Design Recommendations for Multi-Story Frames

THE COMPUTER ANALYSIS OF UNBRACED MULTI-STORY FRAMES

by

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This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Office of Naval Research
Naval Ship Engineering Center
Naval Facilities Engineering Command

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Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

May 1968

Fritz Engineering Laboratory Report No. 345.5
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ABSTRACT

This paper describes a computer program based on the sway-subassemblage method for the strength and drift analysis of unbraced, plastically designed, multi-story frames. It is intended that the program be a quick, reliable means of checking frame strength and drift for the practicing designer. For this reason, the program and a detailed explanation of its use are given as well as the assumptions and basic theories upon which it is based. More detailed explanations of the sway-subassemblage method can be found in the references listed at the end of this paper.

The middle and lower stories of unbraced, multi-story, plane frames of up to eight bays are considered. In this region, the columns are assumed to be bent in almost symmetrical double curvature so that the sway-subassemblage can be used. The program is written in the Fortran IV language so that it is more generally applicable.
1. INTRODUCTION

This report presents a computer program based on the sway-subassemblage method for the strength and drift analysis of unbraced multi-story frames. The program is for rigid, plane frames of up to eight bays. The middle and lower stories of unbraced multi-story frames are the ones generally considered since in this region the columns are bent in almost symmetrical double curvature thus satisfying one of the main assumptions of the sway-subassemblage method. The columns in the top several stories and possibly several of the bottom stories do not always meet this assumption and therefore the sway-subassemblage method and the computer program must be used with care in these regions. Current research is trying to extend the sway-subassemblage method to the top and bottom floors.

A basic outline of present manual design procedures for plastically designing unbraced multi-story frames is given. In this discussion, the importance of being able to check frame strength and drift is shown. Also shown in the necessity of being able to obtain enough information from these checks to act as a basis for possible member size revision. The method of analysis which is considered to give the most information is the subassemblage method. A discussion of the assumptions and basis of the subassemblage method is given.

Using the basic tenets and assumptions of the subassemblage
method, a computer program has been written which analyses unbraced multi-story frames. The use of the program is limited to those floors which can justifiably be analyzed by the subassemblage method. The program starts with the member properties, geometry and loads of the frame and ends with a load deflection curve for each level of the area of the frame in question. To include the moment-rotation behavior of the columns, it was assumed that their moment-rotation curves could be approximated by straight lines. A full discussion of the computer program along with sample input and output are given. A listing of the program as well as flow charts are also given. The use of the program in the revision of member sizes is shown.

The 1963 AISC Specification limits the application of plastic design to one- and two-story buildings of ASTM A7 and A36 structural steels. Recent research has shown that plastic design can be safely extended to taller frames. Until these research advances are included in the AISC Specification, the designer must obtain permission from the proper regulatory body to use the plastic design methods considered in this report.
2. THE DESIGN OF UNBRACED FRAMES

The design of unbraced multi-story frames can be divided into four steps: the calculation of initial design data, the selection of preliminary members, design checks, and the revision of preliminary member sizes. In the first step, an orderly system of frame identification is chosen so that every member and its location is named. The type and amount of loading are selected as dictated by the building's function, the applicable codes, and the designer's judgement. An orderly method of load application and the critical loading cases are selected. For the plastic design of unbraced multi-story frames, the critical loading cases are gravity loading and the combined loading of gravity and wind loads. The individual girder and column loads are calculated and possible live load reductions are applied. Cumulative column loads are calculated for each line of columns. If the design is to be based on plastic theory, the working loads are multiplied by two load factors, one for the gravity load case and one for the combined load case. Horizontal wind loads are calculated for each story and applied as concentrated loads at the joints. The sum of the applied horizontal loads at each level is also found.

Based on the initial design data preliminary girder members

* Superscripts are used to denote reference numbers
Much of the work involved in the preliminary design of unbraced multi-story frames lends itself to computer application. An
existing program is available which tabulates the design loads and performs a moment balance based on the loads and the frame geometry. To aid in the moment balancing procedure a restrictive pattern of plastic hinge formation is assumed. Based on the results of the program, girder and column sections can be chosen.

As in all designs, multi-story frames have certain required design checks. The beams are checked for local and lateral buckling to ensure that their ultimate strength can be reached. Beams are also checked for vertical deflection to see that it is within certain acceptable limits. Columns are checked for strong and weak-axis buckling and lateral-torsional buckling if they are not braced out-of-plane between floors. For the gravity loading condition, the problem of frame buckling can be considered. Recent research has shown that frame buckling is not a problem in most frames. Since the preliminary design of the frame was based on assumed values of relative story drift, it is necessary to analyze the frame to ensure that the members selected are capable of supporting the design loads. It is also necessary to check the actual frame drift of each level at working load to make sure that the drift is within acceptable limits. The drift of each story level can generally be obtained as a byproduct of the analysis for frame strength.

If the results of the analysis for frame strength or the check of frame drift prove unsatisfactory, it is necessary to make revisions to the preliminary members. The results of the frame analysis generally indicate which parts of the frame need revision. The revision of member sizes constitutes a new preliminary design which must be checked for drift and analyzed for strength.
3. THE ANALYSIS OF MULTI-STORY FRAMES

Realizing the importance of being able to analyse a multi-story frame for the effects of wind loading, it is necessary to determine the best method of analysis. For plastically designed frames, two methods are available: the modified slope deflection method and the subassemblage method. In the first method, modified slope deflection equations are used for the analysis of the whole frame as a unit. The result of this slope-deflection type analysis is a load-deflection curve for the frame up to the point of failure of some portion of the frame. No information is obtained about the strength of the remainder of the frame.

The subassemblage method subdivides the frame into small units for analysis. Using the results of the analysis of the individual units, load-deflection curves for each floor level, before and after failure, can be drawn. The subassemblage method thus gives an idea of the behavior of each individual member in the frame. Because of the additional information on frame strength and drift, the subassemblage method was selected as the most suitable for use in the design and analysis of unbraced multi-story frames. The description of the subassemblage method which follows was presented earlier in Ref. 8.

3.1 Frame Subdivisions

In the preliminary design, it is assumed that the columns in the middle and lower stories are bent in double curvature with the inflection point at the mid-height. Using this same assumption, a particular level can be separated from a frame at the inflection points of
the columns above and below the level for the purpose of analyzing for strength and drift. Separation of a level of a frame is shown in Fig. 1. Also shown are the various forces acting on the level. The horizontal shear load is assumed to be evenly distributed to all the columns above and below the level. A simplification of the separated level is obtained by replacing the upper column segments by the loads and moments the segments impose on the level (Fig. 2).

