COMPARATIVE STUDY OF
METHODS OF ESTIMATION OF PRESTRESS LOSSES
FOR PRETENSIONED MEMBERS

by
Hai-Tung Ying

A THESIS
Presented to the Graduate Committee
of Lehigh University
in Candidacy for the Degree of
Master of Science
in Civil Engineering

Lehigh University

November 1972
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COMPARATIVE STUDY OF
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Hai-Tung Ying

ABSTRACT

A rational method of estimating prestress losses in pretensioned concrete members, the "Surface Method", was developed in a research project conducted at the Fritz Engineering Laboratory at Lehigh University. In this thesis, a comparative study is carried out between this method and five other currently available prediction methods.

A description of the development of the prediction procedure in the Surface Method is given, and its application is illustrated. Each of the other five prediction methods is also described briefly.

For comparison purposes, three design examples were solved by each of the six prediction methods. The results obtained by each of the other five methods are then, in turn, compared with those obtained by the Surface Method.

Unlike any other method, the proposed Surface Method provides an upper and a lower bound prediction for the prestress losses, dependent upon concrete characteristics. With this knowledge of the degree of variability, the designer will be in a
better position to exercise his judgment in selecting a reasonable value to use in his design.

The loss component which is initial-steel-prestress-dependent, namely, that due to the shrinkage of concrete and relaxation of steel, predicted by the Surface Method is considerably higher than those by the other prediction methods, while the concrete-stress-dependent loss component, namely, that due to elastic shortening and creep of concrete, is lower.

It is also found that the total prestress losses predicted by most of the other methods fall between the upper and lower bound predictions by the Surface Method.
ACKNOWLEDGMENTS

The comparative study of prediction methods of prestress losses for pretensioned members reported herein is a part of the research project, "Prestress Losses in Pretensioned Concrete Structural Members". The research program was conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania. Dr. Lynn S. Beedle is the Director of Fritz Engineering Laboratory. Dr. D. A. VanHorn, Chairman of the Department, initiated the project in October 1966. The sponsors of this research project are the Pennsylvania Department of Transportation, the U. S. Department of Transportation, Federal Highway Administration and the Reinforced Concrete Research Council.

The supervision and critical review of this study by Dr. Ti Huang, professor in charge of the thesis and director of this research project, are sincerely appreciated and gratefully acknowledged.

The author is indebted to his research colleague, E. G. Schultchen for his assistance at all times. His previous work on the project was basic in carrying out this study.

Sincere thanks are due to Mr. J. M. Gera for his assistance in preparing the figures in this thesis, and to Mrs. Ruth Grimes for typing the entire manuscript.
1. INTRODUCTION

1.1 Components of Prestress Losses in Pretensioned Members

In a pretensioned concrete member, there are four major components of prestress losses, namely, elastic shortening, shrinkage, and creep of concrete, and relaxation of steel (prestressing strands). In order to make a reasonable estimate of prestress losses, the behavior of each of these components should obviously be studied.

Elastic shortening of concrete is an instantaneous effect. Immediately upon transfer of prestress, the concrete is compressed and the prestressing strands shorten, thus relieving some of their prestress. This elastic shortening loss can be accurately determined by using the simple elastic theory of strength of materials.

Creep and shrinkage are time-dependent deformations of concrete. Creep is due to sustained compressive stresses in the concrete, whereas shrinkage is due to losses of liquids in the concrete. The time-dependent natures of creep and shrinkage of concrete have been widely studied. A review of previous research is contained in Ref. 8.

Relaxation of steel is the tendency for the steel to lose stress when subjected to a constant strain. It is also a time-dependent phenomenon. A detailed review of previous research is contained in Ref. 4.
### 1.2 Current Practices for Estimating Prestress Losses

The current practices for estimating prestress losses in pretensioned bridge members are primarily based on a general expression recommended by the ACI-ASCE Joint Committee and AASHO. It simply sums up the aforesaid four components of prestress losses.

\[
\Delta f_s = (\varepsilon_e + \varepsilon_s + \varepsilon_c) E_s + \delta_1 f_{si}
\]  

(1.1)

where \(\Delta f_s\) represents prestress loss and \(f_{si}\) is the initial stress in the prestressing strands. \(\varepsilon_e, \varepsilon_s\) and \(\varepsilon_c\) are strains in concrete due to elastic shortening, shrinkage and creep respectively, and \(\delta_1\) is the percentage loss of steel stress due to relaxation. \(\varepsilon_e, \varepsilon_s, \varepsilon_c\) and \(\delta_1\) are constants determined empirically. However, in their original recommendations, the joint committee did not provide any suggested values for these constants. Instead, a lump sum loss of 35,000 psi was recommended as an acceptable alternative to making detailed estimates of each of these components.

At the present time, both empirical constants and lump sum methods are being used in the designs of pretensioned members by the Pennsylvania Department of Transportation. The general practice is to use the 1954 Bureau of Public Roads' formula (see Section 2.2), with a lower limit of 20.0% loss for box beams and 22.8% loss for I-beams.

More recently, several changes have been incorporated into the design codes. AASHO has recently replaced the lump sum
loss value of 35,000 psi by a more sophisticated expression (see Section 2.3). The Pennsylvania Department of Transportation also has modified the 1954 Bureau of Public Roads' formula for their new standard designs (see Section 2.2).

1.3 Rational Prediction Method

In a prestressed concrete member, all components of prestress losses do not occur independently, but simultaneously. Contrary to the basic implications in creep tests of concrete and relaxation tests of steel, the stress in concrete and the length of the prestressing strands (or the member) do not remain constant. As the concrete creeps and shrinks the length of the member, hence the strand, will shorten; similarly, as the length changes and steel stress decreases, the stresses in concrete will also decrease. Moreover, while the concrete stress decreases with time due to the prestress losses in the strands, there will be a corresponding decrease in the concrete elastic strain. This elastic "rebound" will contribute to some prestress gain in the prestressing strands. Thus, the four components of prestress losses are interdependent on one another.

It follows that the current practices for estimating prestress losses which ignore the interdependence of the components of prestress losses are not adequate. They do not reflect the true nature of the problem. In an effort to develop a more rational method of predicting prestress losses in pretensioned
concrete members, a research project was initiated in 1966 at the Fritz Engineering Laboratory at Lehigh University, under the joint sponsorship of the Pennsylvania Department of Transportation and the U. S. Federal Highway Administration.

The progress of the research project can be summarized into the following phases:

1. Study of concrete strains in pretensioned concrete members (Ref. 8, 12, 16).

2. Study of relaxation losses in prestressing strands (Ref. 4, 13, 14).

3. Development of a stress-strain-time relationship which reflects the true behavior of a pretensioned concrete member (Ref. 16).

4. Formulation of the "Surface Method" for design purposes.

In this report, design examples are used to illustrate the application of the Surface Method, and the results compared to those obtained from some of the currently available methods.
2. METHODS OF ESTIMATING PRESTRESS LOSSES IN PRETENSIONED CONCRETE MEMBERS

2.1 Current Prediction Methods

In this chapter, a brief summary of the currently available methods for estimating prestress losses in pretensioned concrete members is given. The two most widely used methods in this country are those developed by the Bureau of Public Roads and AASHO. Both of these methods have recently been modified. The CEB Method is used as a guideline by many European countries. There are also two recently developed methods, the General Method by the PCI Committee on Prestress Losses and a method proposed by Professor D. Branson of the University of Iowa.

2.2 BPR (Bureau of Public Roads) Method

In 1954 the U. S. Bureau of Public Roads (now Federal Highway Administration) suggested the following expression for estimating prestress losses in pretensioned members

\[
\Delta f_s = 6000 + 16 f_{cs} + 0.04 f_{si}
\]  

(2.1)

where \( f_{cs} \) = Initial stress in concrete at the level of centroid of steel, in psi

\( f_{si} \) = Initial prestress in steel, in psi
In the above expression, the term 6000 represents the prestress loss due to shrinkage of concrete and is obtained by assuming a shrinkage strain of 0.0002 and a steel modulus of 28,000,000 psi. The term $16 f_{cs}$ represents two components, $5 f_{cs}$ accounting for the loss due to elastic shortening, and $11 f_{cs}$ for that due to creep of concrete. For these estimates, a steel-to-concrete modular ratio of 5 and a creep factor of 2.2 have been assumed. The last term, $0.04 f_{si}$, represents the relaxation loss.

As pointed out earlier, this method is currently being used by the Pennsylvania Department of Transportation as the basis for estimating prestress losses in pretensioned members. The standard beam designs, issued in 1964, were also based on this method. However, in the preparation of a new set of standard designs, scheduled for issuance late in 1972, a slightly modified formula is being used.

$$\Delta f_s = 6000 + 16 f_{cs} + 0.08 f_{si}$$

The only change is that a relaxation loss of 8% is assumed, doubling the original value of 4%.

In Appendix B, a design example is used to illustrate the application of this method for estimating prestress losses in pretensioned members.

2.3 AASHO Method and Gamble's Proposal

In the 1971 Interim Specifications developed by the
AASHO Committee on Bridges and Structures the lump sum loss of 35000 psi previously recommended for pretensioned members was replaced by the following expression

\[ \Delta f_s = SH + ES + CR + REL \]  

(2.2)

where \( \Delta f_s \) is the total prestress loss; SH, ES, CR and REL are the four components of prestress losses and they are determined as follows:

(a) Shrinkage loss SH is to be taken from the following table based on the average relative humidity of the geographic area.

<table>
<thead>
<tr>
<th>Relative Humidity (Percent)</th>
<th>SH (psi)</th>
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<tbody>
<tr>
<td>100 - 75</td>
<td>5,000</td>
</tr>
<tr>
<td>75 - 25</td>
<td>10,000</td>
</tr>
<tr>
<td>25 - 0</td>
<td>15,000</td>
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</table>

(b) Elastic Shortening

\[ ES = 7 f_{cr} \]  

(2.3)

where \( f_{cr} \) is the average concrete stress at the centroid of steel at time of release.

(c) Creep of Concrete

\[ CR = 16 f_{cd} \]  

(2.4)
where $f_{cd}$ is the average concrete compressive stress at the centroid of steel under full dead load.

(d) Relaxation of Prestressing Steel

\[ \text{REL} = 20,000 - 0.125 (SH + ES + CR) \]  

(2.5)

The second term in the above equation represents an attempt to reflect the interdependency of the several components of prestress losses.

Recently, a revision to the 1971 AASHO method, described above, has been proposed by Professor W. L. Gamble of the University of Illinois. The proposed changes are as follows:

(a) ES: In equation 2.3, $f_{cr}$ is redefined to represent the concrete stress immediately after transfer, and to include the stress caused by the weight of the member.

(b) CR: Equation 2.4 is replaced by

\[ CR = 12 f_{cr} - 7 (f_{cr} - f_{cd}) \]  

(2.4a)

where $f_{cr} - f_{cd} = \text{Change of concrete stress at the centroid of steel caused by the weight of cast-in-place deck and permanent formwork.}$

The second term in the above equation is used only for composite sections.
(c) REL: Equation 2.5 is replaced by

\[ REL = 20,000 - 0.4 \text{ES} - 0.2 (\text{SH} + \text{CR}) \]  \hspace{1cm} (2.5a)

In Appendix C, a design example is used to illustrate the application of this method for estimating prestress losses in pretensioned members.

2.4 PCI - General Method

The PCI - General Method was recently developed by the PCI Committee on Prestress Losses\(^\text{11}\). This method, in an attempt to approximate the interdependent relationship of the time-dependent components of prestress losses, employs a step-by-step procedure with respect to time. A minimum of four time intervals are recommended for the calculation of the time-dependent components.

For a pretensioned member, the recommended step intervals are as follows:

Step 1. From anchorage of prestressing steel to transfer of prestress.

Step 2. From transfer of prestress until concrete age reaches 30 days, or until the member is subjected to loads in addition to its own weight.

Step 3. From end of step 2 until concrete age reaches one year.

Step 4. Service life of member after the first year.
However, when significant changes in loading are expected, time intervals in addition to these should be used.

The time-dependent components, shrinkage and creep of concrete and relaxation of steel, for each time interval from $t_1$ to $t$ are calculated by Eqs. 2.6, 2.7 and 2.8 respectively, using tabulated coefficients.

