Low Cycle Fatigue Behavior of Joined Structures

REDISTRIBUTION OF STRESS AND STRAIN IN A PLATE WITH A CRACK

by

R. J. Smith
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Fritz Engineering Laboratory Report No. 358.8
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Department of Civil Engineering

Fritz Engineering Laboratory
Lehigh University
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ABSTRACT

This report presents a method of analysis for determining the stress and strain redistribution in a plate with a crack. The study is part of a major research program on the behavior and design of joined structures subjected to low-cycle fatigue.

The analysis investigated the elastic-plastic behavior of a plate with a crack under one loading-unloading cycle. A mathematical model was developed by replacing the continuous material of the plate by an equivalent system of lumped volumes at discrete points, which was then analyzed for deformations.

The analytical method and the results presented in this report are being correlated with experimental studies on the redistribution of stress and strain in plain and welded specimens. It is intended to relate crack propagation to the stress and strain history considering the Manson-Coffin failure criterion as well as
the stress intensity factor in the plate subjected to elastic-plastic stress redistribution.
1. INTRODUCTION

The analysis of stresses, strains and the overall deformation of elasto-plastic structures has been made attainable by the advent of the computer. Such areas of continuum mechanics involve the analysis of stresses and strains where the relationship between them deviates from the linear form of classical elasticity.

Since mathematical difficulties make the solution of non-linear differential equations of the elastic-plastic theory impossible in nearly all cases, another method of approach other than pure mathematical analysis has been used. As an alternate approach to this problem the discrete element methods such as finite element or lumped parameter can be utilized. These methods provide a numerical solution to the problem and can be formulated in simple physical concepts without making use of complex differential equations.

For many crack problems already treated, the theory of fracture mechanics predicts that the stress at the tip of a sharp crack is infinite, and that the stress intensity
reduces inversely with the square root of distance from the crack tip. The inelastic region in front of a crack is relatively small as computed by this theory. In a structural element where elasto-plastic stress distribution exists, the inelastic region at the crack tip may be large. To obtain a satisfactory description of the stress and strain fields near the crack tip prior to its extension thus becomes a necessity for the elasto-plastic analyses of crack problems in low-cycle fatigue.

The choice between the lumped parameter and finite element methods appears to be largely a matter of the problem to be solved and the time required to obtain some satisfactory approximate results. (1) The lumped parameter method can be applied with less computational effort than the finite element method for a given number of elements because its behavior is defined by fewer nodal displacements for a given element. However, the finite element method can be refined to such an extent that it will be more exact than the lumped parameter method.

The finite element method has been used to solve a number of plane elasto-plastic stress problems (2, 3, 4, 5)
whereas the lumped parameter method of analysis has only been employed in the large deformation inelastic flexural analysis of plates (6, 7, 8) and the analysis of shell structures. (9) To date, this method has not been used in the analysis of stress and strain redistribution in a flat plate subject to a uniaxial stress and a cracked condition. It is the objective of this report, therefore, to describe this method in treating an elasto-plastic plane-stress crack problem.
2. ANALYTIC FORMULATION

2.1 Basic Formulation and Assumptions

The analysis of the stress and strain distribution and redistribution in a flat plate was performed by adopting a discrete element or so called "lumped parameter" method. The basic idea of this method consisted in replacing the continuous material of the structural element under investigation by approximating it with that of a mathematical model composed of lumped volumes. These lumped volumes are arranged at discrete points according to a definite pattern or gridwork such that their behavior corresponds to that portion of the continuum which it replaces. The analysis of elasto-plastic stress problems requires:

a) the application of equilibrium conditions to develop a set of simultaneous nonlinear algebraic equations for the deformations of the lumped parameters in the model,

b) the development and programming for a computer application of a numerical method in solving the equations, and
c) the linearizing of the stress-strain relationship.

