compression flange from an initial width, \( t_b \), to some larger width at the edge of the column web. The distance from the beam flange to the edge of the column web is \( k \) (Fig. 2). The stress distribution proportional to \( k \) was developed by curve fitting of test results on 36 ksi steel reported in Ref. 5. The formula was shown to be conservative for high strength steels, as well as for mild steel, by the test reported from previous Lehigh University studies in Ref. 4.

The application of this formula is limited by the AISC Specifications to cases where the column web depth-to-thickness ratio, \( d_c/t \), is small enough to preclude
instability. The limiting ratio is defined by the formula
\[
\frac{d_c}{t} = \frac{180}{\sigma_y}
\] (2)

This formula can be derived using the concept of simply supported edge conditions for the column web panel with a linear elastic solution for the buckling of a simply supported long plate compressed by two equal and opposite forces [4]. The test results of Ref. 4 and Ref. 5 show formula (1) to be conservative for all sections tested regardless of \(d_c/t\) (test set-up is shown in Fig. 3). Furthermore, the present AISC Specifications do not permit consideration of any load carrying capacity in the compression region of sections with \(d_c/t\) ratios greater than \(180/\sigma_y\). Development of a feasible and reliable method of determining ultimate loads for the compression region of sections with \(d_c/t\) ratios greater than \(180/\sigma_y\) will, therefore, be the first objective of this report.

It will be demonstrated herein that strength and stability are not entirely distinct; rather that strength and stability are interrelated, especially when the \(d_c/t\) ratio is near \(180/\sigma_y\). The second objective will be to develop a single formula for predicting the ultimate load carrying capacity of the compression region regardless of the \(d_c/t\) ratio of the column section.
Within the compression region, the column flange simulates a shallow continuous beam. The bending stiffness of the flange as a beam is primarily a function of its thickness. It is the third objective of this report to investigate the contribution of the column flange as a shallow beam to the load carrying capacity of the compression region. It will be shown that the contribution of increased flange thickness is relatively insignificant.

Occasionally, the opposing beams of an interior beam-to-column moment connection will be of unequal depths. This may result in a situation where the loads applied to the compression region are eccentric (Fig. 4). Investigation of the effect of this type of eccentricity on the strength and stability of the compression region will be the fourth objective of this report.

2. ANALYTICAL METHODS

A complete elastic-plastic analysis of a beam-to-column connection using the finite element method has recently been reported by Bose [3]. Both initial buckling and ultimate strength solutions are obtained.

The finite element approach is important to the understanding of the behavior of the connection. However, a practicing engineer is not likely to attempt the use of it in the design of steel structures. Additionally, there
remains the questionable areas of boundary condition stress, residual stress, and degree of accuracy.

In contrast to the very rigorous approach it is current practice to accept the results of physical experiments coupled with drastically simplified statistical analyses as a basis for design rules. This is, indeed, a logical approach for practical use. The design rules are easy to apply and sufficiently conservative to safely permit use of their inherent approximations.

It is evident, however, that, to gain the accuracy needed for more effective and efficient connection design, large amounts of experimental data would be required. The quantity of tests needed to cope with all of the variables affecting connection behavior makes further pursuit of this approach unattractive.

A compromise of the crude and rigorous approaches, that optimizes the benefits of experimental tests in combination with certain idealized theoretical aspects of the problem, is the essence of the approach proposed herein. The problem will be treated as an elastic plate with proper interpretation of boundary conditions for the inelastic range. The desired effect is that of increasing accuracy while retaining simplicity for design use.
One of the major contributions of the flanges is provision of lateral supported edge conditions for the web panel. The flanges provide web edge supports because of the very high bending stiffness of the flange in the plane of the flange. The flanges provide simple supports with 36 ksi material because there is early yielding near the juncture of the web and flange. It was observed that further yielding does not spread throughout the compression panel until just prior to ultimate load when the panel begins to buckle. With the use of high strength materials this early yielding will not occur and the flanges will closely simulate the role of fixed end supports for the web panel.