The level in question can be further subdivided into two types of subassemblages consisting of a lower column section and one or two adjoining beams (Fig. 3). In each of these types of subassemblages, the girders are considered as the restraining members which help the column resist the effects of the applied forces. Two phases of subassemblage behavior are of importance in determining subassemblage strength and drift. The first phase is the behavior of the girders as their end moments increase and their stiffness change during the process of forming girder mechanisms. The second phase is the behavior of the columns as a consequence of the changes in restraint supplied by the girders.

3.2 Behavior of Beams

Girder behavior can be generalized in three stages. The first stage is the initial state of the structure before lateral load is applied. At this stage the end moments and the end stiffness of the girders are usually in the elastic range. In the second stage, changes in end moments and stiffnesses of the girders occur as lateral load is increased and mechanisms form in the girders. In the third stage, all controlling plastic hinges have formed in the girders attached to the
given column. A constant restraining moment will be available to resist drift of the column. The subassemblage would be in a state of uncontrolled plastic flow unless some other subassemblage in the story has not formed a mechanism.

In the initial stage of girder behavior, the moment diagrams of the girders can be found by using the fixed end moments or the moments at the end of one cycle or more of an elastic moment distribution. The limiting moments at this stage can be found by using the moment diagram solutions for girders under uniformly distributed or third point loading.

If an ideal elastic-plastic stress-strain curve is assumed, the slope-deflection equation can be applied to the subassemblage. Based on this equation the resisting moment of each girder in the subassemblage can be found by:

\[ M_{r} = \frac{KEI\theta}{L} \]

where:
- \( M_{r} \) = restraining moment at a certain end
- \( \theta \) = rotation of the beam at the same end
- \( K \) = girder stiffness factor at the same end.

The initial girder stiffness factor assumed in this report is 6.0 for all girders. More exact procedures exist for finding the girder stiffness factor but their application is involved and the final answers are not markedly affected by an initial value of \( K \) different than 6.0.¹¹

After the formation of a plastic hinge in a girder, the girder stiffness is reduced and the restraining moment of that girder is revised depending on the position of the plastic hinge (Fig. 4). The re-
vised restraining coefficients corresponding to the reduced restraining stiffness can be determined from the order of formation of plastic hinges as follows; designating the locations of hinges by their numbers in Fig. 4:

1. **If plastic hinge 1 occurs before plastic hinge 3:** Since additional moment can not be developed at joint A, beam BA may be considered as pinnaed at A. Thus \( K_{BA} \) reduces to 3.0.

2. **3 occurs after 1:** \( K_{BA} \) reduces from 3.0 to 0.

3. **3 occurs before 1:** \( K_{BA} \) reduces to 0.

4. **6 or 7 occur:** \( K_{BC} \) reduces to 3.0.

5. **5 occurs after 6 or 7:** \( K_{BC} \) reduces from 3.0 to 0.

6. **At the time hinge 4 occurs:** \( K_{BA} \) and \( K_{BC} \) remain unchanged from their values at the time 4 develops.

### 3.3 Behavior of Restrained Columns

A restrained column is one whose joint rotation is restrained by the stiffness of adjoining members. A restrained column permitted to sway is one in which lateral displacement of the top of the column relative to its bottom is permitted. A typical restrained column is shown in Fig. 5. The restraining effects of the adjoining girder are represented by a coil spring and the ability to displace laterally is shown by a roller support at the upper end of the column. The forces acting on the restrained column are shown in Fig. 6. From statics, the moment at the **upper end of the column** can be found to be:

\[
M = -\left[ Q \frac{h}{2} + P \frac{A}{2} \right]
\]

where

- \( M \) = column end moment
- \( Q \) = horizontal shear force on the column
- \( h \) = height of the column
Equilibrium of moments at the joint requires that

\[ 2 M + M_r = 0 \]  

(3)

where \( M_r \) is the restraining moment. For small angles of rotation, \( \theta \), \( \gamma \), and \( \Delta/h \) in Fig. 7 are all related by the compatibility condition

\[ \frac{\Delta}{h} = \theta - \gamma \]  

(4)

where \( \theta \) defines the column top (or joint) rotation and \( \gamma \) the column chord rotation.

The sign convention used in establishing these equations is as follows:

1. External moments acting at a joint are positive when clockwise.
2. Internal moments acting at a joint are positive when counterclockwise.
3. Moments and rotations at the ends of members are positive when clockwise.
4. Horizontal shear on a column is positive if it causes a clockwise moment about the opposite joint.

Based on Eqs. 2, 3, and 4, design charts can be plotted to give the non-dimensional load-deflection curve \( Qh/2 M_{pc} \) vs. \( \Delta/h \) for the restrained column, with values of rotational restraint stiffness from zero to infinity. Figure 7 shows a sample design chart. The curves sloping upward to the right are the restrained column curves. If a plastic hinge forms at the top of a restrained column, the curve will follow the second order rigid-plastic mechanism curve shown sloping
3.4 Initial Condition of the Restrained Column

The design chart shown in Fig. 7 assumes that the top of the restrained column has not rotated at the time of zero sway displacement. For the columns in most multi-story frames, this assumption is not correct. The individual bays are of unequal loading and or length which causes the girder end moments of the individual girders to be different. From studying equilibrium at the column top, it can be seen that unless the girder end moments are of equal value and of opposite sign there must be some rotation of the column top. This means that when the girders are loaded with gravity load alone (initial condition) the column tops (or the joints) have some initial value of rotation.

The effects of the initial joint rotation can be taken into account by adjusting the compatibility equation (Eq. 4) by the amount of the initial joint rotation. If the initial joint rotation is taken as $\theta_0$, the compatibility equation can be rewritten as

$$\frac{\Delta}{h} = \theta - \gamma + \theta_0$$  \hspace{1cm} (5)

The effect of this change on the design charts is to shift them parallel to the second-order elastic-plastic curves in a direction compatible with the sign of the initial joint rotation.
4. COMPUTER PROGRAM

Based on the subassemblage method of analysis and restrained column theory, a computer program has been written to find the load-deflection curve for a level of an unbraced multi-story frame. The basic subassemblage used in establishing the subscripts used in the program is shown in Fig. 8. K is used for the level number while J can refer to either a joint number starting with one on the left or a girder starting with one on the left. The program is written in the Fortran language and is limited to rigid, plane, unbraced multi-story frames of up to eight bays.