The plate under investigation was subjected to a uniform uniaxial stress, \( \sigma \), as shown in Fig. 1. In this figure, \( L \) denotes the length, \( W \) denotes the width, and \( t \) indicates the thickness of the plate. The thickness was assumed small so that the resulting stresses in the plate could be considered as representing a case of plane-stress. Since a symmetrical arrangement of the applied stress, of the crack location, and of the plate geometry was assumed, the analysis could be carried out for one quarter of the plate. This region is indicated by the cross-hatched area in Fig. 1.

The mathematical model, representing the cross-hatched area of Fig. 1, is shown in Fig. 2. It consists of rectangular elements discretized to points which lie on grid lines and are represented by the solid circles. In Fig. 2 \( L' \) and \( W' \) represent the distance between lumped parameters along the length (\( Z \) lines) and across the width (\( M \) lines), respectively.
Certain boundary conditions were imposed on the plate in an attempt to ensure that the behavior of the mathematical model would be a close approximation of the behavior of the continuous plate. These boundary conditions are located at Sections a-a, b-b, and b'-b' in Fig. 1. Section a-a is the reference or neutral section at which no change in displacement occurs. The failed or cracked condition of the model was assumed to start at the center of the plate and to propagate symmetrically, element by element, across this reference section. At section b-b (and b'-b') boundary compatibility requires a uniform displacement of the plate.

Other assumptions made in the analysis are:

1. the effect of Poisson's ratio was neglected,
2. the shear modulus, G, was constant for both the elastic and plastic strain range,
3. the stress-strain curve was bi-linearly elastic-strain hardening, and
4. the applicable yield condition was the maximum normal stress theory or Rankine theory.\(^{(10)}\)
2.2 Formulation of Equilibrium Equations

The conventional stiffness method of structural analysis\(^{(11)}\) was employed to establish sets of equilibrium equations from the relative displacements of the lumped parameters in the equivalent model. The equilibrium equations are formed by considering the equilibrium of forces that act on each lumped parameter. Figure 3 shows a typical deformed displacement configuration for a set of lumped parameters used in forming the equilibrium equations. For any nodal point, the distance between the full line circle and the dotted line circle represents the displacement of the lumped parameter from its initial position in the mathematical model. The relative displacement is then the difference in displacement between any two adjacent nodal points.

Figure 4 shows the resulting forces or stress resultants that act on any typical discrete point. These forces, except for the initial residual forces, are dependent upon the relative displacements of four adjacent lumped parameters. They are formed by multiplying the relative displacements by stiffness coefficients. By considering only
the relative displacements of the four adjacent discrete points, the resulting stiffness coefficients lie on five diagonals. All other coefficients in the stiffness matrix are zero.

The resulting stiffness matrix was such that the sum of the stiffness coefficients off the main diagonal was always equal to the stiffness coefficient on the principal diagonal. This implied that no ill-conditioned stiffness matrix would arise and therefore the iteration procedure would always converge. Thus the analysis was based upon the formulation of a stiffness matrix which could be used in the iterative solution procedure to estimate a new set of displacements for the lumped parameters under any given loading and crack condition.

The equilibrium equation for any nodal point may be written as

\[ S_N - S_{N-M} - V_{N+1} + V_N - R_{N-M} + R_N = 0 \]  (1)

Depending upon the state of stress at the nodal point or its location, three situations are possible when formulating the equilibrium equations within the model. These are the elastic,
the plastic or the cracked situation of equilibrium. In any case, the residual stress resultants $R_{N-M}$ and $R_N$ are identical at a nodal point thus can be dropped from Eq. (1).