From observations of the test results in the present and previous tests, it appears reasonably justified to assume that the concentrated load acts only across an effective width, and this width forms a square panel, \( d_c \times d_c \). Thus, the critical buckling stress becomes

\[
\sigma_{cr} = \frac{P_{cr}}{d_c \cdot t} = \frac{33,400}{(d_c/t)^2}
\]  

as developed in Ref. 4.
In Ref. 6 the buckling load of a fixed end long plate compressed by two equal and opposite forces is twice the buckling load of the same plate when it is simply supported. It was also observed in previous tests [4] that sections made of 100 ksi material with $d_c/t$ ratios greater than Eq. 2 did realize stresses approaching twice the critical stresses predicted by the simply supported theory. This is illustrated graphically in comparison with test results in Fig. 5.

It can, therefore, be stated that $\sigma_{cr} = 33,400/(d_c/t)^2$ is a lower bound for 36 ksi material and $2\sigma_{cr}$ is an upper bound for 100 ksi material.

This is closely approximated by making $\sigma_{cr}$ a function of $\sigma_y$ as follows:

$$\sigma_{cr} = \frac{P}{d_c t} = \frac{33,400}{(d_c/t)^2} \left( \frac{\sqrt{\sigma_y}}{6} \right)$$

(4)

If the expression for $\sigma_{cr}$ in Eq. 4 is adjusted to fit the most critical test, test No. 21, the resulting equation

$$P_{cr} = \frac{4100 \ t^3 \sqrt{\sigma_y}}{d_c}$$

(5)

will be safe for all tests.
It should be noted that because \( t \) is cubed while \( d_c \) remains first order, web thickness is a more significant parameter in determining the buckling load, \( P_{cr} \). Graphical comparison of formula (5) is made with test results in Fig. 6 using nominal values of yield stress. A good agreement is observed.

4. DEVELOPMENT OF THE INTERACTION FORMULA

Figure 7 is a non-dimensional comparison of test results with AISC design formulas for strength and stability. There are some inherent drawbacks. The first is that when a section's \( d_c/t \) ratio exceeds the allowable values of \( 180/\sqrt{\sigma_y} \), the specifications declare that the section has no load carrying capacity and is to be stiffened regardless of the magnitude of the applied load.

In the range where \( d_c/t \) is within the allowable limits and the load capacity of the section is controlled by the strength formula, \( P = (t_b + 5k)t_\sigma_y \), other difficulties arise. The test data is much too scattered to make an accurate prediction of the ultimate load capacity. It is readily determined that, although the AISC strength formula is conservative for normal rolled sections, it does not describe what really occurs in the column compression region of a beam-to-column connection.

If we return to the assumption that the compression region of the column is effectively a square web panel with
dimensions $d_c \times d_c$ and thickness, $t$, a different perspective reveals itself. Compressive stress in the columns is now determined as $\sigma = P/d_c t$. From the results plotted against $d_c/t$ on a non-dimensional form in Fig. 8 it is observed that the data is considerably less scattered. However, if the formula $P = d_c t \sigma_y$ was used to predict ultimate load instead of the AISC formula, $P = (t_b + 5k) t \sigma_y$, premature failure would occur as values of $d_c/t$ approach $180/\sqrt{\sigma_y}$. This is observed because of test failures occurring within the limits of the two formulas.

There is a logical conclusion to be drawn from this behavior. It is supported both intuitively and by observation of the plotted results. Interaction between strength and stability criteria does occur near the beginning of the stability criteria range as described by $180/\sqrt{\sigma_y}$. In Fig. 8 this interaction is conservatively described by a straight line from 1.75 on the abscissa to 1.75 on the ordinate. The equation of this line can be written as

$$P = (1.75 - \frac{d_c \sqrt{\sigma_y}}{180t}) d_c t \sigma_y$$ (6)

In comparing values predicted by this formula to test values it was discovered that the equation provided excellent results for all tests except on those specimens made of 100 ksi material. For these specimens the axis
intersection point in Fig. 8 would have to be at least 2.2 or 2.3.

