Composite Assemblages Under Lateral Load

THE INTERACTION OF FLOORS AND FRAMES IN HIGH RISE BUILDINGS

by
Dirk P. du Plessis
J. Hartley Daniels

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This work has been carried out as part of an investigation sponsored by the American Iron and Steel Institute.

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ABSTRACT

The influence of the floor system on the behavior of a three-dimensional multistory steel frame under wind load is studied. The multistory frame consists of a series of parallel unbraced steel frames forming the main wind resisting elements. The floor system consists of solid reinforced concrete slabs attached with steel stud shear connectors to the floor and frame beams. Any type of discontinuities in the floor slabs such as holes and shrinkage gaps at the columns can be accommodated.

The three-dimensional multistory frame is considered to have symmetric geometry and loading as viewed in a direction parallel to the plane of the unbraced frames. Therefore, it is possible to study any one of the unbraced frames with its panel wide floor system to obtain the load-drift curve of the complete structure. Such an unbraced frame with its floor system is called a composite frame in this dissertation.

The composite frame is further reduced to an equivalent plane frame by replacing the floor system with an equivalent slab at each floor level. Each equivalent slab has uniform width along the length of the composite frame and is separately determined for each floor level. In determining the width of the equivalent slab, the effects of flexibility of the shear connectors, of discontinuities in the floor slabs and of cracking of the concrete slabs are included.

The equivalent plane frame is then subjected to a second-order elastic-plastic analysis to obtain the complete load-drift
curve of the composite frame. The plastic moments of the beams are the composite plastic moments of the floors. These were determined by considering each floor as a continuous composite beam.

Two example problems are analyzed to show that the floor system can significantly increase the maximum strength and stiffness of the bare steel frame. If the shear connection between the slabs and the frame beams is flexible instead of rigid, then a significant increase in the service load drift of the composite frame occurs. Cracking of the concrete slabs has a small influence on the service load drift. The service load drift is not affected by discontinuities in the floor slabs. Substantial saving in steel weight is possible by including the floor system in the design of the steel frame to resist wind loads.
1. INTRODUCTION

1.1 Purpose

The steel frame of a multistory building is the primary structural element that is utilized to resist the applied gravity and wind loads. Gravity loads are transmitted to the steel frame through the floor system. The floor system comprises the floor slabs and the floor beams. In this dissertation "floor beams" have a specific meaning as defined in Article 1.2.

The wind loads are transmitted to the steel frame through the exterior wall system. The exterior walls and other systems such as the interior infill panels and the floor system are usually neglected in the design of the steel frame to resist the wind loads. However the increased demand for the earth's limited natural resources and the ever-rising costs of building materials have made it necessary that maximum use be made of more of the components of a building.

Research on the contribution of the exterior and interior walls to the wind resistance of a steel frame is well advanced. (1.1, 1.2) However, little has been done on the contribution of the floor system to the wind resistance of a steel frame. Apart from its role as a horizontal diaphragm to distribute loads from one frame to another, the ability of the floor system to increase the lateral load resistance of the steel frame and to decrease frame drift has virtually been ignored. The primary reason for this situation is that no economical method has yet been devised to
account for the influence of the floor system.

It is the purpose of this dissertation to study the influence of the floor system and to develop a method for including the contribution of the floor system when determining the lateral load resistance of a multistory steel frame. Such a method should result not only in a substantial saving in the weight of the steel structure, but in more reliable calculations of the frame drift under wind loads.

1.2 Composite Frames

Figure 1.1 shows the type of three-dimensional multistory frame that will be studied. The frame consists of several parallel unbraced steel frames forming the main wind resisting elements. The spacing of the unbraced frames is denoted by $B$ and $L_1$ and $L_2$ denote the centerline distances of the steel columns. $W$ represents the distributed floor loads and $H$ the horizontal wind load. The story height is denoted by $h$.

The frame of Fig. 1.1 also has symmetric geometry and loading. This implies that all the unbraced frames are identical and evenly spaced and are subjected to symmetrical distributions of floor loads as viewed in a direction parallel to the plane of the unbraced frames. The behavior of each unbraced frame under the applied gravity and wind loads are therefore the same. It is consequently possible to isolate any unbraced frame with its panel wide floor system and study this frame to obtain the behavior of the complete three-dimensional frame. Such a frame will be called a composite frame and is shown in Fig. 1.2a.
Figure 1.2b shows some of the structural detail of a composite frame. The composite frame comprises the reinforced concrete floor slabs, the floor and frame beams and the steel columns. In this dissertation "frame beams" refer to the beams of the unbraced steel frames. All other beams will be called floor beams.

The reinforced concrete floor slabs are attached to the floor and frame beams with mechanical shear connectors. Rigid connections (AISC Type 1) exist between the frame beams and the columns. The floor beams are attached to the frame beams with rigid or simple connections (AISC Type 2).

1.3 Objectives

The major objective of this dissertation is to develop an analytical method to obtain the second-order load-drift curve of a composite frame. By comparing the load-drift curve of the composite frame with that of the bare steel frame the influence of the floor system will be exposed. Of particular interest is the effect of the floor system on the maximum strength and service load drift of the steel frame.

