FOOTING CONTACT PRESSURE SET BY COMPRESSIBILITY DATA

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SET BY

COMPRESSIBILITY DATA

by

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SYNOPSIS

This Paper presents computer developed design and analysis charts to be used as an aid in designing foundations by a philosophy of equal settlement. The charts may be used to check settlements, design footings, or provide trial values for use in formulas developed previously. Modifications of previous assumptions and formulas are also proposed. Examples are presented to illustrate the suggested procedure.

Equations are developed for stratified soil and a possible computer program is included for these equations. Examples solved by computer are shown and limitations of the method are discussed.

1. INTRODUCTION

1.1 Background

Hough has suggested that settlement be used as a basis for proportioning spread footings. This is because footings are often designed for an allowable contact pressure or presumptive bearing value and later checked for excessive settlement. Professor Hough approached the problem by deriving equations for contact pressure as a function of settlement. His equations provided relationships between depth of significant stress, foundation width, column load, and soil characteristics. However, the equations had to be solved by a repeating trial and error procedure.

The trial and error solution can be eliminated through the use of charts presented herein. If design by charts is not desirable, proposed modified formulas are included. These modifications considerably reduce the amount of work required. Also, the charts can be used for preliminary estimates or checking results and will eliminate much of the time lost in initial (trial)solutions.

The method can be extended to proportion footings on stratified soil. However, the equations become so involved that the most direct solution is by digital computer and a sample program will be included.

(1) Ibid.
Notation - The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in Appendix A.

1.2 Settlement Equation

The basic equation for settlement as compression of a thin layer is the familiar expression:

\[ \Delta H = H \frac{G_c}{1+e} \log \left(1+\frac{\Delta p}{p_i}\right) \quad (1) \]

where:

\[ \Delta H = \text{compression of a thin layer} \]
\[ H = \text{the initial thickness of compressible material} \]
\[ G_c = \text{compression index of the soil} \]
\[ e = \text{initial void ratio} \]
\[ \Delta p = \text{increase in vertical stress} \]
\[ p_i = \text{initial body stress} \]

Equation (1) is readily rewritten to express the vertical stress required for a given settlement:

\[ \Delta p = (10 \frac{G_c}{H} - 1) p_i \quad (2) \]

In which \[ C = \frac{1+e}{G_c} \] is the bearing capacity index.

2. **UNIFORM SOIL**

The equations developed for uniform soil have appeared in similar form in the paper by Hough. However, they are repeated for convenience of reference and the reader should not mistake them for original work. Also, some of the assumptions have been questioned in discussions of the original work and where it is possible to lend validity to either paper, additional material is included.
FIG. I - Body Stress Diagrams, Uniform Soil
(After Hough)
2.1 **Surface Footing - Unstratified Soil**

Equation (1) is developed for a compressible layer and it is necessary to specify the thickness of material included. The depth below which no significant compression occurs, termed the "depth of significant stress", varies with the foundations size and loading but is chiefly a function of stress ratio. The material within this depth will be thought of as a compressible layer of finite thickness.

When \( \Delta p \leq \frac{1}{10} \frac{p_i}{10} \), the \( \log (1 + \frac{\Delta p}{p_i}) \leq 0.0414 \), and \( H \cdot \left( \frac{C}{1+e} \right) \) must be quite large to cause a significant increase in \( \Delta H \) (Eq. 1). Actually, \( H \cdot \left( \frac{C}{1+e} \right) \) is more likely to be in the range of 0.1 to 0.3. This justified setting the depth of significant stress in uniform soils at the level above which \( \Delta p \leq \frac{p_i}{10} \).

The stress conditions within the loaded soil are shown in Fig. 1(a). Note that \( h_s \), the depth of significant stress, includes all soil wherein \( \Delta p \leq \frac{p_i}{10} \). Using a modification of the 60° approximation, the average vertical stress increment at any depth \( h \) (within an imaginary pyramid whose base angle = 63 \( \frac{1}{2} \)) is given by:

\[
\Delta p = \frac{p}{(h+B)^2}
\]  

(3)

where:  
- \( P \) = column load  
- \( B \) = footing width

The initial body stress at depth \( h \) is:

\[
p_i = \gamma h
\]  

(14)

for: \( \gamma \) = unit weight of the soil
At the depth of significant stress:

$$\Delta p = \frac{1}{10} p_1 = \frac{\gamma h_s}{10}$$  \hspace{1cm} (5)

Equating Eqs. (3) and (5)

$$\frac{\gamma h_s}{10} = \frac{p}{(h_s + B)^2}$$

$$\frac{\gamma h_s}{10} (h_s + B)^2 = p$$  \hspace{1cm} (6)

By setting $h_s$ equal to a particular value, $H$, and evaluating pressures at $\frac{1}{3}$ of this depth, we get from Eqs. (2), (3), and (4):

$$\frac{p}{(1 H + B)^2} = \frac{SC}{H} \left( \frac{H}{3} \right) \gamma$$  \hspace{1cm} (7)

$$P = \gamma \frac{H}{27} \left( \frac{SC}{H} - 1 \right) (H+3B)^2$$

Eqs. (6) and (7) are solved concurrently to find a set of conditions which will cause a specific settlement, $S$.