(a) Shrinkage Loss

$$SH = (USH \times SSF) \times (PSH), \text{ psi} \quad (2.6)$$

where

$SH$ = Shrinkage loss over time interval $t_1$ to $t$, in psi

$USH$ = Ultimate loss due to shrinkage of concrete, in psi

$SSF$ = Shrinkage correction factor for size and shape of member

$PSH$ = Fraction of $USH$ over time interval $t_1$ to $t$

(b) Creep Loss

$$CR = (UCR \times SCF \times MCF) \times (PCR) \times (f_{ct}), \text{ psi} \quad (2.7)$$

where

$CR$ = Creep loss over time interval $t_1$ to $t$, in psi

$UCR$ = Ultimate creep coefficient, loss of steel stress per unit concrete stress

$SCF$ = Creep correction factor for size and shape of member
\[ MCF = \text{Correction factor for the concrete age at transfer and length of curing period for moist cured concrete. MCF is taken as 1.0 for steam cured concrete.} \]

\[ PCR = \text{Fraction of UCR over time interval } t_1 \text{ to } t \]

\[ f_{ct} = \text{Net concrete compressive stress at centroid of steel at time } t_1 \text{ (including the effects of all external loads), in psi} \]

The determinations of the various factors used in Eqs. 2.6 and 2.7 are contained in Ref. 11. It should be pointed out that the USH and UCR values recommended are based on an average annual relative humidity of 70%.

\( (c) \text{ Relaxation Loss} \)

\[ RET = f_{st} \left[ \frac{\log (t/t_1)}{K} \right] \left( \frac{f_{st}}{f_y} - 0.55 \right), \text{ psi (2.8)} \]

where \( \left( \frac{f_{st}}{f_y} - 0.55 \right) \) should not be taken as less than 0.05. In Eq. 2.8, \( f_{st} \) is the steel stress at time \( t_1 \) (including the effect of external loading) and \( f_y \) is the yield strength of steel corresponding to 1% total strain. \( K \) is an empirical constant, which has a value of 10 for stress relieved strands and 45 for stabilized strands.

For other types of prestressing steel, relaxation loss estimation should be based upon manufacturer's recommendations supported by test data.

-11-
(d) Elastic Shortening

The prestress loss due to elastic shortening of the concrete is time-independent and can be obtained simply by the conventional elastic theory of materials. The calculation of the elastic shortening of concrete should be based on the modulus of elasticity of concrete at the time of release and the concrete stress at the centroid of the prestressing force at the section of the member under consideration.

The total prestress loss is obtained by summing up the various components over all time intervals.

\[
\Delta f_s = ES + \sum (CR + SH + REL), \text{ psi} \tag{2.9}
\]

where \( N \) is the total number of time intervals considered. If there are any prestress losses due to anchorage of prestressing steel and deflecting device in pretensioned prestressing, they should be added to Eq. 2.9.

In Appendix D, a design example is used to illustrate the application of this method for estimating prestress losses in pretensioned members.

2.5 CEB Method

This method was developed by the European Committee on Concrete and serves as a guide for the several European countries.
Only a brief summary of this method is given here. For detailed discussion, readers are referred to Ref. 7.

(a) Elastic Shortening

Elastic shortening of concrete is calculated by the usual ideally elastic theory of composite materials. CEB recommends that the elastic modulus of normal weight concrete be calculated by the following formula:

\[ E_{cj} = 79,500 \sqrt{f'_{cj}} \]  \hspace{1cm} (2.10)

where \( E_{cj} \) = Elastic modulus of concrete at \( j \) days, psi
\( f'_{cj} \) = Compressive strength of concrete at \( j \) days, psi

It should be mentioned that Eq. 2.10 is valid only for concrete stresses under working conditions not exceeding 0.4 \( f'_{cj} \) and that it yields a value significantly higher than that by the ACI formula \( (E_{cj} = 57,000 \sqrt{f'_{cj}}) \).

(b) Creep

Under a constant stress \( f_c \), the creep strain \( \varepsilon_c \) at any given time is directly proportional to the elastic strain, i.e.

\[ \varepsilon_c = \frac{f_c}{E_{as}} \phi_t \]  \hspace{1cm} (2.11)

In the above equation, \( E_{as} \) is the elastic modulus of concrete at 28 days calculated in accordance with Eq. 2.10, and \( \phi_t \) is the
creep coefficient. This coefficient is evaluated as the product of five partial coefficients, reflecting the effects of different factors:

\[ \phi_t = k_c k_d k_b k_{ec} k_t \]  \hspace{1cm} (2.12)

where

- \( k_c \) depends on the environmental conditions
- \( k_d \) depends on the age of concrete at loading
- \( k_b \) depends on the composition of the concrete
- \( k_{ec} \) depends on the theoretical thickness of the member (which is an equivalent parameter to the volume - surface ratio -- see Appendix E for definition)
- \( k_t \) defines the development of creep with time after load has been applied

Determinations of coefficients \( k_c, k_d, k_b, k_{ec} \) and \( k_t \) are by design charts, as described in Ref. 7.

The recommendations point out that where the concrete stress \( f_c \) varies with time, it would be necessary to use an iterative method or to revert to an appropriate analytical method.

(c) **Shrinkage**

The shrinkage strain \( \varepsilon_s \) at any instant may be determined by the product of five partial coefficients:

\[ \varepsilon_s = \varepsilon_{so} k_b k_{es} k_p k_t \]  \hspace{1cm} (2.13)
where \( k_b \) and \( k_t \) are as defined for Eq. 2.12

\( \epsilon_{so} \) depends on the environment

\( k_{es} \) depends on the theoretical thickness of the member

and \( k_p = \frac{100}{100 + np} \), where \( p \) is the area percentage of longitudinal reinforcement, and \( n \) is taken as 20 to reflect the effect of creep

Determination of coefficients \( \epsilon_{so} \) and \( k_{es} \) are also by design charts, as shown in Ref. 7.

In calculating the prestress losses due to the previous three components, the CEB Method recommends an elastic modulus of \( 1.95 \times 10^7 \) N/cm² (approximately \( 2.82 \times 10^7 \) psi) for prestressing strands.

(d) Relaxation Loss

A linear logarithmic variation of relaxation loss with respect to time is recommended:

\[
\log \left( \frac{\text{RET}}{f_{si}} \right) = k_1 + k_2 \log t \tag{2.14}
\]

where

- \( \text{RET} \) = Relaxation loss at time \( t \), in psi
- \( f_{si} \) = Initial stress, in psi
- \( t \) = Time after tensioning, in hours
- \( k_1, k_2 \) = Empirical constants dependent on the type of steel, and determined based on test results
In the absence of test results, suggested lower bound final relaxation loss values are provided. For an initial stress of 0.80 \( f_{su} \), where \( f_{su} \) is the ultimate tensile strength, these suggested values vary from 0.16 to 0.06 of the initial stress, depending on the type of steel. A parabolic variation with respect to initial stress is recommended (see Appendix E).

To account for the effect of creep and shrinkage on relaxation, an additional multiplication factor is used. Thus

\[
REL = (RLX) \left[ 1 - 3 \frac{(SH + CR)}{f_{si}} \right]
\]

where

- \( REL \) = Actual relaxation loss, in psi
- \( RLX \) = "Pure" relaxation loss, without consideration of creep and shrinkage, in psi
- \( SH \) = Shrinkage loss of steel stress, in psi
- \( CR \) = Creep loss of prestress, in psi
- \( f_{si} \) = Initial steel stress, in psi

In Appendix E, a design example is used to illustrate the application of this method for estimating prestress losses in pretensioned members.

2.6 Branson's Method

In the last several years, a systematic investigation has been conducted by Professor D. E. Branson at the University
of Iowa on the material behavior and structural response of non-composite and composite prestressed concrete members. Based on this study, Professor Branson has proposed a new method for the estimation of prestress losses\textsuperscript{5,6}.

For a pretensioned composite member, the ultimate prestress loss is given by Eq. 2.16.

\[
\Delta f_s = [(nf_c) + (nf_c) (\alpha_s C_u) (1 - \frac{\Delta F_s}{2F_o})] \\
+ (nf_c) (1 - \alpha_s) C_u (1 - \frac{\Delta F_s + \Delta F_u}{2F_o}) \frac{I_s}{I_c} \\
+ \left( \frac{\varepsilon_{su} E_s}{1 + n pk_s} \right) + 0.075 f_{si} - (mf_{cs}) \\
- (mf_{cs}) (\beta_s C_u) \frac{I_s}{I_c} - mf_{cd}
\]

where \( n \) = Steel - concrete modular ratio at the time of transfer

\( f_c \) = Initial concrete stress at the centroid of steel, due to prestress and the weight of the precast portion, in psi

\( \alpha_s = \frac{t^{0.6}}{d + t^{0.6}} \), where \( t \) is time of slab casting in days after transfer

\( C_u \) = Ultimate concrete creep coefficient
\[ \Delta F_s = \text{Total loss of prestress from transfer to slab casting, not including the initial elastic loss, in kips} \]

\[ F_o = \text{Prestress force immediately after transfer, in kips} \]

\[ \Delta F_u = \text{Total loss of prestress after transfer, not including the initial elastic loss, in kips} \]

\[ I_g = \text{Moment of inertia of the precast girder section} \]

\[ I_c = \text{Moment of inertia of the composite section with transformed slab} \]

\[ \varepsilon_{su} = \text{Ultimate shrinkage strain} \]

\[ E_s = \text{Modulus of elasticity of steel, in psi} \]

\[ p = \text{Area ratio of longitudinal steel} \]

\[ k_s = 1 + \frac{e^2}{r^2} \]

\[ e = \text{Eccentricity of prestressing steel, from centroid of gross girder section} \]

\[ r = \text{Radius of gyratic of gross girder section} \]

\[ f_{si} = \text{Initial steel stress, in psi} \]

\[ m = \text{Steel-concrete modular ratio at the time of slab casting} \]

\[ f_{cs} = \text{Concrete stress at centroid of steel caused by the weight of cast-in-place portion, in psi} \]
\[ \beta_s = \text{Creep correction factor for the precast beam concrete age where the slab is cast} \]

\[ f_{cd} = \text{Concrete stress at centroid of steel caused by differential shrinkage, in psi} \]

In Eq. 2.16 term (1) represents the prestress loss due to elastic shortening, and term (5) the total loss due to relaxation of steel.

Term (2) is the prestress loss due to creep of concrete up to the time of slab casting. Here the factor \( 1 - \frac{\Delta F_s}{2F_0} \) is introduced to reflect the effect of a changing stress (prestress force decreasing from \( F_o \) to \( F_o - \Delta F_s \)). The ultimate creep coefficient \( C_u \) is dependent on the loading age, the ambient relative humidity, and shape and size of the member. Typical values of the correction factors for these effects and also \( C_u \) are given in Ref. 5.

Term (3) accounts for the prestress loss due to creep of concrete for the period from slab casting to ultimate. The factor \( 1 - \frac{\Delta F_s + \Delta F_u}{2F_0} \) reflects the decrease of prestress force from \( (F_o - \Delta F_s) \) to \( (F_o - \Delta F_u) \). The factor \( (I_g/I_c) \) accounts for the effect of the composite section in restraining additional creep curvature after slab casting.

Term (4) is the prestress loss due to shrinkage of concrete. The expression \( (1 + npk_s) \) represents the stiffening effect of the steel.
The last three terms represent prestress gain. Term (6) is the elastic prestress gain due to slab dead load. Term (7) is the prestress gain due to creep under slab dead load. Typical value of $\beta_w$ can be found in Ref. 5. For shored construction, the factor $(I_g/I_c)$ should be deleted. Term (8) is the prestress gain due to differential shrinkage. Normally, this term is small and usually neglected.

For a non-composite member, $\alpha_s$ and $\Delta f_s$ in Eq. 2.16 are equal to zero, and terms (6), (7) and (8) vanish. Therefore, Eq. 2.16 becomes

$$\Delta f_s = (n f_c) + (n f_c) c_u (1 - \frac{\Delta F}{2F_0}) + \frac{e_{su} E_s}{(1 + npk_f)} + 0.075 f_{si}$$

(2.16a)

In Appendix F, a design example is used to illustrate the application of this method for estimating prestress losses in pretensioned members.
3. DEVELOPMENT AND APPLICATION OF "SURFACE" METHOD

3.1 Development of Prediction Procedure

The proposed new method developed at Lehigh University is based on two stress-strain-time "surfaces", for steel and concrete, respectively. Detailed description of the experimental program and the development of these two surfaces are contained in several interim reports of the project, Refs. 4, 8, 12, 13, 14, 16. The intersection of these two surfaces will describe the variation with time of stresses and strains in both materials for a pretensioned member. This intersection is obtained by linking the steel and concrete surfaces with three relationships between the corresponding steel and concrete parameters. These three relationships are: (1) the geometric compatibility between the strains of the prestressing strands and the concrete member, (2) the compatibility between the time parameters for the two surfaces, reflected by the time interval from initial tensioning of steel to transfer of prestress, and (3) the equilibrium condition between the forces in concrete and steel. A summary of the two surfaces and the formulation of the above three relationships for the linkage of the two surfaces will be given in the final report of this project.  