The program begins by setting the initial girder stiffness coefficients equal to six and by calculating the girder fixed end moments and the limiting girder moments for the right end of each girder. The coordinates of the starting point of each subassemblage load-deflection curve are also found. Working with one subassemblage at a time, the joint rotation is incremented, checking at each increment for the formation of a plastic hinge. When a hinge has formed, its position on the subassemblage load-deflection curve is found and if the relative story displacement is greater than 0.001, the program interpolates between the hinge point and the initial point to obtain load values for relative story deflection values that are integer functions of 0.001. The program continues to increment the joint rotation until a mechanism forms or a plastic hinge forms in the top of the column. After the formation of each
plastic hinge, the program interpolates for the horizontal loads between the current hinge displacement and the previous hinge displacement for integer values of 0.001. At the formation of the last girder plastic hinge or at the formation of a plastic hinge in the column, the load corresponding to a relative story displacement of 0.025 is found and the program interpolates between this point and the last hinge point. When all the subassemblages in a level have been analyzed, the program adds the individual load values for incremental values of relative story displacement from 0.001 to 0.025 to obtain a load-displacement curve for the level.

4.1 The Main Program

The main program *(Fig. 10-18) begins by reading and printing the number of levels to be analyzed and the number of joints in each level. The remainder of the main program is made into a DO loop starting at one and going to the number of levels. This applies the remainder of the main program to each level to be analyzed. Subroutine OUTIN is called to supply the required data for the level in question. Another DO loop is made of what remains of the main program starting at one and going to the number of joints. This applies the main program from the calling of OUTIN to the end to each subassemblage in the level in question.

Subroutines STIFF, AINIT, and AMLIM are called to calculate required values based on the loads and the frame geometry. Indexes which indicate whether plastic hinges have formed at possible girder

*Portion of the main program are from an earlier program written in LEWIZ by Dr. J. Hartley Daniels.
hinge points (Fig. 9) are set equal to zero to indicate that no hinges have formed. The initial joint rotations are set equal to zero and the moments at the hinge points at the ends of the beams are set equal to the corresponding fixed end moment.

The incremental moments at the hinge locations at the ends of the girders corresponding to the set incremental theta values are found. The location and value of the maximum moments in the interior of the beams are calculated. The distances to the maximum interior moments are checked to ensure that they do not exceed the lengths of the beams. If the beam lengths are exceeded, the moments at the left end of the beams control. Each possible hinge location is then checked for the formation of a plastic hinge. The interior or left end moments, whichever is controlling, are checked with the beam plastic moments while the right end moments are checked with the limiting moments from subroutine AMLIM. If a plastic hinge has not occurred, the total joint rotation is increased by the incremental joint rotation value and the procedure is repeated starting with the calculation of incremental moment values.

If a plastic hinge has occurred, the indicator for that hinge location is set equal to one and the girder end stiffnesses are reduced according to the rules given in Chapter 3. Subroutines POINT and ARRAY are called to find the coordinates of the plastic hinge point on the subassembly load-deflection curve and to interpolate between the current hinge point and the previous hinge point for the horizontal loads corresponding to integers times 0.001 values of relative joint displacement. The total joint rotation is increased by the incremental joint rotation value and the procedure is continued.
starting with the calculation of incremental moment values.

After the incremental moment values and new total values of the moments at the possible hinge locations are found, the new restraining moment on the column is calculated. If the new restraining moment equals or exceeds twice the column elastic moment, the restraining moment is set equal to twice the column plastic moment and subroutines POINT and ARRAY are called. The relative story deflection is set equal to 0.025, the horizontal load is calculated and the load and relative story deflection are printed. Subroutine ARRAY is called and the program continues to the next subassembly.

If enough plastic hinges have formed to cause the girder stiffnesses on both sides of the column to equal zero, a failure mechanism has formed. The restraining moment is held at its value at the formation of the column plastic hinge and the relative story displacement is set equal to 0.025. The horizontal load is calculated and printed along with the relative story deflection. Subroutine ARRAY is called and the program continues to the next subassemblage. If all the subassemblages in the level have been analyzed, subroutine ADD is called and the program continues to the next level to be analyzed.

4.2 The Subroutines

a) OUTIN

Subroutine OUTIN (Fig. 19) reads the member properties, loads, frame geometry, and incremental theta value required for the main program. This subroutine also prints the data it reads in tabular form.
b) STIFF

The initial values of girder stiffnesses are set in this subroutine (Fig. 20). The subroutine assumes that the initial girder stiffness factors are equal to 6.0.

c) AINIT

Subroutine AINIT (Fig. 21) calculates the fixed end moments for all the girders in the level in question. Then for each column, the initial restraining moment is found by adding the adjoining fixed-end moments; the initial relative story displacement is set equal to zero and the initial horizontal load is found using Eqs. 1 and 2. The initial horizontal load and the relative story displacement are printed as output.

d) AMLIM

Subroutine AMLIM (Fig. 22) calculates the limiting girder end moment at the column centerline for the right end of each girder. To find the limiting moments, the subroutine uses either Eq. 14.14 or 14.21 depending upon which is applicable.

e) POINT

Using the restraining moment and the sum of the incremental theta values at the time a plastic hinge occurs, subroutine POINT (Fig. 23) calculates the coordinates of a point on the subassemblage load-deflection curve and prints the coordinates as output. The relative story displacement is found from Eq. 3 and used with Eqs. 1 and 2 to find the horizontal load.

As Eq. 3 is given, it can not be used directly in the computer program. This is because the term $\gamma$ (column chord rotation) is not calculated but is obtained from moment-rotation curves for each column. To include the moment-rotation curves in the program, it was assumed that each moment-rotation curve can be approximated by a straight line through the initial portion of the curve and extending upward till the column moment equals the column plastic moment (Fig.
Based on the data used to plot the moment-rotation curves, values of rotation for $M/M_{pc} = 1$ were obtained for a range of slenderness and axial load to axial yield load ratios. Using these values of rotation, an equation in terms of $P/P_y$ and $h/r$ was developed as follows:

$$\gamma^' x 10^5 = -20 + \frac{h}{5r_x} \left[ \frac{100}{\sigma_y} \cdot \frac{P}{P_y} - (\frac{10}{100}) \right]$$  

(6)

where:

- $\gamma^'$ = rotation at $M/M_{pc} = 1$.
- $P$ = column axial load
- $P_y$ = axial yield load
- $r_x$ = radius of gyration about the strong axis

The rotation for any value of moment can then be expressed as:

$$\gamma = \frac{M_r}{2M_{pc}} \times (\gamma^')$$  

(7)

These two equations are used in subroutine POINT to include the column moment-rotation curves.