Note: Equation (7) uses the pressures $\Delta p$ and $p_1$ existing at level of the centroid of the body stress diagram (Fig. 1(a)). It is assumed that these pressure conditions represent average values within the depth of significant stress. That the centroid can be assumed at $\frac{1}{3} h_s$ will be shown later.

2.2 Footing in Excavation - Unstratified Soil

Footings are usually placed on the floor of an excavation (Fig. 1(b)). Assuming a depth of excavation, $D$, and measuring $h$ from the floor of the excavation, the stress before excavation is:

$$p_1 = \gamma (h+D)$$  \hspace{1cm} (8)
Due to excavation the body stress becomes:

\[ p_1 = \gamma h \quad (9) \]

and with the addition of a foundation:

\[ p_2 = p_1 + \Delta p \quad (10) \]

\[ p_2 = \gamma h + \frac{P}{(B+h)^2} \quad (11) \]

However, settlement depends on the initial and final stress conditions. The net change in stress is:

\[ \Delta P_{\text{net}} = p_2 - p_1 \quad (12) \]

\[ \Delta P_{\text{net}} = \frac{P}{(B+h)^2} - \gamma D \quad (13) \]

Repeating the solution for depth of significant stress and settlement equations:

\[ P = \frac{\gamma}{10} \left( h_s + 11D \right) \left( h_s + B \right)^2 \quad (14) \]

\[ P = \frac{\gamma}{27} \left[ (H + 3D) \frac{SC}{H} - H \right] (H + 3B)^2 \quad (15) \]

Note: substituting \( D = 0 \) in Eq. (14) and Eq. (15) results in Eqs. (6) and (7) respectively.

To find concurrent solutions of Eqs. (14) and (15) by trial and error is definitely an arduous task and this may deter many engineers from applying the method. However, these formulas have been programmed for digital computer solution and the results are presented in the form of design charts. Because the charts are simple and straightforward, the computer program is not included.

3. SOLUTION OF EQUATIONS

3.1 Use of Design Charts

The use of the charts presented in Appendices B and C proceed as follows:
1) Starting with the smallest column load, determine the minimum size footing allowed under applicable codes ($B_{\text{min}} = \sqrt[3]{\frac{P}{P_{\text{allow}}}}$).

2) Knowing the depth of excavation at this footing, select the corresponding analysis chart in Appendix C and read the value of $S_C$ for the computed column load quotient $\frac{P}{A}$. The unit weight of soil, $\gamma$, is assumed to be known.

3) Determine the settlement by dividing $S_C$ by the known value of $C$. $C$ may be determined from compression test data or in absence of all other information may be estimated from the standard penetration resistance of the soil. 1

4) If $S$ is too high, select a larger $B$ and compute a new $S_C$ and $S$. However, the actual value of $S$ is not as restrictive as it is for other design methods since all the footings will settle the same amount.

5) Using the final $S_C$, determine footing sizes with the aid of the design charts given in Appendix B. Remember to use the correct chart if the depth of excavation varies. For depths of excavation other than those plotted, it is possible to interpolate between charts.

If $C$ varies across the building site, corrected values of $S_C$ must be used at each point to provide a constant $S$ from footing to footing. The unit soil weight may also vary and this requires corrected column load quotients $\frac{P}{\gamma}$ at each point.

6) For each column load quotient and footing width, check the depth of significant stress. The soil must be uniform within the depth of significant stress.

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(1) Ibid, pp. 16-17
cant stress (this includes submergence).

3.2 Advantages and Limitations of the Method

The charts presented herein are intended for an isolated footing located in an open excavation (or at the ground surface). Footings near the edge or corner of an excavation must be designed for the abnormal net pressure conditions occurring in that area.* Also a surcharged footing must be handled differently and that type of foundation is beyond the scope of this paper.

a. Settlement - the magnitude of design settlements may be chosen at will and several trial designs compared for economy and practicality. There is very little time required for calculation and the engineer has more opportunity to exercise his own judgement. Any desired number of footings in a group may be checked by formula, but the trial and error solution is not necessary since trial estimates of all values are made with charts. The method is most valuable when column loads vary widely.

b. Allowable Contact Pressure - Because the smallest footing is designed for the maximum pressure under applicable codes, there is no need to check other footings. With all foundations designed for equal settlement, the larger footings must have lower contact pressure. This is proven inversely with two footings of different width but having equal contact pressures. The larger footing settles the most (size effect).

c. Factor of Safety Against Rupture - If the contact pressure selected for the first footing provides a proper factor of safety, all other footings will have a higher factor of safety. However, loading should not exceed pressures beyond which loads and deformations are no longer proportional. The method is valid only in the stress range where load is proportional to settlement.