While the direct application of the surfaces to design problems may not be practical, it is believed that simplifying approximations will be possible after numerical results have been
generated for typical members. With this in mind, a computer program was developed to generate the numerical results necessary for a parametric study. The parameters included in this study were the transfer time (from initial stretching of steel to release) $k_1$, the initial prestress $f_{s1}$, the characteristics of the concrete, the type of prestressing strands and a geometric design parameter $\beta$. The parameter $\beta$ is closely related to the amount of concrete prestress and is defined as

$$\beta = \frac{A_g}{A_{ps}} \left( \frac{I_g}{I_g + A_e \cdot e_g} \right)$$

(3.1)

where $A_g =$ Gross area of the beam section  
$I_g =$ Moment of inertia of the gross beam section  
$A_{ps} =$ Total area of prestressing steel  
$e_g =$ Eccentricity of steel from centroid of the gross beam section

Details of the parametric study is given in Ref. 10.

From the parametric study, it was found that the transfer time had virtually no effect on the final prestress loss, and that the steel characteristics were significant only for members with high $\beta$ values (low concrete prestress). However, the characteristics of the concrete was found to have very significant effect on prestress losses. It was also noticed that for a given
concrete the final prestress loss could be separated into two parts. One part, attributed to shrinkage and relaxation, is entirely dependent on the initial prestress, while the other, attributed to creep and elastic shortening, is primarily dependent on the concrete stress immediately after release (which is closely associated with $\beta$) although the initial prestress also has a secondary effect. Two charts have been developed to predict $(SH + REL)$, shrinkage plus relaxation loss, as a function of initial prestress and $(CR + ES)$, creep plus elastic shortening loss, as a function of concrete stress immediately after release, respectively. (See Figs. 3 and 4.)

In order to utilize Fig. 4, the concrete stress immediately after release must be determined first. This stress is calculated by deducting from the initial prestress the relaxation loss occurring before transfer and also the loss due to elastic shortening of concrete. The relaxation loss before transfer can be estimated as a function of the transfer time and initial prestress in Fig. 2. Loss of prestress due to elastic shortening is estimated by the elastic theory of materials with transformed sections. The whole prediction procedure proposed in this project can be summarized into the following five steps, and is illustrated in Fig. 1.

Step 1: Determination of, $REL_1$, relaxation loss occurring before release from Fig. 2, based on given initial
prestress, \( f_{s1} \), and transfer time, \( k_1 \).

Steel stress immediately before transfer is given by

\[
f_{S2} = f_{s1} - \text{REL}_1 \tag{3.2}
\]

Step 2: Determination of \( ES \), prestress loss due to elastic shortening at transfer based on the geometrical design constant \( \beta \) (defined in Eq. 3.1).

Elastic loss can be calculated, using the basic elastic analysis, or, alternatively,

\[
ES = \left( f_{S2} \right) \left( \frac{n_i}{\beta + n_i - 1} \right) \tag{3.3}
\]

where \( n_i \) = The modular ratio at transfer time

\[
f_{S3} = f_{S2} - ES \tag{3.3a}
\]

\[
f_{C3} = ES/n_i \tag{3.3b}
\]

where \( f_{S3} \) = Steel stress immediately after transfer

\( f_{C3} \) = Concrete stress immediately after transfer

Step 3: Determination of total prestress loss,

\( (SH + \text{REL} + \text{CR} + ES) \), at end of service life (arbitrarily taken as 100 years).

Part 1: From Fig. 3, determine \( (SH + \text{REL}) \) based on
the initial stress, $f_{s1}$, and the concrete characteristics.

Part 2: From Fig. 4, determine (CR + ES) based on the concrete stress immediately after release, $f_{c3}$, and the concrete characteristics.

Step 4: The effect of external loads is evaluated on an elastic analysis basis.

The steel stress due to external loads is calculated as follows:

At transfer time:

$$f_{sti} = \frac{n_i \beta f_{cl}}{\beta + n_i - 1} \quad (3.4)$$

At end of service life:

$$f_{stl} = \frac{(\gamma n_i) \beta f_{cl}}{\beta + (\gamma n_i) - 1} \quad (3.5)$$

where $n_i$ = Modular ratio at transfer

$\gamma n_i$ = Modular ratio at end of service life expressed in terms of $n_i$, where $\gamma$ account for the varying effects of both elastic moduli of steel and concrete. $\gamma$ has an average value of 3.3 for plant AB concrete and 2.9 for plant CD concrete.
\[ f_{ct} = \text{Nominal concrete stress at centroid of steel due to applied loads, based on gross section properties} \]

Equivalent gain of prestress:

\[ \Delta = f_{se} - f_{sti} \quad (3.6) \]

Net loss of prestress (after transfer),

\[ PL = (SH + REL) + (ES + CR) - REL_1 - ES - \Delta \quad (3.7) \]

Step 5: Loss at any intermediate time between transfer and end of service life is calculated by linear interpolation with respect to \( \log (t_c + 1) \)

\[ PL_t = PL \left[ \frac{\log (t_c + 1)}{\log (36501)} \right] \quad (3.8) \]

where \( t_c = \text{time after transfer of prestress, in days} \)

As was pointed out earlier, the characteristics of the concrete material have a very strong influence on the final prestress losses. Therefore, two separate sets of charts and constants have been developed for the two concretes used in this project, representing an upper and a lower bound of prestress losses. The material properties of these two concretes are contained in Ref. 8.

It should be pointed out that, in Eq. 3.7, the net loss of prestress in this method is calculated with respect to \( f_{s3} \).
steel stress immediately after release. This is different from
the common practice of referencing prestress losses to the ini-
tial tensioning stress, $f_{s1}$. However, as the "initial condition"
in prestressed concrete design refers to the condition immediately
after transfer, it is felt that the definition adopted here is a
more logical one.

No attempts have been made to include, in this report,
the developments of the prediction charts and the equations used
in the prediction procedure as well as the determinations of the
material constants (e.g. $n_i$ and $\gamma$). These, together with a dis-
cussion of the characteristics of the prediction method, are given
in Ref. 10.

3.2 Application

In this section, the application of the procedure de-
scribed in the preceding section is demonstrated by the following
design example. For more detailed description of this example,
see problem (1) in Appendix A.

Girder section: gross area, $A_g = 588$ in$^2$
moment of inertia, $I_g = 107,986$ in$^4$
eccentricity of steel, $e_g = 7.31$ in

Composite section: gross area, $A_c = 1008$ in$^2$
moment of inertia, $I_c = 294,443$ in$^4$
eccentricity of steel, $e_c = 18.75$ in
Dead load moments due to:

i) girder weight, \( M_G = 5880 \text{ k-in} \)

ii) slab weight, \( M_S = 4500 \text{ k-in} \)

iii) superimposed dead load, \( M_D = 1440 \text{ k-in} \)

Concrete properties: \( n_i = 6, \gamma = 3.3 \) (pant AB concrete)

Steel properties: total area of steel, \( A_{ps} = 6.08 \text{ in}^2 \)

initial prestress, \( f_{si} = 186 \text{ ksi} = 0.69 f_{pu} \)

ultimate tensile strength of steel, \( f_{pu} = 270 \text{ ksi} \)

Transfer time: \( k_1 = 18 \text{ hours} = 0.75 \text{ days} \)

\[
\beta = \frac{A_g I_g}{A_{ps} (I_g + A_g e^2)}
\]

\[
= \frac{(588)(107,986)}{6.08(107,986 + 588 \times 7.31^2)}
\]

\[
= 74.9
\]

Step 1: From Fig. 2, for \( k_1 = 0.75 \text{ day} \) and \( f_{si} = 0.69 f_{pu} \)

\[ REL_1 = 0.019 f_{pu} = 5.1 \text{ ksi} \]

\[ f_{s2} = f_{si} - REL_1 = 180.9 \text{ ksi} \]
Step 2:

\[ ES = \left( f_{s_2} \right) \left( \frac{n_1}{\beta + n_1 - 1} \right) \]

\[ = (180.9) \left( \frac{6}{74.9 + 6 - 1} \right) \]

\[ = 13.6 \text{ ksi} \]

\[ f_{s_3} = f_{s_2} - ES \]

\[ = 180.9 - 13.6 = 167.3 \text{ ksi} \]

\[ f_{c_3} = \frac{ES}{n_1} \]

\[ = 13.6/6 = 2.27 \text{ ksi} \]

Step 3: From Fig. 3, for plant AB concrete and \( f_{s_1} = 0.69 f_{pu} \)

\[ SH + REL = 0.193 f_{pu} = 52.1 \text{ ksi} \]

From Fig. 4, for plant AB concrete and \( f_{c_3} = 2.27 \text{ ksi} \)

\[ CR + ES = 0.107 f_{pu} = 28.9 \text{ ksi} \]

Step 4: Concrete stress at centroid of steel due to applied loads is calculated with the girder and slab weight carried by the girder section and the superimposed dead load carried by the composite section.

\[ f_{cE} = \left( \frac{5880 + 4000}{107,986} \right) (7.31) + \frac{(1440)(18.75)}{294,443} \]

\[ = 794 \text{ psi} \]
\[ f_{sl_i} = \frac{n_i \beta f_{ct}}{\beta + n_i - 1} \]

\[ = \frac{(6) (74.9) (.794)}{74.9 + 6 - 1} = 4.5 \text{ ksi} \]

\[ f_{sl} = \frac{(\gamma n_i) \beta f_{ct}}{\beta + \gamma n_i - 1} \]

\[ = \frac{(3.3 \times 6) (74.9) (.794)}{74.9 + 3.3 \times 6 - 1} = 12.6 \text{ ksi} \]

\[ \Delta = f_{sl} - f_{sl_i} \]

\[ = 12.6 - 4.5 = 8.1 \text{ ksi} \]

Prestress loss after transfer,

\[ PL = (SH + REL) + (CR + ES) - REL_1 - ES - \Delta \]

\[ = 52.1 + 28.9 - 5.1 - 13.6 - 8.1 \]

\[ = 54.2 \text{ ksi (32.4\% of } f_{s_3}) \]

Step 5: Prestress loss after transfer at the end of 1 year

\[ (PL) \left[ \frac{\log (365 + 1)}{\log (36500 + 1)} \right] = (54.2) \left( \frac{\log 366}{\log 36501} \right) \]

\[ = 30.4 \text{ ksi (18.2\% of } f_{s_3}) \]
4. COMPARISON OF PREDICTED PRESTRESS LOSSES

4.1 Design Examples

For the purpose of comparing the Surface Method with the other prediction methods, three examples have been selected. Each of the prediction methods mentioned in Chapter 2, as well as the Surface Method, has been applied to the design examples in estimating prestress losses. All three examples involve PennDOT standard I precast pretensioned beams with cast-in-place deck slabs (Figs. 5, 6, 7). Unshored construction was assumed, hence the weight of the slab was carried by the precast beam alone. For additional dead and live loads, full composite action between precast and cast-in-place concretes was assumed. Detailed descriptions of the design examples are given in Appendix A. The prestress losses estimated by the various methods are summarized in Tables 1, 2 and 3 for the three example problems.

4.2 Characteristics of the Surface Method

Before going into the actual comparisons of the Surface Method with the other prediction methods, it would be appropriate to mention the following characteristics of the Surface Method:

1. For the Surface Method, two sets of results are calculated for each problem, one for each of the two concretes (plant AB and plant CD) used in the development of the prediction charts. When making the comparisons, these
two sets of results can be regarded as the upper and lower bound values.

2. In section 3.1, it was noted that the net prestress loss by the Surface Method should be calculated with respect to the steel stress immediately after transfer. However, for comparison purposes, total prestress losses with respect to the initial tensioning stress are listed in Tables 1 through 9.

3. In the Surface Method, the components of prestress loss of interest are \((SH + REL)\), \((CR + ES)\) and the total prestress loss. Therefore, only these quantities will be used for comparison purposes. The component \((SH + REL)\), which depends entirely on the initial steel prestress, is not merely the sum of the loss components due to shrinkage of concrete and relaxation of steel. It also contains the reducing influence of shrinkage (decreasing tensile strain) on relaxation. Similarly, the second component \((CR + ES)\), which depends primarily on the concrete stress immediately after transfer, represents not merely the sum of the elastic and creep losses, but also includes the effect of shrinkage and relaxation on these components, as well as the influence of creep and elastic concrete strains on relaxation of steel. A quantitative discussion of the interactions between the different
components of prestress loss will be given in Ref. 10.

As mentioned in section 3.1, in the Surface Method the effect of external loads is considered separately by calculating a quantity $\Delta$ (see Eq. 3.6). Therefore, unlike other methods, the concrete stress dependent loss component (CR + ES) does not include the effect of external loads. However, for comparison purposes, the (CR + ES) losses, estimated by the Surface Method and listed in Tables 1 through 9 (except Table 4), have been adjusted to include the effect of external loads by deducting the quantity $\Delta$.