A. **Elastic Condition**

The forces in Eq. (1) are substituted by the products of relative displacements and stiffness coefficients.

a) For nodes on longitudinal grid line 1,

$$
(X_N - X_{N+M}) C_N + U_N - (X_{N-M} - X_N) C_{N-M} - U_{N-M} 
- (X_{N+1} - X_N) D + (X_N - X_{N-1}) D = 0
$$

where $C_N$ and $D$ are the elastic stiffness coefficients for tension and shear, respectively,

$$
C_N = \frac{E t W (2Z - 1)}{2ML}
$$

$$
D = \frac{2 G t L M}{(2Z - 1)W}
$$

The $U$'s are forces on elements at yielding thus are zero in the elastic range. By symmetry, $X_N = X_{N-1}$ and Eq. (2) can be rearranged as

$$
X_{N+M} (-C_N) + X_{N+1} (-D) + X_N (C_N + C_{N-M} + D) 
+ X_{N-M} (-C_{N-M}) = U_{N-M} - U_N
$$
b) For nodes on longitudinal grid lines 2 to \( M-1 \),

\[
(X_N - X_{N+1}) C_N + U_N - (X_{N-M} - X_N) C_{N-M} - U_{N-M} \\
- (X_{N+1} - X_N) D + (X_N - X_{N-1}) D = 0
\]  

(6)

Rearranging, Eq. (6) can be written as

\[
X_{N+M} (-C_N) + X_{N+1} (-D) + X_N (C_N + C_{N-M} + 2D) \\
+ X_{N-1} (-D) + X_{N-M} (-C_{N-M}) = U_{N-M} - U_N
\]

(7)

c) For nodes on grid line \( M \),

\[
(X_N - X_{N+M}) C_N + U_N - (X_{N-M} - X_N) C_{N-M} - U_{N-M} \\
+ (X_N - X_{N-1}) D = 0
\]

(8)

or

\[
X_{N+M} (-C_N) + X_N (C_N + C_{N-M} + D) + X_{N-1} (-D) \\
+ X_{N-M} (-C_{N-M}) = U_{N-M} - U_N
\]

(9)

B. Plastic Condition

The effect of any particular discrete element becoming yielded is that the stiffness coefficient \( C_N \) is modified and that the forces \( U_N \) are generated for this element. The coefficient change is represented by a substitution of the strain-hardening modulus \( E_s \) for the
modulus of elasticity (E) in the stiffness coefficient \( C_N \) of Eq. (3).

\[
C_N = \frac{E_s t W (2Z-1)}{2 ML} \quad (10)
\]

The forces \( U_N \) are:

\[
U_N = \frac{F_y t W}{2 M} - R_N \quad (11)
\]

where

\[
R_N = \frac{t W S_{res}}{2 M} \quad (12)
\]

Since the shear modulus, \( G \), was assumed to be constant in the analysis, the stiffness coefficient \( D \) remained the same as in the elastic case.

C. Cracked Condition

For the elements adjacent to the cracked section, the stiffness coefficient \( C_N \) is zero and the forces \( U_N \) become

\[
U_N = -R_N \quad (13)
\]

Since Eqs. 5, 7, and 9 have the same form with only difference in constants, the general form of the equilibrium equations can be expressed as:

\[
AA_N (X_{N+M}) + BB_N (X_{N+1}) + CC_N (X_N) + DD_N (X_{N-1}) + EE_N (X_{N-M}) = FF_N \quad (14)
\]
2.3 Solution Procedure

The solution of Eq. 14 employed the Gauss-Seidel iterative method which has been described in detail elsewhere. (12) In general terms, the Gauss-Seidel iteration is given by the formula

\[ x_i = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{n} a_{ij} x_j \right) \text{ for all } i. \]  

This iterative procedure will always converge provided that the coefficient matrix satisfies the "row-sum criterion"

\[ a_{ii} \geq \sum_{j=1}^{n} |a_{ij}| \text{ for all } i \]  

For this analysis the convergence criterion was satisfied.