Further work now in progress may point up a rather simple modification to the method for corner footings. If so, the material will be included in discussions of this paper.
d. Submerged Soils - The method can be used when the entire depth of significant stress is submerged. Since the initial body stress is reduced to submergence, the submerged unit weight is used to compute the column load quotient, \( \frac{P}{g} \).

An excavation may be partially below the water table but the method assumes a uniform weight from the original grade to the depth of significant stress. Hence, an effective depth of excavation, \( D' \), must be computed so that the submerged unit weight multiplied by \( D' \) equals the weight of soil removed per unit area of excavation.

e. Preloaded Clays - A preloaded clay may be treated in one of two ways. One is to limit the maximum applied pressure to the preload value. In this case, a rather low value of \( C_0 \) is used and the stress increment may not exceed the preload.

The second method considers the initial settlement (for loads below preload pressure) to be negligible and treats the clays as an excavation problem. An effective depth of excavation is calculated to equalize the preloading effect with the product of unit weight and effective depth of excavation. In either method the clay must be considered with due caution.

f. Variation of Soil Properties - the charts may be used in some cases where the soil properties vary with depth. The soil must be uniform from the footing grade to the depth of significant stress. If one or more layers are excavated but the footings are placed on a sufficiently thick uniform layer, an effective depth of excavation can be calculated similar to the manner for partially submerged soils.

g. Stress Overlap - If there is an overlap of stress within the depth of significant stress, smaller footings must be used of the design modified by other methods.
3.3 Checking Results

Settlements or pressures are checked by substitution in Eq. (15), but the computation of depth of significant stress under an excavation is much simpler if Eq. (14) is modified.

If the depth of significant stress is redefined as the depth at which $\Delta p_{\text{net}} = 0$, (Fig. 1(b) Eq. (13) becomes:

$$\frac{p}{(B+h_s)^2} - \gamma D = 0$$

$$h_s = \frac{\sqrt{p}}{\sqrt[3]{\gamma d}} - B$$

(16)

Equation (16), being a quadratic rather than a cubic equation, is so much simpler than Eq. (14) that it should be used whenever computations of depth of significant stress are undertaken. Spot checks have shown that values of $h_s$ by Eqs. (14) and (16) are not significantly different and that use of either one in Eq. (15) is equally accurate. This modification is not used as a general formula, though, because it is invalid when $D = 0$.

3.4 Example Problems

The following example problems are included to illustrate some of the applications of the design charts.

Example 1

A particular structure has several columns with widely varying loads, the smallest being 100 kips. The foundations are to be placed in an open excavation four feet deep. The soil is a sandy clay uniform to a rather large depth and having an effective unit wet weight of 11.8 lbs. per cu. ft. The prevailing code specifies an allowable presumptive bearing capacity of 2.0 tons per sq. ft. and it is necessary to find the maximum allowable settlement product, $SC$, to be used for the structure.
Solution

1. \( B_{\text{min}} = \sqrt{\frac{75,000}{2 \times 2,000}} = 4.32', \text{ Say } 4' - 4'' \)

2. \( \frac{P}{\gamma} = \frac{100,000}{118} = 848 \)

3. From Analysis Charts
   For \( B = 4.00 \) \( 6.00 \)
   \( SC = 2.5 \) \( 4.2 \)

4. By Interpolation
   \( SC = 2.8 \) Answer

Using this \( SC \) value, the remainder of the foundation sizes for each column load are found from the design charts in Appendix B.

Example 2

A column carrying 93.8 kips is to be placed in an excavation four feet deep. There is a possibility of a very high water table and the soil has the following properties:

\[ \sqrt{\text{sub}} = 57 \text{ lbs./cu. ft.} \]
\[ C_{\text{c}} = 0.038 \]
\[ e = 0.82 \]

The footing should settle approximately 1/2 inch to be in accord with the remainder of the structure.

Solution

1. \( \frac{P}{\gamma} = \frac{93,800}{57} = 1,645 \)

2. \( C = \frac{1 + e}{C_{\text{c}}} = \frac{1.82}{0.038} = 48 \)

3. \( SC = \frac{48 \times 0.50}{12} = 2 \)

4. From Design Charts
   \( B = 10.7 \text{ ft.} \) Answer
Check:

Using Equations (15) and (16) the error in reading the chart is less than 1%.

Example 3

Given the same conditions as Example 2, find the footing size for an excavation twelve feet deep.