4. In section 3.1, it was noted that the Surface Method is only a simplifying approximation. The more exact solution will be a direct application of the two stress-strain-time surfaces (for steel and concrete) and their intersection. A computer program, PRELOC, has been developed to perform this exact solution and applied to the three example problems. The total prestress losses with respect to the initial steel prestress, obtained by the Surface Method and PRELOC, are shown in Table 4. It can be seen that the Surface Method is a very good approximation to the exact solution.
5. The ultimate prestress losses estimated by the Surface Method are based on a service life of 100 years. For the other prediction methods, the assumed service life is not specified, but their descriptions generally imply a shorter length, in the order of 20 to 50 years.

6. The design constant $\beta$ in the Surface Method is a measure of the concrete prestress in a member. High $\beta$ values indicate low concrete prestress and vice versa. The $\beta$ values for design examples 1, 2 and 3 are 74.9, 50.5 and 57.3, respectively. In other words, problem 1 has the lowest concrete stress, while problem 2 has the highest concrete prestress.

4.3 Comparison with BPR Method (1954 and Revised)

Prestress losses predicted by the BPR methods and the Surface Method for the three design examples are summarized in Table 5. The $(SH + REL)$ losses predicted by the BPR methods were obtained by simply adding the prestress losses due to shrinkage of concrete and relaxation of steel. No interaction of any kind is considered. In spite of this, the $(SH + REL)$ losses predicted by the Surface Method are still considerably higher than those predicted by the BPR methods. This large difference is due to the fact that the basic shrinkage strain and relaxation loss used in developing the Surface Method are much higher than those.
used in developing the BPR methods. Besides, the reducing influence of elastic and creep strains of concrete on the relaxation of steel was not included in (SH + REL) in the Surface method, but was included in (CR + ES).

The (CR + ES) loss predicted by the BPR methods is simply the second term, $16 f_{cs}$, of Eqs. 2.1 and 2.1a. It is the sum of the prestress losses due to creep and elastic shortening of concrete without any consideration being paid to the interactions between the components of prestress loss. Furthermore, it does not include any effect of external loads, since, in the calculation of concrete stress $f_{cs}$, only initial prestress is considered. Consequently, the (CR + ES) losses in all three design examples predicted by the BPR methods are much higher than those predicted by the Surface Method.

It is interesting to note that while the BPR methods predict a much higher concrete stress dependent loss (CR + ES) than the initial steel prestress dependent loss (SH + REL), the reverse is true for the Surface Method. Tables 1, 2 and 3 show that all other methods, except the Branson Method (see section 4.7), also predict higher (CR + ES) than (SH + REL) losses.

The total prestress losses predicted by both BPR methods all fall between the upper and lower bound predictions by the Surface Method, except the one for problem 1 predicted by the 1954 BPR Method, which is lower than the lower bound. It is noted that problem 1 has a relatively low concrete prestress.
(high $\beta$ value), hence the (CR + ES) portion by the BPR methods were correspondingly low.

4.4 Comparison with AASHO Method (Current and Gamble’s Proposals)

The prestress losses predicted by the current AASHO Method, Gamble’s Proposals and the Surface Method for the three design examples are shown in Table 6.

Both the current AASHO Method and Gamble’s Proposals consider influence of shrinkage and creep of concrete on relaxation of steel (see Eqs. 2.5 and 2.5a). However, no interactions are considered for the (CR + ES) loss. Therefore, both methods predict considerably higher (CR + ES) losses than (SH + REL) losses.

The total prestress losses predicted by Gamble’s Proposals agree quite well with the lower bound predictions by the Surface Method. On the other hand, the current AASHO Method yields total prestress losses close to the upper bound predictions, except in problem 2, which has an unusually high concrete prestress.

Unlike other prediction methods which estimate the prestress losses at the midspan section of a member, the AASHO methods refer to the average prestress losses along the length of the member. This explains why problem 3, which has a higher concrete prestress than problem 1, yields lower total prestress loss. Problem 3 has draped prestressing tendons, hence although the
concrete prestress at midspan was very high, the "average along the entire span" was not.

Gamble's Proposals predict lower prestress losses than the current AASHO Method due to the following three changes: (1) it increases the effect of interactions of shrinkage and creep of concrete on relaxation of steel (see Eqs. 2.5 and 2.5a), (2) $f_{cr}$ is redefined to be the concrete stress immediately after, not before, transfer, and (3) the creep loss CR is reduced (see Eqs. 2.4 and 2.4a).

4.5 Comparison with PCI - General Method

The prestress losses predicted by the PCI - General Method and the Surface Method are shown in Table 7. It shows that the PCI - General Method, like the previous two methods, also predicts lower $(SH + REL)$ losses and higher $(CR + ES)$ losses than the Surface Method. However, the differences are not as large, especially for the $(CR + ES)$ losses. This is due to the fact that the PCI - General Methods considers fully the reducing effects of the interactions of the several components of prestress losses.

The total prestress losses predicted by the PCI - General Method for the three design examples are about the same as the lower bound predictions by the Surface Method. This means that for a concrete with average loss characteristics the total prestress losses predicted by the PCI - General Method would be too low.
4.6 **Comparison with CEB Method**

In Table 8 are shown the prestress losses estimated by the CEB and Surface Methods. It is interesting to note that for problem 2, which has higher concrete prestress than the other two example problems, the CEB Method predicts about the same total prestress loss as the lower bound prediction by the Surface Method, while for the other two problems the CEB Method predicts considerably lower prestress losses than the Surface Method. It seems that for members with low concrete prestress the CEB Method tends to predict lower prestress losses than the Surface Method.

The total prestress loss for problem 1 predicted by the CEB Method is especially low (see Table 1). This is mainly due to the rather high concrete modulus yielded by using Eq. 2.10. This results in a considerably lower elastic loss. For the other two problems, the ACI formula for calculating concrete modulus is used.

4.7 **Comparison with Branson's Method**

The prestress losses for the three design examples predicted by the Branson's and Surface Methods are listed in Table 9. The (SH + REL) losses for the Branson's Method are obtained by adding terms (4) and (5) of Eq. 2.16 and the (CR + ES) losses by adding algebraically terms (1), (2), (3), (6) and (7).

For the Branson's Method, the (CR + ES) losses for problems 2 and 3 are higher than the (SH + REL) losses, while the
reverse is true in problem 1. It appears that for members with low concrete prestress, the Branson's Method predicts higher (SH + REL) losses than (CR + ES) losses.

Table 9 also indicates that the total prestress losses predicted by the Branson's Method are lower than those by the Surface Method.
5. CONCLUSIONS

Based on the comparisons in Chapter 4 and the results obtained for the three example problems, the following conclusions can be drawn:

1. The loss component $(SH + REL)$, which depends entirely on the amount of initial steel prestress, predicted by the Surface Method is considerably higher than those by the other prediction methods.

2. The loss component $(CR + ES)$, which depends primarily on the concrete stress immediately after transfer of prestress, predicted by the Surface Method is lower than those by the other prediction methods.

3. Unlike the other prediction methods, except the Branson's Method, the Surface Method predicts higher $(SH + REL)$ losses than $(CR + ES)$ losses. In the Branson's Method, it depends on the amount of concrete prestress. For low concrete prestress (high $\beta$ values) it predicts higher $(SH + REL)$ losses than $(CR + ES)$ losses, and for high concrete prestress (low $\beta$ values) the reverse prevails.

4. The total prestress losses predicted by the other prediction methods, except the CEB and Branson's Methods, all fall between the upper and lower bound predictions.
by the Surface Method. Both the CEB and Branson's Methods predict lower total prestress losses than the Surface Method, although, for members with high concrete prestress, their predictions are close to the lower bound prediction by the Surface Method.

5. The upper and lower bound predictions by the Surface Method are quite different. The difference is due mainly to the loss components (SH + REL). The loss components (CR + ES), including the effect of external loads, are almost the same for the two bounds.

It should be emphasized again that while all other prediction methods do not provide any provisions for considering the loss characteristics (elastic shortening, shrinkage and creep) of the particular concrete used, the Surface Method does so by providing the upper and lower bound predictions. In applying the Surface Method the user, with the information on the loss characteristics of the concrete used and by his own discretion, can estimate the more accurate total prestress loss from the upper and lower bound predictions. For approximate estimations, the upper and lower bound predictions will suffice.
6. TABLES
### TABLE 1: PREDICTED PRESTRESS LOSSES FOR PROBLEM 1

<table>
<thead>
<tr>
<th>Method</th>
<th>Method</th>
<th>SH + REL</th>
<th>CR + ES</th>
<th>$\Delta f_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(psi)</td>
<td>(psi)</td>
<td>(psi) (%)</td>
</tr>
<tr>
<td><strong>BPR (1954)</strong></td>
<td>13,400</td>
<td>39,900</td>
<td>53,300</td>
<td>28.6</td>
</tr>
<tr>
<td><strong>BPR (revised)</strong></td>
<td>20,800</td>
<td>39,900</td>
<td>60,700</td>
<td>32.6</td>
</tr>
<tr>
<td><strong>AASHO</strong></td>
<td>22,600</td>
<td>49,500</td>
<td>72,100</td>
<td>38.8</td>
</tr>
<tr>
<td><strong>AASHO (revised)</strong></td>
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<td>37,900</td>
<td>55,400</td>
<td>29.8</td>
</tr>
<tr>
<td><strong>PCI - General</strong></td>
<td>25,700</td>
<td>31,500</td>
<td>57,200</td>
<td>30.8</td>
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<td><strong>CEB</strong></td>
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<td>39,200</td>
<td>21.0</td>
</tr>
<tr>
<td><strong>BRANSON</strong></td>
<td>27,000</td>
<td>25,100</td>
<td>52,100</td>
<td>28.0</td>
</tr>
<tr>
<td><strong>SURFACE</strong> *</td>
<td>52,100</td>
<td>72,900</td>
<td>39.2</td>
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<tr>
<td></td>
<td>38,000</td>
<td>67,500</td>
<td>31.0</td>
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</tbody>
</table>

+$ f_{si} = \text{initial prestress in steel} = 186,000 \text{ psi}$

*Upper values are for plant AB concrete and lower values for plant CD concrete.
<table>
<thead>
<tr>
<th>Method</th>
<th>SH + REL (psi)</th>
<th>CR + ES (psi)</th>
<th>Δf_s (%)</th>
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<tr>
<td>BPR (1954)</td>
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<td></td>
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<td>36.0</td>
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</table>

+ f_s = initial prestress in steel = 189,000

* Upper values for plant AB concrete and lower values for plant CD concrete
TABLE 3: PREDICTED PRESTRESS LOSSES FOR PROBLEM 3

<table>
<thead>
<tr>
<th>Method</th>
<th>SH + REL (psi)</th>
<th>CR + ES (psi)</th>
<th>Δf_s (psi) (% of f_{si})</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPR (1954)</td>
<td>13,400</td>
<td>51,700</td>
<td>65,100</td>
</tr>
<tr>
<td>BPR (revised)</td>
<td>20,800</td>
<td>51,700</td>
<td>72,500</td>
</tr>
<tr>
<td>AASHO</td>
<td>22,900</td>
<td>47,200</td>
<td>70,100</td>
</tr>
<tr>
<td>AASHO (revised)</td>
<td>17,700</td>
<td>36,600</td>
<td>54,300</td>
</tr>
<tr>
<td>PCI - General</td>
<td>25,100</td>
<td>35,300</td>
<td>60,400</td>
</tr>
<tr>
<td>CEB</td>
<td>9,300</td>
<td>39,900</td>
<td>49,200</td>
</tr>
<tr>
<td>BRANSON</td>
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<td>27,100</td>
<td>53,700</td>
</tr>
<tr>
<td>SURFACE*</td>
<td>52,100</td>
<td>21,700</td>
<td>73,800</td>
</tr>
<tr>
<td></td>
<td>38,000</td>
<td>21,300</td>
<td>59,300</td>
</tr>
</tbody>
</table>

+ f_{si} = initial prestress in steel = 186,000 psi

* Upper values are for plant AB concrete and lower values for plant CD concrete
TABLE 4: COMPARISON OF PREDICTED LOSSES BY PRELOC* AND THE SURFACE METHOD

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Problem</th>
<th>$\Delta f_s$ (% of $f_{si}$)</th>
<th>PRELOC</th>
<th>SURFACE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>38.6</td>
<td>39.2</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>2</td>
<td>43.1</td>
<td>44.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>38.4</td>
<td>39.6</td>
<td></td>
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<tr>
<td>1</td>
<td></td>
<td>30.2</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>2</td>
<td>34.7</td>
<td>36.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>30.6</td>
<td>31.9</td>
<td></td>
</tr>
</tbody>
</table>