Making the constant 1.75 a function of the yield stress presented itself as a possible way of accomplishing the desirable effect of shifting the interaction line upward for high strength steel. Changing 1.75 to $1.75 \left( \sqrt{\frac{\sigma}{36}} \right)$ produced premature failures in 50 ksi materials. Changing 1.75 to $1.70 \left( \frac{\sigma}{36} \right)$ provided the desirable effect. For $\sigma = 100$ ksi, 1.70 is changed to 2.17 and 1.85 for 50 ksi material. The interaction equation takes the form:

$$P = \left( \frac{1.70 \sqrt{\sigma}}{\sqrt{36}} - \frac{d_c \sqrt{\sigma}}{180 t} \right) d_c t \sigma$$

which reduces to

$$P = 61.2 d_c t - 1.20 d_c^2$$  \hspace{1em} (\sigma = 36 \text{ ksi}) \hspace{1em} (7a)$$

and

$$P = 219.5 d_c t - 5.55 d_c^2$$  \hspace{1em} (\sigma = 100 \text{ ksi}) \hspace{1em} (7b)$$

When the formula is solved for $t$ it takes the form:

$$t = \frac{d_c^2 \sqrt{\sigma} + 180 C_1 A_f}{125 d_c \sqrt{36}}$$

(8)
this can be similarly reduced

\[ t = \frac{d_c}{51} + \frac{C_1 A_f}{1.7 d_c} \quad (\sigma_Y = 36 \text{ ksi}) \quad (8a) \]

where \( C_1 \) is the ratio of beam yield stress to column yield stress and \( A_f \) is the area of the beam flange delivering the concentrated load, \( P \). Thus, \( C_1 A_f = P/\sigma_Y \).

Then \( t \) becomes the required web thickness in the column compression zone regardless of \( d_c/t \).

The predicted ultimate loads from this formula for recent Lehigh University tests are tabulated on Table 1 and comparatively plotted against actual values in Fig. 9. Fig. 9 shows the interaction formula (7) to be as accurate as the stability formula (5) and for 100 ksi material the interaction formula provides better accuracy than the stability formula. In the range where the stability formula is not applicable, i.e. \( d_c/t < 180/\sqrt{\sigma_Y} \), the interaction formula is compared with AISC predictions in Fig. 9. Where the stability formula is applicable, AISC makes no prediction.
5. DESCRIPTION OF SPECIAL TESTS

5.1 TEST PROGRAM

Fifteen tests were performed investigating web crippling, in general, as affected by various types of loading conditions and column flange variations. Refer to Fig. 3 for a schematic of the web crippling test set-up.

The first series of tests simulated the compression zone of a column loaded by moments from two opposing beams of unequal depth. This is illustrated schematically in Fig. 4.

To observe the effect of increased column flange thickness, a set of tests were performed on sample specimens with and without cover plates. Cover plates used were 1 inch thick, 20 inches long, and slightly wider than the specimen flanges to permit fillet welding all around.

The role of the column flange as a continuous stiffening beam was analyzed by another set of tests. The specimen flanges were slotted from the outside edges to the web on both sides of the load points. The ends of the cuts near the web were pre-drilled to insure rounded smoothness and prevent notches. On one test the distance between the slots was equivalent to \( t_b + 5k \) and equivalent to \( d' \) on another specimen.
5.2 TEST PROCEDURES

A test set-up was devised which permits rapid testing of specimens. It is basically the same one used by Graham et al [5]. The test set-up is shown in Figs. 2 and 3. In this simulation test, a column is placed horizontally between the loading platens of the testing machine and compressed by two steel bars placed in the same vertical plane on the top and bottom surfaces of the column. The bar was tack-welded to the column flange to simulate a beam flange framing in. The specimens were tested in the Rhicle 800 kip mechanical machine at Fritz Laboratory.

The instrumentation consisted of dial gages to monitor the deflection in the direction of the applied load and another gage to monitor the lateral deflection of column web. This lateral deflection indicated the onset of buckling.

Two tensile specimens were cut from each section in the orientation shown in Fig. 2, in accordance with ASTM standards.
6. **RESULTS**

Table 1 summarizes the measured properties of all test specimens including the tests reported in Ref. 4. Table 1 also summarizes the test results and the theoretical predictions.

6.1 ** ECCENTRIC LOAD TESTS **

It can be observed from the load-deflection curves in Fig. 10 and Fig. 11 that the ultimate load is essentially unaffected by the eccentric load condition. Loading eccentrically has the effect of adding a small amount of stiffness to the web. Design based on the assumption of non-eccentric loading will be conservative. Fig. 12 shows the comparison of the yield pattern of the control specimen with the eccentricity specimen at the end of tests.