For a typical restrained column, joint rotation $\theta$ used in Eq. 4 is measured from a position that assumes the adjoining girders are horizontal or that there is no initial joint rotation. If the initial joint rotation is to be included then Eq. 4 can be written as

$$\frac{\Delta}{h} = \theta + \theta_o - \gamma$$  

(5)

In the computer program, the initial joint rotation is included by considering the actual position of the joint as the initial position instead of the artificial point that assumes girders initially horizontal. The sum of theta in the program (THE) is then equal to the sum of $\theta$ and $\theta_o$ if $\theta$ is measured assuming no initial joint rotation.

f) ARRAY

Subroutine ARRAY (Fig. 24) takes the coordinates of a plastic hinge point from subroutine POINT and places
them into an array for the subassemblage. The subroutine then checks the difference between the relative story displacement of the newest point and the relative story displacement of the previous point to ensure that they differ by at least 0.001. If the newest point does have a relative story displacement sufficiently larger than the last point, the subroutine interpolates between the two points to obtain horizontal load values for integer values of 0.001 of relative story displacement that occur between the two points. The new load coordinates are then placed in a column matrix for the subassemblage. If the difference between the two most recent points is not sufficiently large, the subroutine returns to the main program.

g) ADD

After all the subassemblages in a level have been analyzed, subroutine ADD (Fig. 25) adds the arrays established by subroutine ARRAY to form a final array for the level. The sum gives a set of loads corresponding to integer values of relative story displacement of 0.001 to 0.025. The loads and relative story displacements are printed and are used to plot the level load-deflection curve.
5. RESULTS

Using the computer program presented in Chapter 4 and Appendix I, load-deflection curves for level 20 of Frame C from Reference 1 were obtained for three incremental values of theta. The results of these calculations are shown in Fig. 28. Figure 27 shows a comparison of the computer solution of level 20 (AITH = 0.00020) to the level load-deflection curve obtained manually.

From studying the two sets of curves, it can be seen in Fig. 27 that the computer solution is approximately 6.7% higher than the manual solution. If the incremental theta value is reduced, the load-deflection curves resulting from the two methods of calculations can be brought closer together (Fig. 28). If the incremental theta value is made too large, it has drastic effects upon the load-deflection curve as shown in Fig. 28.

In obtaining the level load-deflection curve manually, available design charts (Ref. 10) were used. Since these charts are for discrete values of slenderness and axial load to axial yield load ratios, the selection of the proper chart for each subassemblage entailed a certain amount of approximation. This approximation was further increased in using the selected chart. Drawing the load-deflection curve was hindered by the discrete number of restraining moment and second order elastic-plastic mechanism curves. This coupled with the difficulty of adding the subassemblage
curves exactly explains part of the difference shown in Fig. 27
The remaining difference lies in the assumption made in writing the
computer program. The program assumes that the column moment-rotation
curves can be approximated by straight lines. For small values of
moment, this approximation is very close but as the column moment
approaches the column plastic moment the approximation becomes less
and less accurate. Since the horizontal load and the relative story
displacement are both functions of the column rotation, the straight
line assumption affects both of these values. The effect of the
straight line moment-rotation curve is to decrease the relative story
displacement and increase the horizontal load for higher values of
column moment.

The computer program was also used to study the effect of
changing member sizes on the level load-deflection curve. Figure
29 shows the effect of increasing and decreasing the column sections
of level 20 to the next larger or smaller section. Increasing the
column sections does not add significantly to the level's strength
or stiffness. The decrease in column sizes has a much greater
effect on the level's maximum strength since some of the columns
fail before the girders have reached their full strength. At the
working load level, the decrease in column sizes slightly reduces the
level's stiffness. The effect of changing girder sections to the
next larger or smaller wide-flange shape is shown in Fig. 30. Both
increasing and decreasing girder sizes have a marked effect on the
maximum level strength but do not change the working load stiffness
much.
If the ultimate horizontal load applied to a level exceeds the maximum load on the level's load-deflection curve, then member sizes in the level must be increased. Figures 29 and 30 indicate that the most effective way to obtain additional level strength is to increase the size of the girders. If level stiffness at working load is such that the relative story displacement exceeds recommended limits, then member sizes must be increase. Figure 29 and 30 show that increasing girder sizes has a greater effect on frame stiffness than increasing the column sizes.
6. CONCLUSIONS

A computer program has been developed for the analysis of one level of an unbraced multi-story frame. The theoretical basis of the program is the sway-subassemblage method of analysis. The program is limited to rigid, plane frames of up to eight bays. Uniformly distributed girder loads and equal story heights are also assumed. The computer program can be used to analyze any level of an unbraced frame but to ensure accurate results it should be applied to those levels to which the sway-subassemblage method is applicable.

The program results were compared to hand-calculated results for level 20 of Frame C from the Lehigh Summer Conference Notes. For the smallest incremental theta value used, the computer solution was 3.8 higher than the manual solution.

Future improvements of the program would be: 1) to include accurate initial girder stiffness values instead of assuming 6.0; 2) to use the results of at least one cycle of an elastic moment distribution for the initial girder end moments instead of using the fixed end moments; 3) to make the program totally dependent on member properties instead of calculating such values as plastic moment before applying the program; and 4) to extend the program to apply to groups of concentrated loads symmetrically placed about the midpoint of the girders as well as to uniformly distributed loads.
The results of the computer program are accurate if the program has been applied to levels which meet the assumptions for the program. The computer-developed load-deflection curve for a level can be used as a final check of the level's strength and stiffness or as a basis for preliminary member size revision.
NOMENCLATURE

AITH = Incremental theta value
E = Modulus of elasticity
H = Story Height
I = Moment of inertia
K = Girder stiffness factor
L = Length of girder
M = Column end moment
M_{pc} = Column plastic moment
M_{r} = Restraining moment at end of girder
P = Axial load on the column
P_{Y} = Axial yield load of the column
R_{x} = Column radius of gyration about the x-axis
\gamma = Column chord rotation
\gamma' = Modified gamma to take into account the assumed column moment-rotation curve.
\Delta = Lateral displacement of the column top relative to the bottom
\theta = Column top (or joint) rotation
\theta_{o} = Initial column top rotation
APPENDIX I

Program Printout
Program Nomenclature
PROGRAM SMOA

SUBASSEMBLAGE ANALYSIS OF UNRACED MULTI-STORY FRAMES

PROJECT 345-1  APRIL 1968
DIMENSION WC(1,9),PLC(1,9),AI(1,9),AMP(1,9)
DIMENSION AMPC(1,9),R(1,9),PY(1,9),PX(1,9)
DIMENSION AD0H(25,9),AO(25,9),AD0H0(9,9),AO(9,9)
DIMENSION AMIR(9),AML(9),AKR(9),AKL(9),TH(9),THE(9),AMP1(9),AMPM(9)