Solution:

1. FROM DESIGN CHARTS (USING SC = 2)

   \[ B = 5.7 \text{ ft. Answer} \]

   Because there is such a significant effect on foundation behavior due to changing depth of excavation, the deepest footings should be designed first. Comparing examples 2 and 3, one can see that the deeper foundation has a much higher pressure. Had the SC value been set for a shallow footing, the deeper one could very well have exceeded allowable code values. It is not intended that this should occur.

Example 4

A building is to be constructed adjacent to an old river bed. The top soil consists of a two feet thick layer of loose organic silt (effective unit weight = 90 lbs/ft.cu.) underlain by a layer of soft highly compressible clay two feet - two inches thick (effective unit weight = 95 lbs./cu.ft.). This is deposited on a dense layer of silty sand and gravel weighing about 135 lbs/cu.ft. The water table is at a depth of six feet and the footings are to be placed in an open excavation at this depth. SC has been established (for the smallest column load) at two. It is required to find the footing size for a 45 ton column given the following properties for the sand and gravel layer: \( G = 2.65, \ e = 0.30, \ C_e = 0.016. \)

Solution: For the sand and gravel layer

\[
\gamma_{\text{sub}} = \frac{G-1}{1+e} \gamma_w = \frac{1.65}{1.30} \times 62.4 = 79.2 \text{ lbs/cu.ft.}
\]
2. PRESSURE REDUCTION DUE TO EXCAVATION
\[ \Delta p = (90 \times 2) + (95 \times 2.17) + (135 \times 1.83) = 633 \text{ lbs/sq.ft.} \]

3. EFFECTIVE DEPTH OF EXCAVATION
\[ D_1 = \frac{633}{79.2} = 8 \text{ ft.} \]

4. \( \frac{P}{\sqrt{\text{sub}}} = \frac{45 \times 2000}{79.2} = 1135 \)

5. FROM DESIGN CHARTS (D = 8 ft.)
\[ B = 5.65 \text{ Ft. Answer} \]

4. STRATIFIED SOIL
4.1 Development of Equations
Soils which are layered, partially submerged, or whose properties vary with depth require special consideration. Equation (1) is evaluated layer by layer but judgement and experience indicate how small a change in properties requires considering separate layers.

The actual trial and error solution bringing depth of significant stress and settlement equations to agreement may be done by hand but is recommended for a digital computer. The somewhat artificial case of a surface footing will be ignored and formulas developed for a footing in an open excavation only.

It is convenient to measure depth, \( h_n \), from the base of the excavation. Thus, the depth to some point \( y \) in layer \( n \) is:
\[ h_y = k t_n + \sum_{i=1}^{n-1} t_i \] (17)
where; \( t_i = \) the thickness of layer \( i \)
\( k = \) the fraction of layer \( n \) above the point
Interior Footing in
Open Excavation

\[ p_i = k_t \gamma_n + \sum_{i=1}^{n-1} \gamma_i t_i + \gamma' D' \]

\[ p_1 = k_t \gamma_n + \sum_{i=1}^{n-1} \gamma_i t_i \]

\[ p_2 = k_t \gamma_n + \sum_{i=1}^{n-1} \gamma_i t_i + \frac{P}{(B+h)^2} \]

FIG. 2 - Body Stress, Stratified Soil
The body stress (Fig. 2) at point y after excavation will be:

\[ P_y = k n y_n + \sum_{i=1}^{n-1} Y_i t_i \tag{18} \]

where \( Y_i \) is the unit weight of soil in layer \( i \).

If \( m \) layers of soil have been excavated, there existed an initial body stress at point \( y \) which was:

\[ P_{iy} = k n y_n + \sum_{i=1}^{n-1} Y_i t_i + \sum_{j=1}^{m} Y_j t_j \tag{19} \]

But the term for excavated depth remains unchanged once depth is set. So,

\[ \sum_{j=1}^{m} Y_j t_j = Y_j D_j \quad \text{(A constant)} \tag{20} \]

Substituting Eqs. (18), (19) and (20) in Eq. (12)

\[ (\Delta P_{\text{net}})_y = \frac{P}{(h_y + B)^2} - Y_1 D_1 \tag{21} \]

For stratified soils it is important to set the depth of significant stress at the depth for which \( \Delta P_{\text{net}} = 0 \). In uniform soils this was shown to be unnecessary since only negligible compression occurred below the depth where \( \Delta p = \frac{1}{10} P_1 \). In layered soil a highly compressible strata could occur immediately below this point and the error caused by disregarding it would no longer be negligible.