The behavior of the composite frame may be affected by several parameters. Among these are the flexibility of the shear connectors between the floor slabs and the frame beams, discontinuities in the floor slabs and cracking of the reinforced concrete slabs. The effects of these parameters on the load-drift curve of a composite frame will be demonstrated.
If the influence of the floor system on the maximum strength of the steel frame is known, then it should be possible to design a composite frame which has the same maximum strength as the steel frame. If, furthermore, the service load drift of the composite frame is less than or equal to that of the steel frame, then a substantial saving in the weight of the steel frame may be possible. That this is indeed so, will be demonstrated.

1.4 Problem Statement

A stiffness analysis of the bare steel frame consists of dividing the structure into one-dimensional beam and column elements, determining the stiffness matrix of each element, assembling all the stiffness matrices to obtain the total stiffness matrix, incorporating the boundary conditions and finally solving the set of simultaneous equations to obtain the nodal displacements. The semibandwidth of the total stiffness matrix is equal to the largest difference in the numbers of adjacent nodes plus one multiplied by the nodal degree of freedom. The size of the total stiffness matrix is equal to the number of nodes times the nodal degree of freedom. The time required to solve the set of simultaneous equations on a computer is proportional to the square of the semibandwidth times the size of the stiffness matrix.

If the procedure as described above is used to analyze a composite frame then the following problems arise:

1) Because the composite frame is three-dimensional both the semibandwidth and the size of the composite frame stiffness
matrix can be from 50 to 100 times greater than that of the steel frame. Execution time on a computer would consequently be \((50)^3\) to \((100)^3\) times greater for the composite frame than for the steel frame. To obtain one linear elastic analysis would therefore involve a very large cost. In addition numerical errors will have a significant influence on the analytical results.

2) To obtain the complete load-drift curve of the composite frame requires a nonlinear analysis involving many linear elastic analyses. Since it was concluded above that the cost of one linear elastic analysis can be very large, it is evident that the cost of a nonlinear analysis can be astronomical. Furthermore, the accumulation of numerical errors would make the analytical results worthless.

For solving large structures such as the composite frame the method of substructures is often used.\(^{(1,4)}\) With this method the composite frame is divided into a number of substructures. The stiffness matrix of each substructure is inverted and the unknown forces at the releases are determined by satisfying compatibility at these locations. Nodal displacements are then determined by back substitution. However, the total computational effort will not decrease and may quite likely increase because of the additional matrix operations.

The problem associated with a composite frame, therefore, is the very large computational effort necessary to obtain the second-order load-drift curve. In this dissertation a method is developed
which greatly reduces this effort thereby providing an economical means for obtaining the load-drift curve. The method yields results which are approximate but compare favorably with available experimental results.

1.5 Scope

The analytical method developed in this dissertation is suitable for all multistory buildings of the type shown in Fig. 1.1. No restriction is placed on the number of stories or the number of panels in both directions. The floors consist of solid reinforced concrete slabs attached to the floor and frame beams with headed steel stud shear connectors. Formed metal deck slabs are not considered although the basic theory developed in this dissertation also applies to those slabs.

In-plane behavior and out-of-plane bending of the concrete slabs are included to accurately determine the stiffness of a composite frame at the working load level. Discontinuities in the reinforced concrete slabs such as holes and shrinkage gaps at the columns are permissible. Cracking of the concrete slabs is studied. Composite action between the concrete slabs and the floor and frame beams including the effect of flexible shear connection is considered.

A nonlinear analysis using the plastic moments of the composite floor sections is included to determine the maximum strength of a composite frame. All other topics related to steel frame analysis are also included. (1.5) Design examples are analyzed to demon-
strate the method of composite frame analysis as developed in this dissertation.

1.6 Review of Previous Work

Reference 1.6 presents an excellent summary of the existing work on the three-dimensional analysis of multistory buildings. A common assumption used is that the floor slabs have infinite in-plane stiffness and zero transverse stiffness. The effect of composite action between the slabs and the frame beams is considered by using the T-beam approach. References 1.7 to 1.11 present linear elastic analyses. References 1.12 and 1.13 also include nonlinear analyses.

The assumption of zero transverse stiffness of the slabs neglects the effect of slab bending on the stiffness of a composite frame. Furthermore, the T-beam approach can only provide an approximation of the composite action between the slabs and the frame beams. The T-beam approach can also not consider the effect of a flexible shear connection between the slabs and the frame beams. Neither of the nonlinear analyses in Refs. 1.12 and 1.13 consider the composite plastic moments of the floors.

References 1.14 to 1.16 present finite element analyses of three-dimensional multistory buildings. The whole building including walls, floor slabs and beams is divided into finite elements and the nodal displacements then determined by the stiffness method. This approach however has all the problems associated with a standard composite frame analysis (Art. 1.4) and will therefore not be used in this dissertation.
Reference 1.17 and 1.18 present an interesting approach to the analysis of buildings composed of floor slabs, shear walls and unbraced frames. It is assumed that the buildings can be separated into two structural systems namely the steel structure and a vertical grillage comprising the floor slabs and shear walls. The two systems are analyzed separately while still satisfying displacement compatibility at the joints. Reference 1.18 also includes an elastic-plastic analysis.