* PRELOC is the computer program which applies directly the stress-strain-time surfaces and their intersection for estimating prestress losses. See section 4.2.
TABLE 5: COMPARISON OF PRESTRESS LOSSES PREDICTED
BY THE BPR AND SURFACE METHOD

<table>
<thead>
<tr>
<th>Problem</th>
<th>Method</th>
<th>SH + REL (psi)</th>
<th>CR + ES (psi)</th>
<th>$\Delta f_s$ (% of $f_{si}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954 BPR</td>
<td>13,400</td>
<td>39,900</td>
<td></td>
<td>28.6</td>
</tr>
<tr>
<td>1</td>
<td>Revised BPR</td>
<td>20,800</td>
<td>39,900</td>
<td>32.6</td>
</tr>
<tr>
<td>SURFACE$^+$</td>
<td>52,100</td>
<td>20,800</td>
<td>19,600</td>
<td>39.2</td>
</tr>
<tr>
<td>1954 BPR</td>
<td>13,600</td>
<td>59,800</td>
<td></td>
<td>38.8</td>
</tr>
<tr>
<td>2</td>
<td>Revised BPR</td>
<td>21,200</td>
<td>59,800</td>
<td>42.8</td>
</tr>
<tr>
<td>SURFACE$^+$</td>
<td>52,600</td>
<td>30,500</td>
<td>29,200</td>
<td>44.0</td>
</tr>
<tr>
<td>1954 BPR</td>
<td>13,400</td>
<td>51,700</td>
<td></td>
<td>35.0</td>
</tr>
<tr>
<td>3</td>
<td>Revised BPR</td>
<td>20,800</td>
<td>51,700</td>
<td>39.0</td>
</tr>
<tr>
<td>SURFACE$^+$</td>
<td>52,100</td>
<td>21,700</td>
<td>21,300</td>
<td>39.6</td>
</tr>
</tbody>
</table>

$^+$ Upper values are for plant AB concrete and lower values for plant CD concrete.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Method</th>
<th>SH + REL</th>
<th>CR + ES</th>
<th>Δf_s (% of f_{si})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AASHO</td>
<td>22,600</td>
<td>49,500</td>
<td>38.8</td>
</tr>
<tr>
<td>1</td>
<td>GAMBLE'S PROPOSALS</td>
<td>17,500</td>
<td>37,900</td>
<td>29.8</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>20,800</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>19,600</td>
<td>31.0</td>
</tr>
<tr>
<td></td>
<td>AASHO</td>
<td>19,400</td>
<td>75,000</td>
<td>50.0</td>
</tr>
<tr>
<td>2</td>
<td>GAMBLE'S PROPOSALS</td>
<td>12,500</td>
<td>56,000</td>
<td>36.2</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,600</td>
<td>30,500</td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,900</td>
<td>29,200</td>
<td>36.0</td>
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<td></td>
<td>AASHO</td>
<td>22,900</td>
<td>47,200</td>
<td>37.7</td>
</tr>
<tr>
<td>3</td>
<td>GAMBLE'S PROPOSALS</td>
<td>17,700</td>
<td>36,600</td>
<td>29.2</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>21,700</td>
<td>39.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>21,300</td>
<td>31.9</td>
</tr>
</tbody>
</table>

+ Upper values are for plant AB concrete and lower values for plant CD concrete.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Method</th>
<th>SH + REL (psi)</th>
<th>CR + ES (psi)</th>
<th>$\Delta f_s$ (% of $f_{si}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCI - GENERAL 1</td>
<td>25,700</td>
<td>31,500</td>
<td>30.8</td>
<td></td>
</tr>
<tr>
<td>SURFACE +</td>
<td>52,100</td>
<td>20,800</td>
<td>39.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38,000</td>
<td>19,600</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>PCI - GENERAL 2</td>
<td>23,300</td>
<td>45,500</td>
<td>36.4</td>
<td></td>
</tr>
<tr>
<td>SURFACE +</td>
<td>52,600</td>
<td>30,500</td>
<td>44.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38,900</td>
<td>29,200</td>
<td>36.0</td>
<td></td>
</tr>
<tr>
<td>PCI - GENERAL 3</td>
<td>25,100</td>
<td>35,300</td>
<td>32.4</td>
<td></td>
</tr>
<tr>
<td>SURFACE +</td>
<td>52,100</td>
<td>21,700</td>
<td>39.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38,000</td>
<td>21,300</td>
<td>31.9</td>
<td></td>
</tr>
</tbody>
</table>

+ Upper values are for plant AB concrete and lower values for plant CD concrete.
TABLE 8: COMPARISON OF PRESTRESS LOSSES PREDICTED BY THE CEB AND SURFACE METHOD

<table>
<thead>
<tr>
<th>Problem</th>
<th>Method</th>
<th>SH + REL</th>
<th>CR + ES</th>
<th>Δf_s (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(psi)</td>
<td>(psi)</td>
<td>(% of f_{si})</td>
</tr>
<tr>
<td>CEB</td>
<td>1</td>
<td>10,200</td>
<td>29,000</td>
<td>21.0</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>20,800</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>19,600</td>
<td>31.0</td>
</tr>
<tr>
<td>CEB</td>
<td>2</td>
<td>7,600</td>
<td>58,600</td>
<td>35.0</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,600</td>
<td>30,500</td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,900</td>
<td>29,200</td>
<td>36.0</td>
</tr>
<tr>
<td>CEB</td>
<td>3</td>
<td>9,300</td>
<td>39,900</td>
<td>26.4</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>21,700</td>
<td>39.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>21,300</td>
<td>31.9</td>
</tr>
</tbody>
</table>

+ Upper values are for plant AB concrete and lower values for plant CD concrete
### TABLE 9: COMPARISON OF PRESTRESS LOSSES PREDICTED BY THE BRANSON'S AND SURFACE METHOD

<table>
<thead>
<tr>
<th>Problem</th>
<th>Method</th>
<th>SH + REL (psi)</th>
<th>CR + ES (psi)</th>
<th>( \Delta f_s ) (% of ( f_{si} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BRANSON</td>
<td>27,000</td>
<td>25,100</td>
<td>28.0</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>20,800</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>19,600</td>
<td>31.0</td>
</tr>
<tr>
<td>2</td>
<td>BRANSON</td>
<td>26,700</td>
<td>36,700</td>
<td>33.6</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,600</td>
<td>30,500</td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,900</td>
<td>29,200</td>
<td>36.0</td>
</tr>
<tr>
<td>3</td>
<td>BRANSON</td>
<td>26,600</td>
<td>27,100</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>SURFACE+</td>
<td>52,100</td>
<td>21,700</td>
<td>39.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38,000</td>
<td>21,300</td>
<td>31.9</td>
</tr>
</tbody>
</table>

+ Upper values are for plant AB concrete and lower values for plant CD concrete
7. FIGURES
Fig. 1 Illustration of Prediction Procedure
Fig. 2 Relaxation Loss Before Transfer as a Function of Initial Tensioning Stress and Transfer Time
Fig. 3 (SH + REL) Loss as a Function of Initial Tensioning Stress
Fig. 4 (ES + CR) Loss as a Function of Concrete Stress Immediately After Transfer.
Fig. 5  Standard I-Beam 24/42 Section of Pennsylvania Department of Transportation for Problem (I)
应付 11 14 II

\[ \begin{align*}
82'' \\
7\frac{1}{2}'' \\
14'' \\
4'' \\
3'' \\
12'' \\
6'' \\
8'' \\
6''
\end{align*} \]

\[ \begin{align*}
20.77'' \\
7.95'' \\
6.41'' \\
20''
\end{align*} \]

- \( \triangle \) Centroid of Steel
- \( \circ \) Centroid of Composite Section
- \( \square \) Centroid of Girder Section

**Fig. 6** Standard I-Beam 20/33 Section of Pennsylvania Department of Transportation for Problem (2)
Fig. 7 Standard I-Beam 24/60 of Pennsylvania Department of Transportation for Problem (3)
8. **APPENDICES**

A. Notation and Description of Design Examples

B. Bureau of Public Roads Method
   (1954 and Revised)

C. AASHO Method
   (Current and Revised)

D. PCI - General Method

E. CEB Method

F. Branson Method
A.1 **Notation**

Listed here are notations used throughout this thesis as well as the several appendices. Special symbols used for one method only are defined locally and not repeated here. The sign conventions used throughout for the concrete stresses are positive for compression and negative for tension.

\( A_c \) = Gross area of composite section, in \( \text{in}^2 \)

\( A_g \) = Gross area of girder section, in \( \text{in}^2 \)

\( A_{ps} \) = Total area of prestressing steel, in \( \text{in}^2 \)

\( CR \) = Prestress loss due to creep of concrete, in psi

\( e_c \) = Eccentricity of steel from centroid of composite section, in inches

\( e_g \) = Eccentricity of steel from centroid of girder section, in inches

\( E_c \) = Elastic modulus of concrete at 28 days, in psi

\( E_{ci} \) = Elastic modulus of concrete at transfer, in psi

\( E_s \) = Elastic modulus of steel, in psi

\( ES \) = Prestress loss due to elastic shortening of concrete, in psi

\( \Delta f_s \) = Total prestress loss after initial tensioning, in psi

\( f'_c \) = Compressive strength of concrete at 28 days, in psi
\( f'_{ci} = \) Compressive strength of concrete at transfer, in psi

\( f_{pu} = \) Ultimate tensile strength of steel, in psi

\( f_{si} = \) Initial prestress in steel, in psi

\( f_y = \) Yield strength (1% elongation) of steel, in psi

\( I_c = \) Moment of inertia of gross composite section, in \( \text{in}^4 \)

\( I_g = \) Moment of inertia of gross girder section, in \( \text{in}^4 \)

\( M_D = \) Moment due to superimposed dead loads, in k-in.

\( M_G = \) Moment due to girder weight, in k-in.

\( M_S = \) Moment due to slab (and diaphragm) weight, in k-in.

\( n = \) Steel-to-concrete modular ratio at 28 days

\( n_i = \) Steel-to-concrete modular ratio at transfer

\( \text{REL} = \) Prestress loss due to relaxation of steel, in psi

\( \text{SH} = \) Prestress loss due to shrinkage of concrete, in psi

A.2 Description of Design Examples

A.2.1 Problem 1

(a) Section properties:

Span = 80 ft; center-to-center beam spacing = 5 ft.

Standard I-beam 24/42 of Pennsylvania Department of Transportation (see Fig. 5);

Slab thickness = 7.5 in (7 in. effective thickness)
(a) Girder section:

\[ A_g = 588 \text{ in}^2 \quad I_g = 107,986 \text{ in}^4 \quad e_g = 7.31 \text{ in.} \]

(ii) Composite section:

\[ A_c = 1,008 \text{ in}^2 \quad I_c = 294,443 \text{ in}^4 \quad e_c = 18.75 \text{ in.} \]

(b) Concrete properties: (same for beam and slab)

i) At transfer:

\[ f'_{ci} = 5,000 \text{ psi} \quad E_{ci} = 4.06 \times 10^6 \text{ psi} \quad n_i = 7 \]

ii) At 28 days:

\[ f_c = 5,500 \text{ psi} \quad E_c = 4.27 \times 10^6 \text{ psi} \quad n = 6.5 \]

(c) Steel properties:

52 straight 7/16" strands, \( A_{ps} = 6.08 \text{ in}^2 \)

\[ f_{pu} = 270,000 \text{ psi} \quad f_y = 226,000 \text{ psi} \]

\[ f_{si} = 186,000 \text{ psi} \]

(d) Sequence of loading:

i) Transfer of prestress - 18 hours

ii) Casting of slab (unshored) - 7 days

iii) Superimposed dead loads - 35 days
(e) Loads and moments:

i) Girder load:

Unit weight of concrete = 150 pcf

Weight of girder section = \((150) \frac{588}{144}\)

\[= 612.5 \text{ plf}\]

\[M_G = \frac{(612.5)(80)^3}{8}(12) = 5,880 \text{ k-in.}\]

ii) Cast-in-place slab:

Weight of slab section = \(\frac{(150)(7.5)(5)}{12}\)

\[= 468.7 \text{ plf}\]

\[M_S = 4,500 \text{ k-in.}\]

iii) Superimposed dead loads:

Additional dead load = 30 psf = 150 plf

\[M_D = 1,440 \text{ k-in.}\]

A.2.2 Problem 2

(a) Section properties:

Span = 60 ft.; center-to-center beam spacing

\[= 6 \text{ ft. 10 in.}\]

Standard I-beam 20/33 of Pennsylvania Department of Transportation (see Fig. 6);

Slab thickness = 7.5 in. (7 in. effective thickness)
i) Girder section:
\[ A_g = 417 \text{ in}^2 \quad I_g = 44,754 \text{ in}^4 \quad e_g = 7.95 \text{ in.} \]

ii) Composite section:
\[ A_c = 991 \text{ in}^2 \quad I_c = 165,492 \text{ in}^4 \quad e_c = 20.77 \text{ in.} \]

(b) Concrete properties: Same as in problem 1 for both girder and slab

(c) Steel properties:
34 straight 1/2" strands, \( A_{ps} = 5.20 \text{ in}^2 \)
\[ f_{pu} = 270,000 \text{ psi} \quad f_y = 226,000 \text{ psi} \]
\[ f_{si} = 189,000 \text{ psi} \]

(d) Sequence of loading: Same as in problem 1

(e) Loads and moments:

i) Girder load:
Weight of girder section = 435 plf
\[ M_G = 2,350 \text{ k-in.} \]

ii) Cast-in-place slab:
Weight of slab section = 640 plf
\[ M_S = 3,460 \text{ k-in.} \]
iii) Superimposed dead loads:

Superimposed dead load = 150 plf

\[ M_D = 1,110 \text{ k-in.} \]

A.2.3 Problem 3

(a) Section properties:

Span = 103 ft.; center-to-center beam spacing

= 85 in.