COMMON WC,BL,BLG,AL,AMP,AMPC,P,PY,RX,AD0H,AL,AMIR,AML,AKR,AKL,TH,THE,AMP1,AMPM,NI,NJ,I,J,K,L,N,F,PM,AM,AMR,THE,AMPL,THEY,AMPR,AML,THER
READ 1,NI,NJ
1 FORMAT(215)
   PRINT 2,NI,NJ
   DO 77 K=1,NI
      K=K1
      PRINT 3,K
   77 CONTINUE
   PRINT 3,NI
   CALL OUT1
   DO 77 J=1,NJ
      J=J1
      CALL STIFF
      CALL AINIT
      CALL AMLIM
      N=1
      PRINT 151,J
   151 FORMAT(1H,13HSUBASSEMBLAGE,15/)
      PRINT 152
   152 FORMAT(1H,38HDELTA OVER HEIGHT)
      PRINT 153,AD0H(N,J),AO(N,J)
   153 FORMAT(1H,F13.7,12H,F10.5)
      H1=0
      HX=0
      H2=0
      H3=0
      HY=0
      H4=0
      THE(J)=0.0
      AMIR=0.0
      AMP=0.0
      IF(J-NJ)33,34,34
   33 AM1=AMIR(J)
      AM2=AML(J)
      TH(0)=0.0
      TH(J)=0.0
      TH(J+1)=0.0
   34 IF(J-1)40,40,35
   35 AM3=AM1R(J-1)
      AM4=AML(J-1)
      TH(J)=0.0
      TH(J-1)=0.0
   40 IF(J-NJ)41,45,45
   41 AM1=AKL(J+1)*E:AI(K,J)*TH(J+1)/(D1(K,J):1+4.0)
      AM2=AKR(J)*E:AI(K,J)*TH(J)/(D1(K,J):1+4.0)
      AM1=AM1+AM1
345.5
GO TO 68
61 IF(HY)62,62,64
62 AMYA=ABS(AMY)
63 AKL(J)=3.0
AKR(J-1)=0.0
HY=1
GO TO 68
64 TH(J)=AITH
THE(J)=THE(J)+AITH
IF(AKR(J))66,65,66
65 IF(AKL(J))67,72,67
66 TH(J+1)=AITH:(AKR(J)-2.0)/(AKL(J+1)-2.0)
IF(AKL(J))67,40,67
67 TH(J-1)=AITH:(AKL(J)-2.0)/(AKR(J-1)-2.0)
GO TO 40
68 CALL POINT
CALL ARRAY
TH(J)=AITH
THE(J)=THE(J)+AITH
IF(AKR(J))70,69,70
69 IF(AKL(J))71,72,71
70 TH(J+1)=AITH:(AKR(J)-2.0)/(AKL(J+1)-2.0)
IF(AKL(J))71,40,71
71 TH(J-1)=AITH:(AKL(J)-2.0)/(AKR(J-1)-2.0)
GO TO 40
72 DOH=0.025
O=RM/H-PM(K,J):DOH
GO TO 74
73 RM=2.0:AMPC(K,J)
CALL POINT
CALL ARRAY
DOH=0.025
O=RM/H-PM(K,J):DOH
74 PRINT 75,DOH,0
75 FORMAT(1H,F13.7,12H,F10.5//)
76 CALL ADD
GO TO 78
77 CONTINUE
78 CONTINUE
END
SUBROUTINE OUTIN
DIMENSION W(1,8),BL(1,8),BLC(1,8),AI-1,8),AMPc18)
DIMENSION AMPC(1,9),PM(1,9),PY(1,9),RX(1,9)
DIMENSION AD0H(25,9),AQ(25,9),AD0H(6,9),AAQ6,9)
0DIMENSION AMIR(9),AMIL(9),AKP(9),AKL(9),TH¢(9),THE(9),AMP1(9),AMPM(49)
0COMMON W,BL,BLG,AL,AMP,AMPc,PM,PY,RX,AD0H,AQ,AMIR,AMIL,AKP,AKL,TH,T
HE,AMP1,AMPM,NE,NJ,I,J,K,L,E,N,N,NDH,RM,AM,M,AMPR,THE1,AMRL,THEY
4,AMPH1,AMRLY,AD0H,AAQ,ATH
READ 4,(W(K,J1),J1=1,L)
READ 5,(D0(K,J1),J1=1,1)
5 FORMAT(8F12.3)
READ 6,(BLG(K,J1),J1=1,1)
6 FORMAT(8F12.3)
READ 7,(AI(K,J1),J1=1,1)
7 FORMAT(8F12.3)
READ 8,(AMP(K,J1),J1=1,1)
8 FORMAT(8F12.3)
PRINT 9
9 FORMAT(1H ,74HNUMBER UNIFORM LOAD BEAM LENGTH CLEAR LENGTH
4H I MP//
DO 10 J2=1,L
10 PRINT 11,J2,W(K,J2),BL(K,J2),BLG(K,J2),AI(K,J2),AMP(K,J2)
110 FORMAT(1H ,I3,10H ,F6.3,9H ,F6.3,10H ,F6
4.3,7H ,F8.3,4H ,F8.3)
READ 13,(AMPC(K,J1),J1=1,NJ)
13 FORMAT(9F12.3)
READ 14,(P(K,J1),J1=1,NJ)
14 FORMAT(9F12.3)
READ 15,(P(K,J1),J1=1,NJ)
15 FORMAT(9F12.3)
READ 160,(RX(K,J1),J1=1,NJ)
160 FORMAT(9F12.3)
PRINT 161
161 FORMAT(1H ,///)
PRINT 16
160 FORMAT(1H ,51HJOINT MPC PY P RX /
4/)
DO 17 J2=1,NJ
17 PRINT 18,J2,AMPC(K,J2),PY(K,J2),P(K,J2),RX(K,J2)
18 FORMAT(1H ,I3,6H ,F8.3,4H ,F8.3,4H ,F8.3,4H ,F6.3)
READ 19,E,H
19 FORMAT(2F12.3)
PRINT 20,E,H
20 FORMAT(1H ,2HEE=F10.3,17H STORY HEIGHT,F8.3)
READ 21,AITH
21 FORMAT(F12.6)
PRINT 22,AITH
22 FORMAT(1H ,17HINCREMENTAL THETA,F12.6)
RETURN
END
SUBROUTINE STIFF
DIMENSION W(1,8),BL(1,3),BLG(1,8),AI(1,8),AMP(1,8)
DIMENSION AMPC(1,9),P(1,9),PY(1,9),PX(1,9)
DIMENSION ADOH(25,9),AQ(25,9),AADOH(6,9),AAO(6,9)
0DIMENSION AMIR(9),AMIL(9),AKR(9),AKL(9),TII(9),THE(9),AMP1(9),AMPM(9)
0COMMON W, BL, BLG, AI, AMP, AMPC, P, PY, RX, ADOH, AQ, AMIP, AMIL, AKP, AKL, TH, T
4HE, AMP1, AMPM, NF, N1, J, K, L, N, E, H, O, DOH, PM, AM, AMPR, THE1, AMRL, THEY
C THIS SUBROUTINE IS FOR CALCULATING THE INITIAL RESTRAINING
C COEFFICIENTS. IN THIS PROGRAM THE IRC ARE ASSUMED EQUAL TO 6.00.
C IF MORE EXACT VALUES ARE DESIRED THE SUBROUTINE CAN BE CHANGED
C AS REQUIRED
AKL(1)=0.0
AKR(1)=6.0
DO 28 J3=2,L
AKR(J3)=6.0
28 AKL(J3)=6.0
AKL(N1)=6.0
AKR(N1)=0.0
RETURN
END
SUBROUTINE AINIT