From Eq. 14, the equation for the Gauss-Seidel iteration can be expressed by

\[ X_N = \frac{[FF_N - AA_N (X_{N+M}) - BB_N (X_{N+1}) - DD_N (X_{N-1}) - EE_N (X_{N-M})]}{CC_N} \]  

where \( AA_N, BB_N, CC_N, DD_N, EE_N, \) and \( FF_N \) are the known constants, \( X_{N+M}, X_{N+1}, X_{N-1}, \) and \( X_{N-M} \) are initial displacements, and \( X_N \) is the new displacement calculated by the iteration procedure.
The solution procedure, carried out by the use of a computer, was incremental as well as iterative. The incremental approach was employed to maintain equilibrium at the cracked section. It may be described as follows. For any deformed configuration the difference between the sum of the nodal equilibrating forces across the reference section a-a and the applied stress resultant on the plate was treated as an "unbalanced" force. The unbalanced force was then reduced to an arbitrary small quantity by successive corrections to the uniform displacement condition at section b-b. The magnitude of the correction was made arbitrarily small (under-relaxation) and was applied recurrently to the nodal points of the mathematical model after a specified number of iterations had occurred. The displacement configuration was therefore made to approach the equilibrium configuration to within any arbitrarily chosen limit on the total unbalanced force.

The initial displacements of the nodal points were estimated by an initial calculation of the uniform displacement which was based on the applied loading condition. This displacement was then distributed over the mesh in
proportion to the distance from the reference section.

The procedure was programmed for a CDC 6400 computer.
3. RESULTS AND DISCUSSION

The analysis was applied to a crack propagation problem in an attempt to study the stress and strain redistribution associated with gradual plastification of structural elements in a cracked medium. The plate model consisted of rectangular elements in a mesh division of 20 by 20. The residual stress pattern, the maximum and minimum applied nominal stresses, and the material properties used in the example were taken from a cracked beam test. Some of the results of the analysis are presented below.

Figure 5 is a plot of stress, $S$, versus the half width of the plate. The stress distribution has been plotted for only half of the plate ($W/2$) because the stress distribution and redistribution has been assumed to be symmetrical about the center. In Fig. 5, the dashed line represents the initial residual stress pattern, $S_{res}$, and the solid lines (0-1 to 4-1) depict the consecutive stress redistribution in the first row of elements from the line
of the crack for the conditions of 0, 1, 2, 3, and 4 cracked elements under an applied stress of 36.2 ksi. These curves were truncated at the yield stress which was 110 ksi.

Similarly, the redistribution of stress in the first row of elements for 0, 4, 6, 8, and 10 cracked elements are shown in Fig. 6. The flat portion of the stress diagram (the plastic zone size), which is large at first due to high residual stresses, decreases upon initial crack propagation. It then increases in size with further crack propagation indicating that the size of the plastic region in front of the crack tip grows with increased crack propagation. It may be noted here that 10 cracked elements represent a crack length of one-half of the width of the plate.

The solid curves of Fig. 7 show the redistribution of stress for rows 1, 2, 6 and 20 for a 5 element cracked condition under an applied stress of 36.2 ksi. The figure indicates that at increasing distances (rows) removed from the cracked section of the plate, the stress distribution pattern will approach that of the redistribution of the initial residual stress under an applied nominal stress of
36.2 ksi. This condition is closely approximated by curve marked 5-20 of Fig. 7.

In the example studied, the minimum applied stress was zero, thus stress distributions after unloading from maximum load are residual stresses. Figure 8 shows the residual stress pattern, $S_{res}$, and the stress distribution patterns in the first row of elements under load (L) and after unloading (U) when there was no crack. Redistribution of stress occurred mainly in the region that reached yielding and in its immediate vacinity. Very little redistribution occurred in the remainder of the plate. For a crack length of ten elements, the corresponding stress distribution patterns are given in Fig. 9. It is apparent that there was stress reversal at the crack tip when unloading took place.

While stresses are limited by the yield point of the plate material, strains are not. The strain distributions in Fig. 10 correspond to the stress patterns of Fig. 5 for the first row of elements with 0, 1, 2, 3, and 4 cracked elements. These strain distributions do not include the initial residual strains. The solid portions of the curves
represent average strains in the elements whereas the dashed portions represent average strains which includes the crack opening displacement as part of the change in element length for calculating the strains. The strain distributions of Fig. 11 are for 10 cracked elements for both the loaded and unloaded condition. The distance between the two curves represents the change in strain due to an applied stress range of 36.2 ksi.