Standard I-beam 24/60 of Pennsylvania Department of Transportation (see Fig. 7);

Slab thickness = 7\(\frac{1}{2}\) in. (7 in. effective thickness)

i) Girder section:

\[ A_g = 848 \text{ in}^2 \quad I_g = 355,185 \text{ in}^4 \]

\[ e_g = 19.51 \text{ in. at drape point} \]

\[ = 11.49 \text{ in. at supports} \]

ii) Composite section:

\[ A_c = 1,443 \text{ in}^2 \quad I_c = 790,734 \text{ in}^4 \]

\[ e_c = 34.02 \text{ in.} \]

(b) Concrete properties: Same as in problem 1

(c) Steel properties:

Drape points for prestressing tendons at 0.35 span from supports;
66 - 7/16" strands, \( A_{ps} = 7.72 \) in.\(^2\)

\[ f_{pu} = 270,000 \text{ psi} \quad f_{y} = 226,000 \text{ psi} \]

\[ f_{si} = 186,000 \text{ psi} \]

(d) Sequence of load: Same as problem 1

(e) Dead load moments:

i) Girder load:
\[ M_G = 14,100 \text{ k-in.} \]

ii) Cast-in-place slab and diaphragm loads:
\[ M_S = 11,700 \text{ k-in.} \]

iii) Superimposed dead loads:
\[ M_D = 3,370 \text{ k-in.} \]
APPENDIX B

BUREAU OF PUBLIC ROADS METHOD
(1954 and Revised)

Design Example (see problem 1 in Appendix A)

\[ f_{cs} = (f_{si}) \left( A_{ps} \right) \left( \frac{1}{A_g} + \frac{e_g^a}{f_g} \right) \]

\[ = (186,000) \times (6.08) \times \left( \frac{1}{588} + \frac{7.31^a}{107,986} \right) \]

\[ = 2,490 \text{ psi} \]

\[ ES = 5 f_{cs} = 12,500 \text{ psi} \checkmark \]

\[ SH = 6,000 \text{ psi} \]

\[ CR = 11 f_{cs} = 27,400 \text{ psi} \checkmark \]

\[ REL = 0.04 f_{si} = 7,400 \text{ psi} \]

\[ \Delta f_s = 53,900 \text{ psi} \]

\[ = 28.6\% \text{ of } f_{si} \]

In the above calculations, the 1954 Bureau of Public Roads' formula is used. The results for the revised version of the formula are exactly the same as those above, except that the relaxation losses are doubled and the total losses increased by 4% of \( f_{si} \). Therefore, the total loss becomes 60,700 psi, or 32.6% of \( f_{si} \).
APPENDIX C

AASHO METHOD

(Current and Revised)

C.1 Notation

\( f_{cd} \) = Average concrete stress at centroid of steel under full dead loads, in psi

\( f_{cr} \) = Average concrete stress at centroid of steel after transfer of prestress, in psi (see following comment)

Comment: Concrete stress \( f_{cr} \) is calculated differently in the current and revised AASHO Method. The current method uses the initial stress in steel, while the revised method uses the steel stress immediately after transfer. This difference is illustrated in the following calculations.

C.2 Design Example (see example 1 in Appendix A)

Concrete stresses at centroid of steel:

i) Initial prestress (carried by the girder section)

\[
(f_{si})(A_{ps})\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right)
\]

\[
= (186,000)(6.08)\left(\frac{1}{588} + \frac{7.31^2}{107,986}\right) = 2,490 \text{ psi}
\]
ii) Girder load (carried by the girder section)

\[
\text{At midspan } \frac{M_{eG}}{I_g} = - \frac{(5880) (7.31)}{107,986} = -398 \text{ psi}
\]

\[
\text{At supports } \frac{M_{eG}}{I_g} = 0
\]

iii) Cast-in-place slab load (carried by the composite section) and superimposed dead loads (carried by the composite section)

\[
\text{At midspan } \frac{M_{eS}}{I_g} + \frac{M_{cD}}{I_c} = - \frac{(4500) (7.31)}{107,986} - \frac{(1440) (18.75)}{294,443} = -396 \text{ psi}
\]

\[
\text{At supports } \frac{M_{eS}}{I_g} + \frac{M_{cD}}{I_c} = 0
\]

A. Current AASHO Method

Elastic shortening

\[
\text{At midspan } f_{cr} = 2490 - 398 = 2092 \text{ psi}
\]

\[
\text{At supports } f_{cr} = 2490 - 0 = 2490 \text{ psi}
\]

Average \( f_{cr} = \frac{1}{2} (2092 + 2490) \)

\[
ES = 7 f_{cr} = (7) (2291) = 16,000 \text{ psi}
\]
Shrinkage loss

In the state of Pennsylvania,
relative humidity = 70 - 75%

\[ \therefore \text{SH} = 10,000 \text{ psi} \]

Creep loss

At midspan  \( f_{cd} = 2490 - 398 - 396 = 1696 \text{ psi} \)

At supports  \( f_{cd} = 2490 - 0 - 0 = 2490 \text{ psi} \)

Average  \( f_{cd} = \frac{1}{2} (1696 + 2490) = 2093 \text{ psi} \)

\[ \text{CR} = 16 f_{cd} = (16) (2093) = 33,500 \text{ psi} \]

Relaxation loss

\[ \text{REL} = 20,000 - 0.125 \left( \text{SH} + \text{ES} + \text{CR} \right) \]
\[ = 20,000 - 0.125 \left( 10,000 + 16,000 + 33,500 \right) \]
\[ = 12,600 \text{ psi} \]

Total prestress loss

\[ \Delta f_s = \text{ES} + \text{SH} + \text{CR} + \text{REL} \]
\[ = 16,000 + 10,000 + 33,500 + 12,600 \]
\[ = 72,100 \text{ psi} \]
\[ = 38.8\% \text{ of } f_{si} \]
B. Revised AASHO Method (after Gamble's Proposal)

Elastic shortening

Steel stress after transfer of prestress,

\[ f_{so} = f_{si} \left[ \left( 1 + n_1 A_{ps} \left( \frac{1}{A_g} + \frac{e_g^2}{I_g} \right) \right) \right] \]

\[ = \frac{186,000}{\left( 1 + (7) (6.08) \left( \frac{1}{588} + \frac{7.31^2}{107,986} \right) \right)} \]

\[ = 170,000 \text{ psi} \]

Concrete stress at centroid of steel after transfer,

\[ f_{co} = (f_{so}) (A_{ps}) \left( \frac{1}{A_g} + \frac{e_g^2}{I_g} \right) \]

\[ = (170,000) (6.08) \left( \frac{1}{588} + \frac{7.31^2}{107,986} \right) = 2270 \text{ psi} \]

At midspan \( f_{cr} = 2270 - 398 = 1872 \text{ psi} \)

At supports \( f_{cr} = 2270 - 0 = 2270 \text{ psi} \)

Average \( f_{cd} = \frac{1}{2} (1872 + 2270) = 2071 \text{ psi} \)

ES = 7 \( f_{cr} = (7) (2071) = 14,500 \text{ psi} \)

Shrinkage loss

\( SH = 10,000 \text{ psi for R.H.} \approx 70 - 75\% \)
Creep loss

At midspan \( f_{cr} = 1872 \text{ psi} \), \( f_{cr} - f_{cd} = 396 \text{ psi} \)

At supports \( f_{cr} = 2270 \text{ psi} \), \( f_{cr} - f_{cd} = 0 \text{ psi} \)

Average \( f_{cr} = \frac{1}{2} (1872 + 2270) = 2071 \text{ psi} \)

Average \( (f_{cr} - f_{cd}) = \frac{1}{2} (396 + 0) = 198 \text{ psi} \)

\[ CR = 12 \left( f_{cr} - f_{cd} \right) = (12) (2071) - (7) (198) = 23,400 \text{ psi} \]

Relaxation loss

\[ REL = 20,000 - 0.4 \text{ ES} - 0.2 \text{ (SH + CR)} \]

\[ = 20,000 - (0.4) (14,500) - (0.2) (10,000 + 23,400) \]

\[ = 7,500 \text{ psi} \]

Total prestress loss

\[ \Delta f_s = \text{ES} + \text{SH} + \text{CR} + \text{REL} \]

\[ = 14,500 + 10,000 + 23,400 + 7,500 \]

\[ = 55,400 \text{ psi} \]

\[ = 29.8\% \text{ of } f_{si} \]
APPENDIX D

PCI - GENERAL METHOD

D.1 Notation

AUC = Coefficient defining variation of creep with time

AUS = Coefficient defining variation of shrinkage with time

CR = Prestress loss due to creep over time interval \( t_1 \) to \( t \), in psi

\( f_{cp} \) = Concrete stress at centroid of steel due to prestress alone, in psi

\( f_{ct} \) = Concrete stress at centroid of steel at time \( t_1 \), in psi

\( f_{s_3} \) = Steel stress immediately before transfer, in psi

\( f_{s_3} \) = Steel stress immediately after transfer (without the effect of girder weight), in psi

\( f_{sp} \) = Steel stress due to prestress alone, in psi

\( f_{st} \) = Total steel stress at time \( t_1 \), in psi

PCR = \((\text{AUC at } t) - (\text{AUC at } t_1)\)

PSH = \((\text{AUS at } t) - (\text{AUS at } t_1)\)

RET = Prestress loss due to relaxation over time interval \( t_1 \) to \( t \), in psi
SCF = Coefficient accounting for the effect of size and shape of the member on creep

SH = Prestress loss due to shrinkage of concrete over time interval $t_1$ to $t$, in psi

SSF = Coefficient accounting for the effect of size and shape of the member on shrinkage

UCR = Ultimate creep coefficient

USH = Ultimate loss of prestress due to shrinkage, in psi

V/S = Volume to surface ratio of the member

$t$ = Time after transfer of prestress at the end of a time interval, in days

$t_1$ = Time after transfer of prestress at the beginning of a time interval, in days

D.2 Design Example (see problem 1 in Appendix A)

Concrete stresses at centroid of steel and steel stresses due to various loads:

i) Girder weight (carried by the girder section)

\[
\frac{M_{eg}}{I_g} = \frac{(5880)(7.31)}{107,986} = -398 \text{ psi}
\]

Steel stress = $(7)(398) = 2786 \text{ psi}$
ii) Slab weight (carried by the girder section)

\[
\frac{M_{s}\epsilon_{g}}{I_g} = - \frac{(4500)(7.31)}{(107,986)} = -304 \text{ psi}
\]

Steel stress = (6.5)(304) = 1980 psi

iii) Superimposed dead load (carried by composite section)

\[
\frac{M_{D}\epsilon_{c}}{I_c} = - \frac{(1440)(18.75)}{294,443} = -92 \text{ psi}
\]

Steel stress = (6.5)(92) = 600 psi

Basic Creep and Shrinkage Values (see Ref. 11)

Creep

Steam curing, normal weight concrete UCR = 16.5

\[
SCF = .74 \quad (V/S = 4.31 \text{ in.})
\]

(UCR)(SCF) = (16.5)(.74) = 12.2

\[
CR = (12.2)(PCR)(f_{ct})
\]

Shrinkage

Normal weight concrete

\[
USH = 27,000 - \frac{3000 F_o}{10} = 14200
\]

\[
SSF = .74 \quad (V/S = 4.31 \text{ in.})
\]