DIMENSION U(1,8),BL(1,8),BLG(1,8),AI(1,8),AMP(1,8)
DIMENSION AMPC(1,9),P(1,9),PY(1,9),PX(1,9)
DIMENSION ADOH(25,9),AQ(25,9),AADOH(6,9),AAQ(6,9)
DIMENSION AMIR(9),AMIL(9),AKR(9),AKL(9),TH(9),THE(9),AMP1(9),AMPM(49)

COMMON W,BL,BLG,Al,AMP,AMPC,P,PY,PX,ADOH,AQ,AMIR,AMIL,AKR,AKL,TH,T
THE,AMP1,AMPM,NF,NJ,I,J,K,L,N,E,H,N,DOH,RM,AM,M,AMRR,THE1,AMRL,THEY
4,AMRP1,AMRLY,AADOH,AAQ

C THIS SUBROUTINE Calculates THE GIRDER FIXED END Moments AND THE
C Initial JOINT ROTATIONS.

DO 30 J4=1,L
AMIR(J4)=W(K,J4)**BL(K,J4)**BL(K,J4)/12.0
AMIL(J4)=-AMIR(J4)
30 CONTINUE

DO 81 J5=1,NJ
IF(J5-1)130,130,131
130 RM=AMIL(J5)
GO TO 134
131 IF(J5-NJ)133,132,132
132 RM=AMIR(J5-1)
GO TO 134
133 RM=AMIL(J5)+AMIR(J5-1)
134 N=1
AADOH(N,J5)=0.0
AAQ(N,J5)=RM/H
81 CONTINUE
RETURN
END
SUBROUTINE A1LIM

DIMENSION W(1,8),BL(1,8),BLG(1,8),AI(1,8),AMP(1,8)
DIMENSION AMPC(1,9),P(1,9),PY(1,9),PX(1,9)
DIMENSION ADOH(25,9),AQ(25,9),AAOHD(6,9),AAO(6,9)
0DIMENSION AMIR(9),AML(9),AKR(9),AKL(9),TH(9),THE(9),AMP1(9),AMPM(49)
0COMMON W, BL, BLG, AI, AMP, AMPC, P, PY, RX, ADOH, AQ, AMIR, AML, AKR, AKL, TH, T4HE, AMP1, AMPM, NF, NJ, I, J, K, L, N, E, H, O, DOH, RM, AM
C THIS SUBROUTINE CALCULATES THE LIMITING GIRDER END MOMENTS USING
C THE EQUATIONS FOR THE SOLUTION OF THE GENERAL MOMENT DIAGRAM FOR
C A GIRDER SUBJECTED TO UNIFORM LOAD.
DO 32 J6=1,L
AMPM(J6)=W(K,J6)**BL(K,J6)**BL(K,J6)/16.0
F1=1.3**AMP(K,J6)/AMPM(J6)
Z=F1/1.3
IF(F1-4.0)30,30,31
300AMP1(J6)=Z**AMPM(J6)+4.0**AMPM(J6)**((BL(K,J6)-BLG(K,J6))/BLG(K,J6))**4SJRZ(TZ)
GO TO 32
31 AP=(BL(K,J6)-BLG(K,J6))/BL(K,J6)
AMP1(J6)=AMPM(J6)**Z/(1.0-AP)+4.0**AMPM(J6)**AP/(1.0-AP)
32 CONTINUE
RETURN
END
SUBROUTINE POINT
DIMENSION W(1,8),BL(1,8),BLG(1,8),AI(1,8),AMP(1,8)
DIMENSION AMPC(1,9),P(1,9),PY(1,9),RX(1,9)
DIMENSION ADWH(25,9),AQ(25,9),AADOH(6,9),AAQ(6,9)
DIMENSION AMIR(9),AMIL(9),AKR(9),AKL(9),TH(9),THE(9),AMP1(9),AMPM(49)

COMMON U,VL,BLG,AL,AMP,AMPC,P,PY,RX,ADOH,AQ,AMIR,AMIL,AKR,AKL,TH,T
HE,AMP1,AMPM,NF,NJ,1,J,K,L,N,E,H,Q,DOH,AM

C THIS SUBROUTINE FINDS THE COORDINATES OF A POINT ON THE
C SUBASSEMBLAGE LOAD-DEFLECTION CURVE USING THE BASIC EQUATIONS
C OF RESTRAINED COLUMN THEORY
AM=(RM/2.0)
RCM=AM/AMPC(K,J)
GAP=-20.0+264.0*H/RX(K,J)-240.0*H*P(K,J)/(RX(K,J)*PY(K,J))
GAM=(RCM*GAP*0.00001)
DOH=TH(K)+GAM
Q=(-AM*2.0/H)-P(K,J)*DOH
PRINT 90,DOH,Q
90 FORMAT(1h,F13.7,12h,F10.5)
RETURN
END
SUBROUTINE ARRAY
DIMENSION W(I,8),BL(1,8),BLG(1,8),AI(1,8),AMP(1,8)
DIMENSION AMPC(1,9),P(1,9),PYC(1,9),PX(1,9)
DIMENSION AADOH(25,9),AO(25,9),AADOH(6,9),AAQ(6,9)
DIMENSION AMIR(9),AML(9),AKR(9),AKL(9),TH(9),THE(9),AMP1(9),AMP2(9)
COMMON U, BL, BLG, AI, AMP, AMPC, P, PY, RX, ADOH, AO, AMIR, AMIL, AKR, AKL, TH, THE, AMP1, AMP2, NF, N, J, I, K, L, M, H, N, DOH, RM, AM, M, AMP, THE1, AMRL, THEY
C
C THIS SUBROUTINE INTERPOLATES BETWEEN THE HINGE POINTS AND PLACES
C THE ANSWER IN A MATRIX.
140 N=N+1
AAQ(N,J)=0
AADOH(N,J)=DOH
CHK=AADOH(N,J)-AADOH(N-1,J)
IF(N-2)102,102,103
102 M=1
103 IF(CHK-0.001)106,104,104
104 FN=M
FN=0.001-M
IF(AADOH(N,J)-FN)106,105,105
1050 AO(M,J)=AAQ(N-1,J)+(AAQ(N,J)-AAQ(N-1,J))*(FN-AADOH(N-1,J))/(AADOH(N,J)-AADOH(N-1,J))
M=M+1
GO TO 104
106 RETURN
END
SUBROUTINE ADD