With the depth of significant stress at \( \Delta P_{\text{net}} = 0 \):

\[ h_s = \sqrt{\frac{P}{Y_1 D_1}} - B \tag{22} \]

The compression of layer \( n \) is found as in Eq. (1).

\[ \Delta H_n = \frac{t_n}{c_n} \log \left[ 1 + \frac{\Delta P_{\text{net}}}{P_i} \right] \tag{23} \]
Interior Footing in Open Excavation

\[ \Delta p_{\text{net}} = \frac{P}{(B+z)^2} - \gamma'D' \]

FIG. 3 - Body Stress Diagram Centroid
The net pressure change and initial body stress must be evaluated at the level of the centroid of the body stress diagram for the particular layer. The centroid is assumed at some fraction, \( A \), of the layer thickness. Thus from Eqs. (19) and (21):

\[
p_i = \gamma_i D_i + A_{th} Y_n + \sum_{i=1}^{n-1} \gamma_{it_i} \tag{24}
\]

\[
\Delta P_{\text{net}} = \frac{P}{(A_{th} + \sum_{i=1}^{n-1} t_i + B)^2} \tag{25}
\]

The total settlement under a particular footing load may now be expressed as the sum of the compressions of individual layers.

\[
S = \sum_{i=1}^{n} \Delta H_i \tag{26}
\]

### 4.2 Centroid of Stress Diagram

Equation (23) must be evaluated at the level of the centroid of the net pressure diagram for the layer in question. In uniform soil this was assumed to be \( 1/3 \) of the depth of significant stress. The stratified soil development requires a more rigorous check.

Referring to Fig. 3, the centroid of the net pressure diagram for the \( n^{th} \) layer is found as the first moment of the area divided by the area:

\[
\overline{z} = \frac{\int (\Delta P_{\text{net}} zdz)}{\int \Delta P_{\text{net}} dz} \tag{27}
\]

\[
\overline{z} = \frac{\int_{z_t}^{z_B} \left[ \frac{P}{zt(B + z)^2} - \gamma_i D_i \right] zdz}{\int_{z_t}^{z_B} \left[ \frac{P}{zt(B + z)^2} - \gamma_i D_i \right] dz} \tag{28}
\]
\[ \frac{\ln (B+z) + \frac{B}{B+z} - \frac{\gamma D'z^2}{2P}}{z_t} \]

\[ - \frac{1}{(B+z)} - \frac{\gamma D'z}{P} \]

\[ \frac{\ln \left[ \frac{B+z_b}{B+z_t} \right]}{(B+z_b)(B+z_t)} - \frac{Bt}{2P} \left( 2z_t + t \right) \]

\[ \frac{\gamma D't}{P} \]

Equation (29) may be evaluated for each layer as compression of individual layers are being calculated. However, for methods of solution other than by digital computer, this is overly rigorous. Considering the value of \( A \), where \( A = \frac{z-z_t}{t} \), it is obviously impossible that this could exceed 0.5. On the other hand, a large number of computer solutions of Eq. (29) for varying conditions and layer thickness show that \( A \) will probably not be less than 0.25, even in a relatively thin layer close to the surface of the excavation.

For uniform soil, investigation of the entire depth of significant stress shows that the position of the centroid of the body stress diagram varies around \( A = 0.33 \). This value generally decreases as the depth of excavation becomes large and again reaches a minimum \( A \) of 0.25. However, to follow previous work the value of \( A = 0.33 \) was used in developing the included charts.

One can see that if the layers of soil are thin with respect to the depth of significant stress, the order of accuracy of \( A \) is not critical. On the other hand, when the layers are relatively thick, \( A \) is more important since the stresses vary considerably in the upper portion of the depth of significant stress.

Equation (29) was not included as part of the computer solution suggested because there is provision for a large number of layers (thereby inferring thin layers) and instead the constant value 0.4 was used for \( A \) in programming. If the
Fig. 4 - COMPUTER FLOW DIAGRAM
suggested program is to be used with thicker layers, this should be considered as a minor limitation. However, assuming that each thick layer consists of two or more thinner layers whose properties are similar will avoid this difficulty.

4.3 **Computer Solution**

The computer flow diagram for a possible LOP-30 ACT III program is presented as Fig. 4. The program itself is included as Appendix D and the following general notes will help to interpret symbols and operations specified.

An initial maximum and minimum limit is provided for footing widths and an allowable differential settlement is chosen for the type of building. This may be as small as desired but must always be non-zero. After reading the dimensions and properties of the excavated and unexcavated soil layers, the computer selects a trial footing size for the allowable contact pressure.

The next computation is for depth of significant stress and the layers it includes. A test is provided to assure that the contact pressure allowed is high enough to match the depth of significant stress. Each layer is then examined for stress conditions and the resulting settlement is calculated. For the first footing (smallest column load) the resulting settlement is taken as a mean value and the maximum and minimum settlements calculated with reference to this mean. This is similar to the procedure for uniform soils.

Settlements of all other footings are first compared with the maximum allowable. If they exceed this, the "trial B" represents a lower bound on the footing size and the footing must be enlarged. However, it must not be larger than some upper bound (chosen maximum or one which will provide minimum contact pressure) or the computer rejects the conditions and starts the next footing.