6.2 **INCREASED FLANGE THICKNESS**

The load deflection curve of the control test, Fig. 13, exhibits the usual properties of a beam of this size made of 36 ksi material. From no load to approximately half of ultimate load the curve is almost linear and reasonably steep. The upper half of the curve to ultimate load is at a lesser slope indicating the occurrence of some yielding and redistribution of stresses from the increasing load. The maximum design load, as determined by the AISC formula \((t_b + 5k)\sigma_y\) is reached soon after the initial yield point on the load deflection curve with considerable reserve capacity remaining.
With the addition of a heavy cover plate, reasonably long, a significantly different situation exists. The load deflection curve is essentially linear all the way to ultimate load with no stress redistribution exhibited. The \( (t_b + 5k)\sigma_y \) formula leaves only 4.8% of ultimate load as reserve capacity as compared to 43% in the W10 x 29 section without cover plate. These figures are 4.8% and 33% respectively for the W12 x 27 section. Also, though the flange thickness is tripled, the ultimate load is increased only by a factor of 1.3.

It can be concluded that for very thick flanges the \( t_b + 5k \) formula does not meet present standards of reliability. Thus, in the design of beam-to-column connections the presence of a cover plate on a column flange should not be considered as part of the \( k \) dimension. These results further support the relative insignificance of the column flange thickness as compared with web dimensions.

6.3 DEFORMATION CAPACITY

In Ref. 5 it is developed that the required rotation at the ends of a fixed ended beam uniformly loaded along its length, so that it will be able to form a mechanism, is \( \frac{M_p L}{6EI} \). In the practical case of a W16 x 36 beam spanning 24 feet the required rotation is calculated in Ref. 5 to be \( 7.2 \times 10^{-3} \) radians. If this deformation was to be absorbed by an interior column, the required, compression deformation would be \( 2 \times 8 \times (7.2 \times 10^{-3}) \) or about 0.12 inches.
In the test case represented in Fig. 13 the control column develops the required deformation within the load requirements of the interaction formula with considerable reserve capacity. Increasing the flange thickness without changing the web properties produces a stiff section, barely capable of developing deformation, that fails suddenly.

6.4 SLOTTED FLANGE TEST

Slotting the flanges as shown in Fig. 14 had very little effect on the load deflection curve. Stiffness was essentially unchanged and ultimate load decreased only slightly. This adds support to the theory that compression region analysis is basically a local problem. Specimens at the end of the test are shown in Figs. 15 and 16.

7. SUMMARY AND RECOMMENDATIONS

7.1 PARAMETERS

It has been shown that the parameters most pertinent to the strength and stability of the column compression zone in a beam-to-column connection are four fold. They are web thickness, $t$, column depth, $d_c$, yield stress, $\sigma_y$, and the role of the column flanges as supports for the web panel. The column flanges vary in their support effect from a lower bound of simple edge supports to an upper bound of fixed edge supports with increasing yield stress. Flange thickness has been shown to be a parameter not especially significant or needed. Inclusion
of flange thickness effects would unduly complicate the design equations and is therefore unwarranted.

7.2 FORMULAS

The formulas developed or under consideration in this study are summarized below. They are shown both in a form readily useful to the designer and in a form for predicting the maximum permissible load that can be carried by the column compression zone in a beam-to-column connection.

Strength governs when \( t > \frac{d_c \sqrt{\sigma_y}}{180} \)

Stability governs when \( t < \frac{d_c \sqrt{\sigma_y}}{180} \)