DIMENSION V(1,9), BL(1,8), BLG(1,8), AI(1,8), AMP(1,8)
DIMENSION AMPC(1,9), P(1,9), PY(1,9), RX(1,9)
DIMENSION ADOH(25,9), AO(25,9), AADOH(6,9), AAO(6,9)

0DIMENSION AMIR(9), AMIL(9), AKR(9), AKL(9), TH(9), THE(9), AMP1(9), AMPM(49)

COMMON W, BL, BLG, AI, AMP, AMPC, P, PY, RX, ADOH, AO, AMIR, AMIL, AKR, AKL, TH, T
4HE, AMP1, AMPM, NF, NJ, I, J, K, L, N, E, H, O, DOH, RM, AM

C THIS SUBROUTINE ADDS THE COLUMN MATRIX FOR EACH SUBASSEMBLAGETO
C GET POINTS FOR THE LOAD DEFLECTION CURVE FOR THE LEVEL
I=25
PRINT 150
150 FORMAT(IH1, 32HDELTA OVER HEIGHT
DO 88 M1=1,I
DO 87 J7=1,L
AQ(M1, J7+1) = AQ(M1, J7) + AQ(M1, J7+1)
IF(J7-L)87, 85, 87
85 FM=M1
FM=FM*0.001
PRINT 86, FM, AQ(M1, J7+1)
86 FORMAT(IH, F12.4, 10H
GO TO 88
87 CONTINUE
88 CONTINUE
RETURN
END
## PROGRAM NOMENCLATURE

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AADOH</td>
<td>Relative story deflection over story height for a plastic hinge point on the subassemblage curve</td>
</tr>
<tr>
<td>AAQ</td>
<td>Horizontal load at plastic hinge point</td>
</tr>
<tr>
<td>ADD</td>
<td>Subroutine which adds the individual subassemblage load-deflection curves to get a level load-deflection curve</td>
</tr>
<tr>
<td>ADOH</td>
<td>Array value of relative story deflection over story height</td>
</tr>
<tr>
<td>AI</td>
<td>Girder moment of inertia</td>
</tr>
<tr>
<td>AIM1</td>
<td>Increment of moment at hinge location one</td>
</tr>
<tr>
<td>AIM2</td>
<td>Increment of moment at hinge location two</td>
</tr>
<tr>
<td>AIM3</td>
<td>Increment of moment at hinge location three</td>
</tr>
<tr>
<td>AIM4</td>
<td>Increment of moment at hinge location four</td>
</tr>
<tr>
<td>AINIT</td>
<td>Subroutine which finds the initial girder end moments, initial joint rotation, and initial point on the subassemblage load-deflection curve</td>
</tr>
<tr>
<td>AITH</td>
<td>Incremental value of theta</td>
</tr>
<tr>
<td>AKL</td>
<td>Girder stiffness factor on left end</td>
</tr>
<tr>
<td>AKR</td>
<td>Girder stiffness factor on right end</td>
</tr>
<tr>
<td>AM</td>
<td>Column end moment</td>
</tr>
<tr>
<td>AM1</td>
<td>Moment at hinge location one</td>
</tr>
<tr>
<td>AM2</td>
<td>Moment at hinge location two</td>
</tr>
<tr>
<td>AM3</td>
<td>Moment at hinge location three</td>
</tr>
<tr>
<td>AM4</td>
<td>Moment at hinge location four</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>AMIL</td>
<td>Initial restraining moment on left end of girder</td>
</tr>
<tr>
<td>AMIR</td>
<td>Initial restraining moment on right end of girder</td>
</tr>
<tr>
<td>AMLIM</td>
<td>Subroutine which calculates the maximum attainable moment on the right end of the beam</td>
</tr>
<tr>
<td>AMP</td>
<td>Girder plastic moment</td>
</tr>
<tr>
<td>AMPL</td>
<td>Limiting moment on right end of girder</td>
</tr>
<tr>
<td>AMPC</td>
<td>Column plastic moment</td>
</tr>
<tr>
<td>AMRL</td>
<td>Restraining moment on left side of joint</td>
</tr>
<tr>
<td>AMRLY</td>
<td>Restraining moment on left side of joint at the formation of a plastic hinge at Y</td>
</tr>
<tr>
<td>AMRR</td>
<td>Restraining moment on right side of joint</td>
</tr>
<tr>
<td>AMRR1</td>
<td>Restraining moment on right side of joint at the formation of a plastic hinge at 1</td>
</tr>
<tr>
<td>AMX</td>
<td>Moment at hinge location X</td>
</tr>
<tr>
<td>AMY</td>
<td>Moment at hinge location Y</td>
</tr>
<tr>
<td>AQ</td>
<td>Horizontal load in array for subassemblage</td>
</tr>
<tr>
<td>ARRAY</td>
<td>Subroutine which interpolates between hinge points and places the answers in an array for each subassemblage</td>
</tr>
<tr>
<td>BL</td>
<td>Beam Length</td>
</tr>
<tr>
<td>BLG</td>
<td>Clear span length of beam</td>
</tr>
<tr>
<td>DOH</td>
<td>Relative horizontal story displacement divided by the story height.</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>FM</td>
<td>A dummy variable to change Ml to 0.001 times Ml</td>
</tr>
<tr>
<td>FN</td>
<td>A dummy variable to reduce M from 1 to 0.001</td>
</tr>
<tr>
<td>GAM</td>
<td>Column rotation based on straight line moment-rotation curve</td>
</tr>
</tbody>
</table>
GAP = Column rotation when column moment equals the column plastic moment

H = Story height

H1 = Hinge location one, at right end of right girder. H1 is set equal to one when a plastic hinge forms at location one

H2 = Hinge location two, at left end of right girder

H3 = Hinge location three, at right end of left girder

H4 = Hinge location four, at left end of left girder

HX = Hinge location X, somewhere in right girder

HY = Hinge location Y, somewhere in left girder

I = Number of rows in array for each subassemblage

J = Number of joints, column below and girder to right of joint.

K = Number of levels to be studied

L = Number of bays in each level

M = Number of rows in array for each subassemblage

N = Hinge number in order of formation, first hinge is two, second hinge three etc.