The settlement is then compared with the minimum allowable and if it is greater than this (thereby being between the maximum and minimum), the computer
prints the results and begins another footing. If the settlement is less than
the minimum, the "trial B" is an upper bound and must be reduced. If it cannot
be reduced without being less than the chosen minimum, the computer rejects the
conditions and starts the next footing.

Finally, in presenting results, the computer prints the footing number,
load, size, contact pressure, depth of significant stress and settlement. These
may all be used to evaluate results in the light of method limitations.

4.4 Limitations

The first footing (smallest column load) determines the settlement properties
of the remainder of the structure. If the printed results for this footing are
not satisfactory, the program must be interrupted and reset for revised data.
Except where noted, the same general limitations apply to stratified soil as
were outlined for uniform soil.

a. Settlement - The magnitude of differential settlement inversely affects
the time required for computation but computer time is relatively un-
important. It may be an advantage to select differential settlements
in some cases and the program is more flexible if the option is included.

b. Allowable Contact Pressure - Where small differential settlements are
allowed, contact pressure must be noted. This is somewhat dissimilar
to the method for uniform soil since a contact pressure slightly higher
than that of the smallest footing is possible.

c. Submergence - The water table is treated as a division between layerers.

d. Factor of Safety Against Rupture - Where important, the factor of safety
against rupture must be calculated from the printed bearing pressures.
However, there will be few footings with higher contact pressure than
the smallest footing (unless a change in soil type allows more than one allowable contact pressure across the structure) and checking this one will usually be sufficient.

The computer solution is included only to illustrate one way in which the equations could be solved. Because those interested in using this information will have diverse applications of it, a general program was written. It is to be expected that many refinements could be made in the method depending on the objectives of the individual. However, the explanation of this program will help define the objectives of the design philosophy and the future programmer should keep this in mind.

4.5 Examples

A structure is to be constructed on a formation of stratified soil. The nature of the building will allow differential settlement from column-to-column to be $1/4$ but no more than $1/2$. The footing loads and elevations are given in Table 1. The applicable code allows a presumptive bearing capacity of four tons per square foot. The soil profile is shown in Table 2.

<table>
<thead>
<tr>
<th>Column Number</th>
<th>Column Load</th>
<th>Depth of Excavation</th>
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<tbody>
<tr>
<td>1</td>
<td>$79^k$</td>
<td>10'</td>
</tr>
<tr>
<td>2</td>
<td>$96^k$</td>
<td>10'</td>
</tr>
<tr>
<td>3</td>
<td>$140^k$</td>
<td>10</td>
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<tr>
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<td>$270^k$</td>
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<tr>
<td>8</td>
<td>$270^k$</td>
<td>5'</td>
</tr>
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Table 1
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<tr>
<th>Elevation</th>
<th>Description</th>
<th>Standard Penetration Resistance N</th>
<th>Effective Unit Weight</th>
<th>Bearing Capacity Index C</th>
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<tr>
<td>100</td>
<td>Loose Organic Silt</td>
<td>9</td>
<td>95</td>
<td>25</td>
</tr>
<tr>
<td>98.5</td>
<td>Medium Organic Clay</td>
<td>15</td>
<td>107</td>
<td>20</td>
</tr>
<tr>
<td>95.0</td>
<td>Hard Inorganic Sandy Clay</td>
<td>31</td>
<td>126</td>
<td>64</td>
</tr>
<tr>
<td>92.5</td>
<td>Medium Silty Clay</td>
<td>15</td>
<td>115</td>
<td>45</td>
</tr>
<tr>
<td>90.0</td>
<td>Compact Silty Sand</td>
<td>45</td>
<td>129</td>
<td>106</td>
</tr>
<tr>
<td>86.0</td>
<td>Loose Clean Sand</td>
<td>8</td>
<td>89</td>
<td>52</td>
</tr>
<tr>
<td>84.0</td>
<td>Compact Sand and Gravel</td>
<td>40</td>
<td>120</td>
<td>134</td>
</tr>
<tr>
<td>83.0</td>
<td>Water Table</td>
<td></td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>69.0</td>
<td>Boring Stopped</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The computer was used to design the eight footings and separate trials were completed for differential settlements, ΔS, of 0.04', 0.02' and 0.005'. The computation time for ΔS = 0.04' was about ten minutes while for ΔS = 0.005' the time was almost 30 minutes.