The results of the analysis have been compared to the results of an experimental fatigue crack growth test on a flange of a welded beam. Further details concerning the cracked beam test can be obtained from a previous report. (14)

The theoretical stress distributions under load and after unloading for different crack sizes are plotted in Fig. 12 for a flange crack. The loading distribution curve corresponding to the initial crack size at the beginning of the experiment was used as a reference line to plot stresses evaluated from strains recorded during the test. The solid lines represent the results of the theoretical analysis and the plotted points (open and
closed dots) represent the results of the experiment. The theoretical values and recorded data compare satisfactorily. Deviation between results, particularly after unloading, may be related to the restraint of the web on the flange. An extension of this analysis to a flange-web, three-ended crack\(^{(15)}\) provides significant information on flange-web interaction.

It may be important for the correlation between crack length and stresses to have information on the magnitude of strains at the leading edge of a propagating crack. Theoretically derived strains under load (L) and after unloading (U) are presented in Fig. 13. These are average strains over the length of an element close to the crack tip. The distance between the curves represents the strain range to which the element in front of the crack was subjected. Further investigation in this respect should be conducted.
4. SUMMARY AND CONCLUSIONS

The analytical investigation described in this report is concerned with the distribution and redistribution of stresses and strains in a plate under the influence of a cracked condition. The lumped parameter method was employed with the Gauss-Seidel iterative procedure in the formulation and solution of a mathematical model representing the plate.

Results were obtained by means of a computer program. The material properties, magnitudes of the applied stresses and some of the cracked conditions in the example of analysis were taken from an experiment on a cracked beam. It was found that the experimental results compared quite well with those obtained analytically.

The following comments are drawn from the study and the results:

1. In the analysis by the adopted procedure, the size of elements is of great practical
importance. A large number of small elements are necessary for a satisfactory estimate of the stresses and the strains in the plate, but it requires extensive computational efforts. For stress and especially strain distributions at locations closer to the crack tip than herein chosen, expansion of the mesh division of the mathematical model is necessary.

2. The program is functional for elastic-plastic but not fully plastic sections where very large deformations take place. The program needs to be extended to satisfy plastic flow requirements when the plate is subjected to very high loads above yielding.

3. The analysis is being extended to the situation of a three-ended crack such as that in the flange-to-web junction of a beam.\textsuperscript{(15)}

It may be concluded that the results of the study are very encouraging to the investigators. It is hoped that
further study and correlation of these results and the crack propagation rates will bring about better understanding of the low-cycle fatigue phenomenon.
5. ACKNOWLEDGEMENTS

The investigation is part of a major research program on low-cycle fatigue and was conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The Office of Naval Research, Department of Defense, sponsored the research under contract N 00014-68-A-514; NR 064-509.

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### 6. NOMENCLATURE

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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$C_T$</td>
<td>Stiffness Coefficient for Tension (kip/in)</td>
</tr>
<tr>
<td>$D$</td>
<td>Stiffness Coefficient for Shear (kip/in)</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's Modulus (ksi)</td>
</tr>
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<td>$E_s$</td>
<td>Strain-Hardening Modulus (ksi)</td>
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<td>$F_Y$</td>
<td>Static Yield Strength (ksi)</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear Modulus (ksi)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of Plate Under Investigation (in)</td>
</tr>
<tr>
<td>$L$</td>
<td>Distribution Under Load</td>
</tr>
<tr>
<td>$L'$</td>
<td>Longitudinal Distance Between Transverse Grid Lines (in)</td>
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<tr>
<td>$L_f$</td>
<td>Total Length of Element due to Lumped Parameter Displacement (in)</td>
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<td>$M$</td>
<td>Number of Grid Lines or Divisions Across Model Width</td>
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<td>$P$</td>
<td>Applied Load (kips)</td>
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<td>$R_N$</td>
<td>Residual Stress Resultant (kips)</td>
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<td>$S_{res}$</td>
<td>Residual Stress (ksi)</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of Plate (in)</td>
</tr>
<tr>
<td>$U$</td>
<td>Distribution After Unloading</td>
</tr>
</tbody>
</table>
$U_N$  
Force on Element at Yielding (kips)