(USH)(SSF) = 10,500

\[
SH = (10,500)(PSH) \text{ psi}
\]
Time Interval I: From Anchorage to Transfer

\[ t_1 = \frac{1}{24} \text{ day} \quad t = \frac{18}{24} \text{ day} \quad t/t_1 = 18/2 \]

\[ f_{st} = 186,000 \text{ psi} \quad f_{st}/f_y = 186/226 = .823 \]

\[ \text{RET} = f_{st} \left[ \frac{\log (t/t_1)}{10} \right] \left( \frac{f_{st}}{f_y} - 0.55 \right) \]

\[ = 186,000 \left( \frac{\log 18}{10} \right) (0.823 - 0.55) \]

\[ = 6360 \text{ psi} \]

SH = CR = 0

\[ : \text{At the end of time interval I (before transfer),} \]

\[ f_{sp} = f_{st} = 179,640 \text{ psi} \]

Time Interval II: From Transfer to Casting of Slab

\[ t_1 = \frac{18}{24} \text{ day} \quad t = 7 \text{ days} \quad t/t_1 = 9.3 = 3.33 \]

(a) Transfer loss:

\[ f_{s3} = f_{s2} \left[ 1 + \frac{n_p}{A_p} \left( \frac{1}{A_p} + \frac{c_g^2}{f_g} \right) \right] \]

\[ = 179,640 \left[ 1 + (7) (6.08) \left( \frac{1}{588} + \frac{7.31^2}{107,986} \right) \right] \]

\[ = 164,000 \text{ psi} \]

ES = \[ f_{s2} - f_{s3} = 15,600 \text{ psi} \]
Immediately after transfer,

\[ f_{sp} = 164,000 \text{ psi} \]

\[ f_{cp} = (f_{sp}) (A_{ps}) \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) \]

\[ = (164,000) (6.08) \left( \frac{1}{588} + \frac{7.31^2}{107,986} \right) \]

\[ = 2190 \text{ psi} \]

(b) Time-Dependent loss:

\[ f_{st} = 164,000 + 2,786 = 166,790 \text{ ksi} \]

\[ f_{st}/f_y = 166.79/226 = .74 \]

\[ f_{ct} = 2,190 - 398 = 1,792 \text{ ksi} \]

At time \( t_1 \), AUC = 0, AUS = 0  (see Ref. 11)

At time \( t_2 \), AUC = 0.23, AUS = .22  (see Ref. 11)

\[ PCR = 0.23 - 0 = 0.23 \]

\[ CR = (12.2) (0.23) (1,792) = 5030 \text{ psi} \]

\[ PSH = 0.22 - 0 = 0.22 \]

\[ SH = (10,500) (0.22) = 2310 \text{ psi} \]

\[ RET = (166,790) \left( \frac{\log 9.3}{10} \right) (.74 - .55) \]

\[ = 3080 \text{ psi} \]
Total loss in time interval II

\[ = ES + CR + SH + RET \]

\[ = 15,600 + 5,030 + 2,310 + 3,080 \]

\[ = 26,000 \text{ psi} \]

:. at the end of interval II

\[ f_{sp} = \frac{179,640 - 26,000}{15,030} = 153,640 \text{ psi} \]

\[ f_{cp} = 2,190 \times \frac{153,640}{164,000} = 2,050 \text{ psi} \]

**Time Interval III:** From Casting of Slab to Application of Superimposed Dead Loads

\[ t_1 = 7 \text{ days} \quad t = 35 \text{ days} \quad t/t_1 = 5 \]

\[ f_{st} = 153,640 + 2,786 + 1,980 = 158,410 \text{ ksi} \]

\[ f_{ct} = 2,050 - 398 - 304 = 1,348 \text{ ksi} \]

At time \( t_1 \), AUC = 0.23, AUS = 0.22 (see Ref. 11)

At time \( t \), AUC = .367, AUS = 0.441 (see Ref. 11)

\[ CR = (12.2) (.367 - .23) (1,348) = 2,250 \text{ psi} \]

\[ SH = (10,500) (.441 - .22) = 2,320 \text{ psi} \]

\[ RET = (158,410) \left( \frac{\log 5}{10} \right) (.704 - .55) = 1,670 \text{ psi} \]

Total loss in time interval III = 6,240 psi
At the end of interval III

\[ f_{sp} = 153,640 - 6,240 = 147,400 \text{ ksi} \]

\[ f_{cp} = 2,190 \times \frac{147,400}{164,000} = 1,970 \text{ ksi} \]

**Time Interval IV:** From Application of Superimposed Dead Loads to End of One Year

\[ t_1 = 35 \text{ days} \quad t = 365 \text{ days} \quad t/t_1 = 10.41 \]

\[ f_{st} = 147,400 + 2,786 + 1,980 + 600 = 152,770 \text{ ksi} \]

\[ f_{st}/f = 152.77/226 = 0.677 \]

\[ f_{ct} = 1,970 - 398 - 304 - 92 = 1,176 \text{ ksi} \]

At time \( t_1 \), AUC = .367, AUS = .441 (see Ref. 11)

At time \( t \), AUC = .740, AUS = .860 (see Ref. 11)

\[ CR = (12.2) (\cdot740 - .367)(1,176) = 5,350 \text{ psi} \]

\[ SH = (10,500) (.860 -.441) = 4,400 \text{ psi} \]

\[ \text{RET} = (152,770) \left( \frac{\log 10.41}{10} \right) (.677 - .55) = 1,970 \text{ psi} \]

Total loss in time interval IV = 11,720 psi

∴ at the end of time interval IV

\[ f_{sp} = 147,400 - 11,720 = 135,680 \text{ ksi} \]

\[ f_{cp} = 2,190 \times \frac{135,680}{164,000} = 1,810 \text{ ksi} \]
Time Interval V: From End of One Year to End of One Hundred Years

\[ t = 365 \text{ days} \]
\[ t = 36,500 \text{ days} \]
\[ t/t_1 = 100 \]

\[ f_{st} = 135,680 + 2,786 + 1,980 + 600 = 141,050 \text{ ksi} \]

\[ f_{st}/f_y = 141.05/226 = 0.625 \]

\[ f_{ct} = 1,810 - 398 - 304 - 92 = 1,016 \text{ ksi} \]

At time \( t_1 \), \( AUC = .740, \ AUS = .860 \) (see Ref. 11)

At time \( t \), \( AUC = 1.000, \ AUS = 1.000 \) (see Ref. 11)

\[ \text{CR} = (12.2) (.26) (1,016) = 3,220 \text{ psi} \]

\[ \text{SH} = (10,500) (.14) = 1,470 \text{ psi} \]

\[ \text{RET} = (141,050) \left( \frac{\log 100}{10} \right) (.625 - .55) = 2,120 \text{ psi} \]

Total loss in time interval V = 6,810 psi

\[ f_{sp} = 135,680 - 6,810 = 128,870 \text{ ksi} \]
### Summary

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>SH</th>
<th>CR</th>
<th>RET</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
<td>9,440</td>
</tr>
<tr>
<td>II</td>
<td>2,100</td>
<td>5,030</td>
<td>1,500</td>
</tr>
<tr>
<td>III</td>
<td>2,110</td>
<td>2,270</td>
<td>1,420</td>
</tr>
<tr>
<td>IV</td>
<td>4,400</td>
<td>5,350</td>
<td>1,970</td>
</tr>
<tr>
<td>V</td>
<td>1,470</td>
<td>3,220</td>
<td>2,120</td>
</tr>
<tr>
<td>Σ</td>
<td>10,500</td>
<td>16,020</td>
<td>15,550</td>
</tr>
</tbody>
</table>

\[ Δf_s = ES + \Sigma (SH + CR + RET) \]

\[ N = 13,100 \]

\[ = 15,600 + 10,500 + 15,900 + 15,200 \]

\[ = 57,200 \text{ psi} \]

\[ 29.1\% \]

\[ = 30.8\% \text{ of } f_{\text{si}} \]
APPENDIX E

CEB METHOD

E.1 Notation

\( \varepsilon_c \) = Creep strain in a given time interval

\( \varepsilon_s \) = Shrinkage strain in a given time interval

\( \varepsilon_{so} \) = Shrinkage coefficient depending on the relative humidity

\( \varepsilon_m \) = Theoretical thickness of a section, dividing area of the section by half of its exposed perimeter

\( k_b \) = Coefficient depending on water-cement ratio and cement content of concrete

\( k_c \) = Creep coefficient depending on the relative humidity

\( k_d \) = Coefficient depending on the age of concrete at time of loading

\( k_{ec} \) = Coefficient for creep, depending on the theoretical thickness, \( e_m \)

\( k_{es} \) = Coefficient for shrinkage, depending on the theoretical thickness, \( e_m \)

\( k_p \) = \( 100/(100 + np) \), where \( n = 20 \) and \( p \) is the steel percentage

\( k_t \) = Coefficient for time variations

\( \varphi_t \) = \( k_c k_d k_b k_{ec} k_t \)
\( f_c \) = Constant concrete stress at centroid of steel producing creep in a given time interval, in psi

\( f_{cr} \) = Concrete stress at centroid of steel after transfer, in psi

\( f_{so} \) = Steel stress after transfer, in psi

\( E_{28} \) = Elastic modulus of concrete at 28 days, in psi

\( RLX \) = Total "pure" relaxation loss (without interaction of creep and shrinkage), in psi

E.2 Design Example (see example 1 in Appendix A)

Determination of coefficients (see sections R12.31 and R12.32 of Ref. 7).

i) Creep coefficients:

\[ k_c = 2.30 \quad (R.H. = 70\%) \]

\[ k_b = 0.8 \quad (\text{water-cement ratio} = 0.38 \text{ by weight; cement content} = 420 \text{ kg/m}^3) \]

\[ k_{ec} = 0.8 \quad (e_m = 25.2 \text{ cm.}) \]

For high early strength cement,

\[ k_d = 1.7 \quad \text{at } t = 1 \text{ day} \]

\[ k_d = 1.1 \quad \text{at } t = 7 \text{ days} \]

\[ k_d = 0.7 \quad \text{at } t = 35 \text{ days} \]

\[ k_d = 0.3 \quad \text{at } t = 1 \text{ year} \]
ii) Shrinkage coefficients:

\[ \varepsilon_{so} = 27.5 \times 10^{-6} \quad \text{(R.H. = 70\%)} \]

\[ k_b = 0.8 \quad \text{(same as } k_b \text{ in creep)} \]

\[ k_{es} = 0.72 \quad \text{(e}_m = 25.2 \text{ cm.)} \]

\[ k_p = \frac{100}{100 + np} = \frac{100}{100 + (20)(1.03)} = 0.83 \]

iii) Coefficients \( k_t \) for \( e_m = 25.2 \text{ cm.} \)

\[ k_t = 0.06 \quad \text{at } t = 7 \text{ days} \]

\[ k_t = 0.20 \quad \text{at } t = 35 \text{ days} \]

\[ k_t = 0.70 \quad \text{at } t = 1 \text{ year} \]

\[ k_t = 1.00 \quad \text{at } t = \text{end of service life} \]

Elastic Shortening

\[ E_{ci} = 79,500 \sqrt{\frac{f'}{f_{ci}}} \quad \text{(equation 2.10)} \]

\[ = (79,500) \sqrt{5000} = 5.62 \times 10^6 \text{ psi} \]

\[ n_i = \frac{E_s}{E_{ci}} = \frac{28 \times 10^6}{5.62 \times 10^6} = 5 \]
Steel stress after transfer of prestress,

\[ f_{so} = \frac{f_{si}}{1 + n_i A_{ps} \left( \frac{1}{A} + \frac{e_i^a}{I} \right) = 186,000/[1 + (5) (6.08) (\frac{1}{588} + \frac{7.31^a}{107,986})] = 174,000 \text{ psi} \]

Concrete stress at centroid of steel after transfer,

\[ f_{cr} = (f_{so}) (A_{ps}) \left( \frac{1}{A} + \frac{e_i^a}{I} \right) + \frac{M_{eg} e_i}{I} = (174,000) (6.08) (\frac{1}{588} + \frac{7.31^a}{107,986}) - (5880) (7.31) = 1922 \text{ psi} \quad \text{(See Note 1)} \]

\[ ES = (n_i) (f_{cr}) = (5) (1922) = 9610 \text{ psi} \]

Basic Shrinkage, Creep and Relaxation Loss Coefficients

(Note 2)

At end of service life,

\[ \varepsilon_s = \varepsilon_{so} k_p k_{es} k_p k_t \]

\[ = (27.5 \times 10^{-6}) (0.8) (0.72) (0.83) (1) = 0.132 \times 10^{-3} \]

\[ SH = (\varepsilon_s) (E_s) = (0.132 \times 10^{-3}) (28.0 \times 10^6) = 3,690 \text{ psi} \]
For $f_{si} = 0.8 f_{pu} = 216,000$ psi,

$$RLX = 12\% \text{ of } f_{si} = 25,900 \text{ psi}$$

(See section R11.22, Ref. 11)