<table>
<thead>
<tr>
<th>Footing No.</th>
<th>Column Load P</th>
<th>Footing Width B</th>
<th>Contact Pressure P₀</th>
<th>Depth of Significant Stress hₛ</th>
<th>Settlement S</th>
<th>Depth of Excavation D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79000</td>
<td>3.14</td>
<td>8000</td>
<td>5.26</td>
<td>0.018</td>
<td>10'</td>
</tr>
<tr>
<td>2</td>
<td>96000</td>
<td>3.46</td>
<td>8000</td>
<td>5.80</td>
<td>0.020</td>
<td>10'</td>
</tr>
<tr>
<td>3</td>
<td>110000</td>
<td>5.23</td>
<td>5110</td>
<td>5.95</td>
<td>0.017</td>
<td>10'</td>
</tr>
<tr>
<td>4</td>
<td>270000</td>
<td>7.71</td>
<td>4544</td>
<td>7.82</td>
<td>0.021</td>
<td>10'</td>
</tr>
<tr>
<td>5</td>
<td>79000</td>
<td>7.05</td>
<td>1590</td>
<td>5.31</td>
<td>0.017</td>
<td>5'</td>
</tr>
<tr>
<td>6</td>
<td>124000</td>
<td>9.27</td>
<td>1143</td>
<td>6.22</td>
<td>0.018</td>
<td>5'</td>
</tr>
<tr>
<td>7</td>
<td>185000</td>
<td>11.89</td>
<td>1308</td>
<td>7.02</td>
<td>0.018</td>
<td>5'</td>
</tr>
<tr>
<td>8</td>
<td>270000</td>
<td>15.30</td>
<td>1153</td>
<td>7.55</td>
<td>0.016</td>
<td>5'</td>
</tr>
</tbody>
</table>
One should not infer that selecting a very small $\Delta S$ will necessarily give results which are equally as accurate. It is not intended to give the impression that this is even reasonable. However, it has been pointed out that computer time is relatively unimportant and though the solution will take slightly longer, a smaller value of $\Delta S$ provides the most accurate results for the given date.

Table 3 illustrates an important point. Note that, to provide equal settlement, footing contact pressure must be varied not only with column load but also with depth of excavation. By allowing increased contact pressures under deeper footings, more reasonable contact pressures would result under shallow footings. Where there is no such allowable increase, the deeper footings should be designed first since these will most likely be the controlling footings. When this is done the shallow footings may become unreasonably large (as shown). On the other hand, if the shallow footings are designed first, the deeper footings might have to be unreasonably small to provide equal settlement.
5. CONCLUSIONS

The paper by Hough, on which this work has been based, was a major step towards developing a new design philosophy for spread footings. Being a first step, it was necessarily limited to simple cases but the way was prepared for further investigations. Many of the discussions expressed hope for further work while others criticized the limited applicability of some of the results.

The charts presented herein will help to increase the range of applications. It is more important, though, that by eliminating the trial and error solutions, these charts allow the engineer to quickly find and compare several designs. While there are many who object to handbook procedures they certainly could not object to using charts for selecting trial values or making preliminary design estimates. Furthermore, the analysis charts provide a simple means of checking footings designed by other methods.

The following general conclusions are drawn from the text of the paper:
1. Footings designed by the procedure outlined for use with the included charts will automatically provide equal settlement at the maximum bearing pressure possible (most economical footing size). A reasonable factor of safety against rupture is provided and only that of the smallest footing need be checked.

2. Calculations of depth of significant stress for footings in open excavations can be significantly shortened by including the entire depth within which \( \Delta p_{net} \) is greater than zero.

3. The depth of excavation has a direct effect on the allowable bearing pressures. Example problems in this paper show that for equal column loads, the footing in the deeper excavation must be smaller to provide

(1) Ibid.
equal settlement. Thus, it shows an increase in contact pressure as the depth of excavation is increased.

4. The philosophy of design based on equal settlement can be extended to include footings in stratified soils. However, the solution is so time consuming that, with the equations presented, a digital computer is the only practical means of application. With the inclusion of this material many of the original objections have been overcome.

5. The included design charts can be used for footings placed on submerged soil as long as the water table is no lower than footing grade. An effective depth of excavation is calculated to adapt the charts to this situation. This procedure could also have been used in formulas presented previously\(^1\).

6. In stratified soil formations, as long as the soil layer from footing grade to the depth of significant stress is uniform, an effective depth of excavation can be calculated to allow use of the charts or equations. Where there is stratification within the depth of significant stress, the computer type solution must be utilized.

\(^{1}\) Ibid.
ACKNOWLEDGMENTS

The author wishes to express appreciation to B. K. Hough for his guidance and suggestions, without which this paper could not have been written.

Credit is due also to Prof. William A. Smith and Mr. Gary E. Whitehouse of the Industrial Engineering Department at Lehigh University, Bethlehem, Pennsylvania, who helped prepare computer programs and data used in the paper.
Appendix A

LIST OF VARIABLES

A  -- Fraction of thickness to the centroid of the body stress diagram for a particular layer.