<table>
<thead>
<tr>
<th>Ultimate Load Form</th>
<th>Design Form</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strength</strong></td>
<td></td>
</tr>
<tr>
<td>( P = (t_b + 5k)t\sigma_y )</td>
<td>( t \leq \frac{C_1 A_f}{t_b + 5k} )</td>
</tr>
<tr>
<td><strong>Stability</strong></td>
<td><strong>Stiffener Required</strong></td>
</tr>
<tr>
<td>( P = 0 )</td>
<td></td>
</tr>
<tr>
<td><strong>Strength &amp; Stability</strong></td>
<td></td>
</tr>
<tr>
<td>( P = \frac{\sigma_y^{3/2} d_c}{180} \left[ \frac{125t}{\sqrt{\sigma_y}} - d_c \right] )</td>
<td>( t \leq \frac{d_c^2 \sqrt{\sigma_y} + 180 C_1 A_f}{125 \frac{d_c}{\sqrt{\sigma_y}} \cdot d_c} )</td>
</tr>
<tr>
<td><strong>Strength</strong></td>
<td><strong>Strength</strong></td>
</tr>
<tr>
<td>( P = (t_b + 5k)t\sigma_y )</td>
<td>( t \leq \frac{C_1 A_f}{t_b + 5k} )</td>
</tr>
<tr>
<td><strong>Stability</strong></td>
<td><strong>Stiffener Required</strong></td>
</tr>
<tr>
<td>( p = \frac{4100 t^3}{d_c \sqrt{\sigma_y}} )</td>
<td>( d_c &gt; \frac{4100 t^3}{C_1 A_f \sqrt{\sigma_y}} )</td>
</tr>
</tbody>
</table>
The present AISC formulas are conservative. This has been shown previously and is reconfirmed in this report. The AISC formulas are incomplete in that they offer no estimate of the load capacity of the compression zone when \( \frac{d_c}{t} \) exceeds \( 180/\sqrt{\sigma_y} \). The \( (t_b + 5k)\sqrt{\sigma_y} \) formula is not an accurate expression of strength and, in the case of very thick flanges, is unconservative.

The interaction formula is considerably more accurate. It has the advantage of being a conservative fit to data that is far less scattered than the data pertaining to the AISC formula. This fact alone makes it more pertinent than the AISC formula. Another important advantage is that it permits the designer to make a one step analysis of the compression zone of a connection to determine whether a stiffener is advisable.

The last set of formulas, herein referred to as Modified AISC, adds to the present AISC approach the advantage of being able to predict ultimate loads in the stability range very accurately. When the constant in the formula \( P = 4100t^3 \sqrt{\sigma_y/d_c} \) is increased to 4400 this equation is an excellent fit to the test results of specimens made of 36 ksi and 50 ksi material and is conservative for 100 ksi material. When the constant is left at the conservative 4100 value, it is a reliable design aid.
7.3 RECOMMENDATIONS

After thorough evaluation of the test results set forth in this report, it is the considered opinion of the authors that the proposed "interaction formula" offers a decided improvement to the present AISC approach on the basis of simplicity, safety, accuracy, and thoroughness.

Also the addition of the stability formula
\[ P = 4100t^3 \sqrt{\sigma_y/d_c} \]
to the AISC Commentary would be an asset to that text and to persons interested in greater accuracy for determining buckling loads of rolled sections of 36 or 50 ksi material.

8. ACKNOWLEDGMENTS

The work is a part of the general investigation on "Beam-to-Column Connections," sponsored by AISI and WRC at the Fritz Engineering Laboratory, Lehigh University.

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10. NOMENCLATURE

\[ A_f \] area of one flange (of the beam framing in);
\[ C_1 \] ratio of the beam yield stress to the column yield stress;
\[ d_c \] column web depth between column k-lines or between toes of fillets;
\[ d_b \] depth of beam;
\[ d' \] distance between column flanges, Fig. 2;
\[ E \] Young's modulus of elasticity;
\[ k \] distance from outer face of flange to web toe of fillet, Fig. 2;
\[ P \] concentrated load;
\[ t_b \] thickness of the beam flange;
\[ t \] column web thickness;
\[ \sigma \] normal stress;
\[ \sigma_y \] yield stress in ksi;
\[ \Delta \] vertical displacement, Fig. 3.
Table 1