For $f_{si} = 0.69 f_{pu} = 186,000$ psi

$$RLX = \frac{(25,900)(0.69 - 0.5)^2}{0.09} \text{ (See Note 3)}$$

$$= 10,400 \text{ psi}$$

For a constant concrete stress $f_c$,

$$\varepsilon_c = \frac{f_c}{E_{2g}} \phi_t$$

(equation 2.11)

where $\phi_t = k_c k_d k_b k_{ec} k_t$

$$= (2.30)(k_d)(0.8)(0.8)k_t = 1.47 k_d k_t$$

$$E_{2g} = 79,500 \sqrt{f_{ci}}$$

$$= (79,500)\sqrt{5,500} = 5.9 \times 10^6 \text{ psi}$$

**Interval I: Transfer Time to Casting of Slab**

At transfer time,

$$k_t = 0, \quad k_d = 1.7$$

$$f_c = f_{cr} = 1,922 \text{ psi} \text{ (See Elastic Shortening)}$$

At casting of slab, $k_t = 0.06$

$$\varepsilon_c = (1922)(1.47)(1.7)(0.06 - 0) = 4.88 \times 10^{-4}$$
In Interval I:

Creep loss = \( \left( \frac{4.88 \times 10^{-4}}{28 \times 10^6} \right) \times 28 \times 10^6 \) = 1,370 psi

Shrinkage loss = \( (0.06 - 0) \times 3,690 \) = 222 psi (See Note 4)

Relaxation loss = \( (0.06 - 0) \times 10,400 \) = 624 psi (See Note 4)

Total loss = 2,216 psi

Interval II: Casting of Slab to Application of Superimposed Loads

At casting of slab,

\( k_t = 0.06, \quad k_d = 1.1 \)

At time of application of superimposed loads,

\( k_t = 0.20 \)

Concrete stress at centroid of steel due to loss from interval I and slab weight

\[
\varepsilon_c = \frac{(1922)(1.47)(1.7)(0.20 - 0.06) - (334)(1.47)(1.1)(0.20 - 0)}{5.9 \times 10^6}
\]

\[
\varepsilon_c = \frac{-334}{5.9 \times 10^6}
\]

\[
\varepsilon_c = .956 \times 10^{-4}
\]
In interval II:

Creep loss = \((.956 \times 10^{-4}) (28 \times 10^6)\) = 2,680 psi

Shrinkage loss = \((0.20 - 0.06) (3,690)\) = 516 psi

Relaxation loss = \((0.20 - 0.06) (10,400)\) = 1,460 psi

Total loss = 4,656 psi

**Interval III:** Application of Superimposed Dead Loads to End of One Year

At time of application of superimposed dead loads

\[k_t = 0.20, \quad k_d = 0.7\]

At end of one year,

\[k_t = 0.70\]

Concrete stress at centroid of steel due to loss from interval II and superimposed dead loads

\[\varepsilon_c = \left\{\left[\left(1922\right) \left(1.47\right) \left(1.7\right) - \left(334\right) \left(1.47\right) \left(1.1\right)\right] \left(0.7 - 0.2\right)\right.\]

\[- \left(154\right) \left(1.47\right) \left(0.7\right) \left(0.7 - 0.0\right)\] \(\left(\frac{1}{5.9 \times 10^6}\right)\)

\[= 3.42 \times 10^{-6}\]
In interval III:

Creep loss \( = (3.42 \times 10^{-4}) (28 \times 10^6) \) = 9,570 psi

Shrinkage loss \( = (0.7 - 0.2) (3,690) \) = 1,845 psi

Relaxation loss \( = (0.7 - 0.2) (10,400) \) = 5,200 psi

Total loss = 16,615 psi

**Interval IV: End of One Year to End of Service Life**

At end of one year,

\( k_t = 0.70, \quad k_d = 0.3 \)

At end of service life,

\( k_t = 1.00 \)

Concrete stress at centroid of steel due to loss from interval III

\[
\varepsilon_c = \left\{ [(1.1) - (1.1) - (1.147) (1.0 - 0.7)] \quad (1.47) (1.0 - 0.7) \\
- (220) (1.47) (0.3) (1 - 0) \right\} \frac{1}{5.9 \times 10^6} \]

\( = 2.08 \times 10^{-4} \)
In interval IV:

Creep loss = (2.08 \times 10^{-4}) (28 \times 10^6) = 5,820 psi

Shrinkage loss = (1.0 - 0.7) (3,690) = 1,110 psi

Relaxation loss = (1.0 - 0.7) (10,400) = 3,120 psi

Total loss = 10,050 psi

**Total Losses:**

CR = 1,370 + 2,680 + 9,570 + 5,820 = 19,440 psi

SH = 3,690 psi

Modified total relaxation loss, (See Note 3)

\[
REL = RLX \left[ 1 - 3 \left( \frac{SH + CR}{f_{si}} \right) \right]
\]

\[
= (10,400) \left[ 1 - 3 \left( \frac{3,690 + 19,440}{186,000} \right) \right]
\]

\[
= 6,500 \text{ psi}
\]

**Summary**

ES = 9,600 psi

SH = 3,700 psi

CR = 19,400 psi

REL = 6,500 psi

Total prestress loss = 39,200 psi (21% of \( f_{si} \))
Note 1: As mentioned in section 2.5, Eq. 2.10 only applies for working concrete stresses less than $0.4 f'_{ci}$. In this problem,

$$f_{cr} = 1,922 \text{ psi}$$

$$0.4 f'_{ci} = (0.4)(5,000) = 2,000 \text{ psi}$$

$$\therefore f_{cr} < 0.4 f'_{ci}$$

However, for problems 2 and 3 in Appendix A, $f_{cr}$ is greater than $0.4 f'_{ci}$. In these two problems, the ACI Code formula for determining the elastic modulus of concrete was used.

Note 2: In order to take into account the effect of varying concrete stresses on creep, the determination of prestress loss due to creep is done in four separate time intervals.

i) Interval I: From transfer time to casting of slab at 7 days

ii) Interval II: From casting of slab to application of superimposed dead loads at 35 days

iii) Interval III: From application of superimposed dead loads to end of 1 year

iv) Interval IV: From end of 1 year to end of service life
Note 3: RLX is the pure relaxation loss without any interaction from shrinkage and creep of concrete. REL is the actual relaxation loss, considering interactions from shrinkage and creep, and is calculated by Eq. 2.15.

Note 4: Time variations of shrinkage and relaxation losses are assumed to be the same as that of creep loss.
APPENDIX F

BRANSON METHOD

F.1 Notation

\[ \alpha_s = \frac{t^{0.6}}{(10 + t^{0.6})} \] where \( t \) is time after initial loading in days

\[ \beta_s = \text{Creep correction factor for the age of concrete when the slab is cast} \]

\[ \varepsilon_{su} = \text{Ultimate shrinkage strain} \]

\[ f_c = \left( \frac{F_o}{A_g} \right) + \left( \frac{F_o \varepsilon_g^2}{I_g} \right) - \left( \frac{M_c e_g}{I_g} \right), \text{in psi} \]

\[ f_{cd} = \text{Concrete stress at centroid of steel due to differential shrinkage, in psi} \]

\[ f_{cs} = \text{Concrete stress at centroid of steel due to slab weight, in psi} \]

\[ \Delta F_s = \text{Total prestress loss at slab casting minus the initial elastic loss, in kips} \]

\[ \Delta F_u = \text{Total ultimate prestress loss minus the initial elastic loss, in kips} \]

\[ F_i = \text{Initial tensioning force, in kips} \]

\[ F_o = \text{Prestress force after transfer, in kips} \]

\[ k_s = 1 + \left[ \left( \frac{e_g}{I_g/A_g} \right) \right] \]
\[ m = \text{Steel-concrete modular ratio at the time of slab casting} \]
\[ n = \text{Steel-concrete modular ratio at transfer time} \]
\[ p = \text{Steel percentage, } \frac{A_{ps}}{A_g - A_{ps}} \]

F.2 Design Example  (see example 1 in Appendix A)

Required parameters:

For 35 days between initial prestress and application of dead loads, with normal weight concrete

\[ \frac{\Delta F_s}{F_o} = 0.11, \quad \frac{\Delta F_u}{F_o} = 0.22 \]
\[ \alpha_s = 0.44, \quad \beta_s = 0.83 \]

For an ambient relative humidity of 70%

\[ C_u = 1.88, \quad \varepsilon_{su} = 510 \times 10^{-6} \text{ in./in.} \]

(1) Instantaneous Elastic Loss:

\[ p = \frac{A_{ps}}{A_g - A_{ps}} = \frac{6.08}{588 - 6.08} = 0.0104 \]

\[ F_i = (f_{si}) \left( A_{ps} \right) = (186,000)(6.08) \]
\[ = 1,130 \text{ k} \]

\[ F_o = F_i (1 - n p) = (1130)(1 - 0.7 \times 0.0104) \]
\[ = 1,050 \text{ k} \]

\[ f_c = \frac{F_o}{A_g} + \frac{F_0 e_g^2}{I_g} - \frac{M e_g}{I_g} \]
\[
f_c = \frac{1050}{588} + \frac{(1050)(7.31)^2}{107,986} - \frac{(5880)(7.31)^2}{107,986} = 1,910 \text{ psi}
\]
\[
nf_c = (7)(1,910) = 13,400 \text{ psi}
\]

(2) Creep Loss Until Casting of Slab:

\[
(nf_c) \left( \alpha_s c u \right) \left( 1 - \frac{\Delta F_s}{2F_o} \right)
\]

\[
= (13,400)(.827)(.945) = 10,500 \text{ psi}
\]

(3) Creep Loss After Casting of Slab:

\[
(nf_c) \left( 1 - \alpha_s \right) C_u \left( 1 - \frac{\Delta F_s - \Delta F_u}{2F_o} \right) \frac{I_g}{I_c}
\]

\[
= (13,400)(.56)(1.88)(.835) \left( \frac{107,986}{294,443} \right) = 4,310 \text{ psi}
\]

(4) Shrinkage Loss:

\[
k_s = 1 + \frac{e^a}{(I_g/A_g)} = 1 + (7.31^a)(\frac{588}{107,986})
\]

\[
= 1.29
\]

\[
\frac{(\varepsilon_{su} E_s)}{1 + npk_s} = \frac{(510 \times 10^{-6}) (28 \times 10^9)}{1 + (7)(0.0104)(1.29)} = 13,100 \text{ psi}
\]

(5) Relaxation loss:

\[
0.075 f_{si} = (0.075)(186,000) = 13,900 \text{ psi}
\]
(6) Elastic Rebound Due to Slab Weight:

\[ f_{cs} = \frac{M_e \cdot E}{I} = \frac{(4500)(7.31)}{107,986} = 304 \text{ psi} \]

\[ mf_{cs} = - (6.5)(304) = - 1,975 \text{ psi} \]

(7) Creep Rebound Due to Slab Weight:

\[ \left( mf_{cs} \right) \left( \beta_s C_u \right) \left( I_g / I_c \right) \]

\[ = - (1975)(1.56) \left( \frac{107,986}{294,443} \right) \]

\[ = - 1,130 \text{ psi} \]

(8) Elastic Rebound Due to Differential Shrinkage:

This term is neglected.

**Summary**

<table>
<thead>
<tr>
<th>Term</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13,400</td>
</tr>
<tr>
<td>2</td>
<td>10,500</td>
</tr>
<tr>
<td>3</td>
<td>4,300</td>
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<tr>
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<tr>
<td>5</td>
<td>13,900</td>
</tr>
<tr>
<td>6</td>
<td>-2,000</td>
</tr>
<tr>
<td>7</td>
<td>-1,100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>52,100 psi</strong> (28.0% of ( f_{si} ))</td>
</tr>
</tbody>
</table>
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16. Ying, H., Schultchen, E. and Huang, T.
Hai-Tung Ying was born on September 2, 1947 in Shanghai, China, the youngest son of Foh-Mei Ying and Yuk-Hang Tam.

In 1949 his family moved to Hong Kong, where he attended elementary and high school. In September 1966 he enrolled at the University of California, Berkeley, California, U. S. A., where he received his Baccalaureate Degree in the Department of Civil Engineering in March 1970.

In September 1970 he joined the research staff in the Fritz Engineering Laboratory, Department of Civil Engineering, at Lehigh University. He was associated with the research project on "Prestress Losses in Pretensioned Concrete Structural Members" in the Structural Concrete Division of the Fritz Engineering Laboratory.