B  -- Foundation width

Blb  -- Computed lower bound on B

B^max  -- Maximum allowable B(selected at running time)

B^min  -- Minimum allowable B(selected at running time)

B^spec  -- Specification (code value)B

B^ub  -- Computed upper bound on B

C  -- Bearing capacity index

C_c  -- Compression index

D  -- Depth of excavation

D'e  -- Effective depth of excavation

e  -- Void ratio

F  -- Footing number

G  -- Specific gravity of soil particles

'H  -- Thickness of a compressible layer

ΔH  -- Compression or reduction in thickness of a thin layer

h  -- Depth of a point from the base of a footing

h_c  -- Depth to pressure diagram centroid for a particular layer

h_s  -- Depth of significant stress

i  -- Subscripts for unexcavated layers

j  -- Subscripts for excavated layers
k  -- Fraction of layer thickness to some particular point
n  -- Subscript for layer being checked
P  -- Column load
Pc  -- Contact pressure under a footing
p1  -- Initial body stress
p1  -- Body stress after excavation
p2  -- Final body stress
\Delta p  -- Increment of vertical stress
\Delta p_{net}  -- Net difference of initial and final body stress
S  -- Settlement
S_{max}  -- Maximum allowable settlement
S_{min}  -- Minimum allowable settlement
t  -- Thickness of a compressible layer
z  -- Depth of a particular point
\bar{z}  -- Depth of body stress diagram centroid measured from the the bottom of the footing
z_b  -- Depth to the bottom of a layer from the bottom of the footing
z_t  -- Depth to the top of a layer from the bottom of the footing
\gamma  -- Unit weight of soil
\gamma_{up}  -- Submerged unit weight of soil
\gamma_D  -- Stress reduction due to excavation
\gamma_w  -- Unit weight of water
Appendix B

DESIGN CHARTS
Design Chart
Depth = 4 ft.
Uniform Soil

Column Load Quotient, P/y (cu. ft.)

Foundation Width, B (ft.)
DESIGN CHART
Depth = 8 ft.
Uniform Soil

Foundation Width, B (ft.)

Column Load Quotient, $P/\gamma$
(cu. ft.)

200 300 400 500 600 700 800 900 1000 2000 3000 4000 5000 6000 7000 8000 9000 10,000
Appendix C

ANALYSIS CHARTS
ANALYSIS CHART
Depth = 4 ft.
Uniform Soil
ANALYSIS CHART
Depth = 8 ft.
Uniform Soil

Column Load Quotient, $P/\gamma$

Settlement Factor, $SC$ (ft.)

200 300 400 500 600 700 800 900 1,000 1,100 1,200 1,300 1,400 1,500 1,600 1,700 1,800 1,900 2,000

2 3 4 5 6 7 8 9 10 11 12

8 ft.
8 ft.
8 ft.
8 ft.

Depth = 8 ft.
Uniform Soil
ANALYSIS CHART
Depth = 12 ft.
Uniform Soil

Column Load Quotient, \( P/\gamma \) (cu. ft.)

Settlement Factor, SC (ft.)
Appendix D

SUGGESTED COMPUTER PROGRAM

(LGP30 ACT III)
```
sl!read"bmax!read"bmin!read"ds!iread"finf"
0!;"f"
s2!read"p!read"pall!read"a"
s3!dim"te"6"ge"6"
index"m
iread"lastm"
0!;"m"
s4!read"te"m!read"ge"m"
iter"m!"lastm"sl"
s5!dim"tu"15"c"15"
index"n"
iread"lastn"
0!;"n"
s6!read"tu"n!read"gu"n!read"c"n"
iter"n!"lastn"s6"
s26!sqrt"(!p!/pall!)"bspec
bspec"!"1!"e"1!"h"!b"
bspec!"bib"
bmin!"bub"
bmax!"bub"
0!;"m"
prev"/gd"
s7!"(te"m!x"ge"m!)"+!gd"!gd"
iter"m!"lastm"s7"
s8!"sqrt"(!p!/gd!)"-"h!"hs"
if"(!hs!)"neg"s21"
s9!0!;"n"
prev"/h"
s10!if"(!tu"n!+!h!-!hs!)"pos"s11"
tu"n!+!h!+!h"
iter"n!"lastn"s10"
sl!hs!="h!; tstop"
gu"n!; tstop"
c"n!; cstop"
n!i!-!1!; tstopn"
dim"hc"15"
index"n"
0!;"h"
prev";s"
prev";n"
prev";gt"
sl2!h!+!a!x!tu"n!; hc"n"
tu"n!+!h!+!h"
iter"n!; tstop"s12"
h!+!a!x!tstop!; hstop"
```
FIG. 1 - Body Stress Diagrams, Uniform Soil
(After Hough)
Interior Footing in
Open Excavation

Fig. 2 - Body Stress, Stratified Soil
Interior Footing in Open Excavation

\[ \Delta p_{\text{net}} = \frac{P}{(B+Z)^2} - \gamma'D' \]

FIG. 3 - Body Stress Diagram Centroid
Fig. 4 - COMPUTER FLOW DIAGRAM