SECTION PROPERTIES AND PREDICTED CRITICAL LOADS
WITH NOMINAL STRESS VALUES

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Section</th>
<th>$\sigma_y$ (Nom.) ksi</th>
<th>$\sigma_y$ Actual ksi</th>
<th>Measured Dimensions</th>
<th>AISC $P_{cr}$ Eqs.1,2</th>
<th>Buckling $P_{cr}$ Eqs.5,2</th>
<th>Interaction $P_{cr}$ Eq. 7</th>
<th>Tests $F_{ult}$ kips</th>
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<tbody>
<tr>
<td>W-3*</td>
<td>W10 x 39</td>
<td>100</td>
<td>121.9</td>
<td>8.15 0.344 0.91</td>
<td>0</td>
<td>205</td>
<td>297</td>
<td>253</td>
</tr>
<tr>
<td>W-4*</td>
<td>W12 x 45</td>
<td>100</td>
<td>118.2</td>
<td>9.87 0.344 1.11</td>
<td>0</td>
<td>169</td>
<td>204</td>
<td>260</td>
</tr>
<tr>
<td>W-5*</td>
<td>W12 x 31</td>
<td>36</td>
<td>39.8</td>
<td>10.59 0.270 0.70</td>
<td>0</td>
<td>46</td>
<td>40</td>
<td>61</td>
</tr>
<tr>
<td>W-6*</td>
<td>W10 x 29</td>
<td>36</td>
<td>41.6</td>
<td>8.91 0.308 0.73</td>
<td>0</td>
<td>81</td>
<td>73</td>
<td>90</td>
</tr>
<tr>
<td>W-7*</td>
<td>W10 x 54</td>
<td>50</td>
<td>57.8</td>
<td>8.05 0.380 1.02</td>
<td>106</td>
<td>---</td>
<td>155</td>
<td>215</td>
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<td>W-8*</td>
<td>W 8 x 67</td>
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<td>6.60 0.575 1.22</td>
<td>137</td>
<td>---</td>
<td>180</td>
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<td>W-9*</td>
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<td>97.7</td>
<td>9.95 0.700 1.57</td>
<td>585</td>
<td>---</td>
<td>978</td>
<td>980</td>
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<td>W-10</td>
<td>H10 x 62</td>
<td>36</td>
<td>33.7</td>
<td>7.82 0.504 1.33</td>
<td>130</td>
<td>---</td>
<td>168</td>
<td>237</td>
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<tr>
<td>W-11</td>
<td>W12 x 45</td>
<td>50</td>
<td>54.0</td>
<td>10.02 0.377 1.00</td>
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<td>151</td>
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<tr>
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<td>10.74 0.324 0.82</td>
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<td>123</td>
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<td>0</td>
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<td>0</td>
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<td>159</td>
<td>168</td>
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*Reported in Ref. 4
Fig. 1 Schematic of Typical Interior Beam-To-Column Moment Connection

Fig. 2 Simulation of the Compression Region
Fig. 4 Schematic of Eccentricity Problem

Fig. 3 Test Set-up
Fig. 5 Comparison of Theory Developed in Ref. 5 Using Tests of Specimens with \( \frac{d_c}{t} \) Ratio Greater than or Close to Allowable
Fig. 6 Comparison of Stability Formula with Tests of Specimens with $d_c/t$ Ratio Greater than or Close to Allowable
\[ \sigma = \frac{P}{(t_b + 5k) t} \]

\[ \left( \frac{d_c}{t} \right)_a = \frac{180}{\sqrt{\sigma_\text{y}}} \]

Fig. 7 Comparison of Test Results with AISC Formulas
\[
\frac{\sigma}{\sigma_y} = \frac{P}{d_c \cdot \sigma_y} = \frac{1}{(d_c/t)^2} \left( \frac{d_c}{t_a} \right)
\]

Fig. 8 Comparison of Test Results with Interaction Formula. Ref. 5 Points not Numbered
Fig. 9 Composite Comparison of Ultimate Test Load with Predicted Ultimate Load as Determined by the Various Formulas
Fig. 10
Fig. 11 Eccentric Loading Results
Fig. 12 Yield Pattern at the End of Tests W-11 and W-12

a) Eccentricity Test

b) Control Test
Fig. 13 Contribution of Increased Flange Thickness, t
Test 1 - Control Test
Test 2 - \( L = d - 2t_f \)
Test 3 - \( L = t_f + 5k \)

\[ \begin{align*}
P & \text{KIPS} \\
\Delta, \text{INCH} & \\
0 & 20 \\
0.2 & 40 \\
0.4 & 60 \\
0.6 & 80 \\
0.8 & 100 \\
1.0 & 120 \\
1.2 & 140
\end{align*} \]

Fig. 14
a) Side View Slotted Flange Test

b) Top View, Width - $t_b + 5k$

Fig. 15 Slotted Flange Specimen after Test
a) Side View Slotted Flange Test

b) Top View, Width = d'

Fig. 16 Slotted Flange Specimen After Test