$V_N$  
Shear Stress Resultant (kips)

$W$  
Width of Plate Under Investigation (in)

$W'$  
Transverse Distance Between Longitudinal Grid Lines (in)

$X_N$  
Displacement of Nodal Point (Lumped Parameter) from Initial Position in Mathematical Model (in)

$Z$  
Number of Grid Lines or Divisions Along Model Length

$\varepsilon$  
Strain (in/in)

$\varepsilon_y$  
Yield Strain (in/in)

$\sigma$  
Uniform Uniaxially Applied Stress (ksi)

$\sigma_{\text{max}}$  
Maximum Applied Stress (ksi)

$\sigma_{\text{min}}$  
Minimum Applied Stress (ksi)
7. FIGURES
Fig. 1 Plate Under Investigation
Fig. 2 Mathematical Model
$X_N = \text{Displacement of Nodal Point from Initial Position in Mathematical Model}$

Fig. 3 Typical Lumped Parameter Displacement Configuration
Fig. 4 Stress Resultants of a Typical Discrete Point
Fig. 5 Stress Distribution for 0, 1, 2, 3, and 4 Cracked Elements
\( \sigma_{\text{MAX}} = 36.2 \text{ KSI} \)
\( \sigma_{\text{MIN}} = 0 \text{ KSI} \)

MESH DIVISION (20 x 20)

Fig. 6 Stress Distribution for 0, 4, 6, 8, and 10 Cracked Elements
\[ \sigma_{\text{MAX}} = 36.2 \text{ KSI} \]
\[ \sigma_{\text{MIN}} = 0 \text{ KSI} \]

MESH DIVISION (20 x 20)

Fig. 7. Stress Distribution for Different Rows
Fig. 8 Stress Distribution Under Load and After Unloading, No Crack
Fig. 9 Stress Distribution Under Load and After Unloading - Ten Cracked Elements
\[ \sigma_{\text{MAX}} = 36.2 \text{ ksi} \]
\[ \sigma_{\text{MIN}} = 0 \text{ ksi} \]

Mesh Division (20 x 20)

\( \epsilon \times 10^{-3} \text{IN}_{\text{MIN}} \)

Progressive Crack Length

Fig. 10 Strain Distribution for 0, 1, 2, 3, and 4 Cracked Elements
Fig. 11 Strain Distribution Under Load and After Unloading - Ten Cracked Elements

\( \sigma_{\text{MAX}} = 36.2 \text{ ksi} \)
\( \sigma_{\text{MIN}} = 0 \text{ ksi} \)

Mesh Division (20x20)

Crack Length

\( \varepsilon \) (x10^{-3} IN/IN)

\( \varepsilon_y \)
\[ \sigma_{\text{MAX}} = 36.2 \text{ KSI} \]
\[ \sigma_{\text{MIN}} = 0 \text{ KSI} \]

MESH DIVISION (20x20)

Fig. 12 Theoretical and Experimental Stress Distribution
\[ \sigma_{\text{MAX}} = 36.2 \text{ KSI} \]
\[ \sigma_{\text{MIN}} = 0 \text{ KSI} \]

**Mesh Division (20x20)**

\[ \epsilon = \frac{L_f - L'}{L'} \]

Fig. 13 Average Strains Near the Crack Tip
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REDISTRIBUTION OF STRESS AND STRAIN IN A BEAM WITH A CRACK, Fritz Engineering Laboratory Report No. 358.9, June, 1970.
This report presents a method of analysis for determining the stress and strain redistribution in a plate with a crack. The study is part of a major research program on the behavior and design of joined structures subjected to low-cycle fatigue.

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<table>
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<th>KEY WORDS</th>
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<td></td>
<td>HOLE</td>
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Security Classification