NF = Number of levels in frame to be analyzed

NJ = Number of joints in each level

P = Column axial load

POINT = Subroutine which funds the coordinates of a point on the subassemblage load-deflection curve for a given theta

PY = Column yield load

Q = Horizontal load

RCM = Restrained column end moment

RM = Restraining moment at a joint
RX = Column radius of gyration about X axis
STIFF = Subroutine which calculates the initial restraining coefficient
TH = Rotation of joint due to applied horizontal load
THE = Sum of the joint rotation
THEY = Sum of joint rotation at the formation of a plastic hinge at Y
THEI = Sum of joint rotation at the formation of a plastic hinge at I
W = Uniformly distributed beam load
X = Distance from right end of right beam to hinge location X
Y = Distance from right end of left beam to hinge location Y
Z = Effective load factor for gravity case divided by load factor for combined loading case
APPENDIX II

Input Format
Sample Input
Sample Output
**INPUT FORMAT**

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Data</th>
<th>Symbol</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number of floors</td>
<td>NF, NJ</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>Number of joints in floor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2F*</td>
<td>Uniformly distributed load for combined loading case (kip/ft.)</td>
<td>W</td>
<td>8F12.3</td>
</tr>
<tr>
<td>3F</td>
<td>Girder length (ft.)</td>
<td>BL</td>
<td>8F12.3</td>
</tr>
<tr>
<td>4F</td>
<td>Clear span length (ft.)</td>
<td>BLG</td>
<td>8F12.3</td>
</tr>
<tr>
<td>5F</td>
<td>Moment of inertia of girders (in$^3$)</td>
<td>AI</td>
<td>8F12.3</td>
</tr>
<tr>
<td>6F</td>
<td>Girder Plastic Moment (kip-ft.)</td>
<td>AMP</td>
<td>8F12.3</td>
</tr>
<tr>
<td>7F</td>
<td>Column Plastic Moment (kip-ft.)</td>
<td>AMPC</td>
<td>9F12.3</td>
</tr>
<tr>
<td>8F</td>
<td>Column Axial yield load (kip)</td>
<td>PY</td>
<td>9F12.3</td>
</tr>
<tr>
<td>9F</td>
<td>Column Load (kips)</td>
<td>P</td>
<td>9F12.3</td>
</tr>
<tr>
<td>10F</td>
<td>Column radius of gyration (in.)</td>
<td>RX</td>
<td>9F12.3</td>
</tr>
<tr>
<td>11</td>
<td>Modulus of Elasticity (kip/in$^2$)</td>
<td>E,H</td>
<td>2F12.3</td>
</tr>
<tr>
<td>12</td>
<td>Incremental theta</td>
<td>AITH</td>
<td>F12.6</td>
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</table>

*F refers to "and following cards as required"
### Sample Card Input

<table>
<thead>
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<tbody>
<tr>
<td>5.500</td>
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<tr>
<td>5.270</td>
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<td>20,000</td>
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<tr>
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<td>450.200</td>
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<tr>
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<td>12.000</td>
</tr>
<tr>
<td>5.810</td>
<td>12.000</td>
</tr>
</tbody>
</table>

Note: The sample card input reads correctly from the top of the page downward but the cards themselves are stacked in the reverse order.
SAMPLE INPUT AND OUTPUT

The computer program presented in this report was prepared using a Quiktran terminal. This type of terminal consists of a typewriter and the control equipment necessary for transmission over telephone lines. Since the terminal used differs from a regular terminal the appearance of the input and output is somewhat different. The output is interspersed with the printout of the input data where a regular terminal would give the printout of the output only. The input formats and the appearance of results given in this section apply to any type of terminal which will accept Fortran.

In the example shown here, all lines of print which have = I in the sixth and seventh spaces are input typed by the operator. This would be the information supplied on input data cards in a batch processing machine. The lines of print labelled = 0 in the sixth and seventh spaces are output which would be the only output furnished by a batch processing machine.
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FIGURES
Fig. 1 One-Story Assemblage
Fig. 2 Half-Story Assemblage
Fig. 3 Typical Subassemblage
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Fig. 16 Flow Chart of Main Program (continued)
Fig. 17 Flow Chart of Main Program (continued)
Fig. 18 Flow Chart of Main Program (continued)
Fig. 19 Flow Chart of Subroutine OUTIN
Fig. 20 Flow Chart of Subroutine STIFF
Fig. 21 Flow Chart of Subroutine AINIT
\[ AP = \frac{(BL(K, J6) - BLG(K, J6))}{BLG(K, J6)} \]

\[ AMP1(J6) = \frac{Z \cdot AMPM(J6) + 4.0 \cdot AMPM(J6) \cdot ((BL(K, J6) - BLG(K, J6)) / BLG(K, J6))^2 \cdot \sqrt{Z}}{1.3} \]

\[ F1 = 1.3 \cdot AMP(K, J6) / AMPM(J6) \]

\[ Z = F1 / 1.3 \]

\[ AMPM(J6) = \frac{W(K, J6) \cdot BL(K, J6) \cdot BL(K, J6)}{16.0} \]

\[ F1 = 4.0 \]

Fig. 22 Flow Chart of Subroutine AMLIM
AM = -RM/2.0
RCM = AM/AMPC(K, J)
GAP = -20 + (22.0*H*12.0)/RX(K, J)
   - (20*H*12.0*P(K, J))/(RX(K, J)*PY(K, J))
GAM = -RCM*GAP*0.00001
DOH = THE(J) + GAM
Q  = -(AM*2.0/H) - P(K, J)*DOH

Fig. 23 Flow Chart of Subroutine POINT
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Fig. 27 Comparison of Manual and Computer Solution
Fig. 28 Effect of Changing Incremental Theta
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Fig. 30 Effect of Changing Girder Sizes
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ACKNOWLEDGMENTS

This report is part of a study into design recommendations for the plastic design of unbraced multi-story frames. The work was carried out at Fritz Engineering Laboratory of the Civil Engineering Department, Lehigh University. Dr. D. A. VanHorn is Chairman of the Civil Engineering Department and Dr. Lynn S. Beedle is Director of Fritz Engineering Laboratory.

The investigation was sponsored jointly by the Welding Research Council, American Iron and Steel Institute and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, American Iron and Steel Institute, Naval Ships Systems Command, Naval Facilities Engineering Command, and the Welding Research Council. Technical guidance was provided by the Lehigh Project Subcommittee of the Structural Steel Committee of the Welding Research Council. Dr. T. R. Higgins is the Chairman of the Lehigh Project Subcommittee.

The authors would like to acknowledge the helpful suggestions given by Professor J. Hartley Daniels and Mr. Sung-Woo Kim. Sincere thanks are also due to Mr. John Gera, Jr. for preparing the drawings and to Misses M. Courtright and Karen Philbin for their care in typing this report.