The approach described above has the disadvantage that it cannot be applied to a structure which has no shear walls such as the composite frame. In addition the basic concept of separating the steel structure from the floor slabs ignores composite action between the slabs and the steel beams. This approach is therefore not suitable for the analysis of a composite frame.

Reference 1.19 presents a method whereby a slab and frame system is analyzed using the force method. The system is released at the top and bottom of the columns, the flexibility coefficients for the columns and slabs determined and the unknown forces at the releases obtained by satisfying displacement compatibility. The flexibility coefficients of a slab are obtained by solving the governing differential equation for plate bending using finite differences.

The approach described above is essentially the method of substructures and its disadvantages were discussed in Art. 1.4. In addition the finite difference method has its own problems and disadvantages (Art. 4.3.2). The approach of Ref. 1.19 will therefore not be used in this dissertation.
When the three-dimensional building frame has both symmetric geometry and loading then the structure can be reduced to an equivalent plane frame. In this process equivalent slab widths are determined for the floor slabs. (1.20-1.22) In Refs. 1.20 and 1.21 equivalent slab widths are determined for slabs subjected to concentrated moments as applied by the columns. In Ref. 1.22 equivalent slab widths are determined for slabs connecting shear walls.

The equivalent slab widths as determined above do not involve any beams and consequently no composite behavior and are therefore not applicable to a composite frame. Furthermore, no attempt is made to determine the effect of concrete cracking on the equivalent slab width. However, the concept of an equivalent slab width under the action of lateral loads is important since it will also form the basic approach to be used in this dissertation to represent the floor system of a composite frame.

All the references quoted so far have either inadequately treated or completely neglected composite action between the slab and the steel beams. None of the references discussed the effect of a flexible shear connection between the slab and the steel beams. Composite beams with flexible shear connection were studied in Refs. 1.23 to 1.25. The composite beam is treated as a two-dimensional member to set up the governing differential equation or the equivalent set of simultaneous equations. Because of the two-dimensional approach this method can not be applied to three-dimensional composite floors.

References 1.26 and 1.27 present finite element treatments of
composite action in beam-slab systems. In Ref. 1.26 special linkage elements are introduced to simulate the shear connectors. However, the additional linkage elements require additional nodes leading to a larger bandwidth and size for the total stiffness matrix. This method will consequently not be used.

In Ref. 1.27 the effect of composite action between the slab and a steel beam is considered by deriving an equivalent stiffness matrix for the steel beam. The stiffness matrix is however only valid for a rigid shear connection between the slab and the steel beam. In Ch. 3 of this dissertation this stiffness matrix is extended to also include the effect of a flexible shear connection between the slab and the steel beam.

Regarding experimental work the results reported in Refs. 1.28 and 1.29 are significant. Reference 1.28 reports the results of an experimental investigation that was conducted on a small scale four story building. The model was tested with and without the concrete floors in position. There was no mechanical shear connection between the slabs and the steel beams. Composite action was due to friction only caused by the dead weight of the slabs. The test results showed that the presence of the concrete slabs increased the lateral stiffness of the steel frame by 67 percent. The results of the investigation in Ref. 1.29 showed the same effect although an increase of only 15 percent was observed.
2. FACTORS AFFECTING THE BEHAVIOR OF COMPOSITE FRAMES

2.1 Introduction.

The behavior of a composite frame under combined gravity and wind loads is represented by its load-drift curve. Figure 2.1 shows an assumed load-drift curve for a composite frame. The lateral load $H$ is plotted against the horizontal deflection or drift $\Delta$ at the top of the frame.

Two portions of the load-drift curve are of prime importance. The first is the drift at service loads which is used to determine the comfort of the occupants. The second is the peak value of the curve called the maximum strength of the composite frame. The maximum strength determines the safety against overloads. The maximum strength must be greater than or equal to the service load times a certain factor called the load factor.

Several factors affect the maximum strength and service load drift of a composite frame and will be discussed in the next sections. These factors will be included in subsequent chapters where the maximum strength and stiffness of composite frames are determined.

2.2 Cracking and Crushing of Plain Concrete

2.2.1 Uniaxial Behavior.

Figure 2.2a shows the uniaxial behavior of plain concrete. In this figure $\sigma_1$ is the stress in the concrete and $\varepsilon$ the corre-
sponding strain. The maximum compressive stress is $f'_c$ and the maximum tensile stress is $f_t$.

It can be seen from Fig. 2.2a that concrete does not maintain its maximum strength as the strain increases. The maximum stress $f'_c$ can therefore not be used for ultimate strength design. For the purpose of ultimate strength design an elastic-plastic behavior is assumed with the concrete having a maximum strength of $0.85f'_c$ as shown in the figure. (2.2)

When a multistory building as shown in Fig. 1.1 (Art. 1.2) is subjected to lateral loads only, then all the floors are essentially subjected to uniaxial bending except for regions close to the columns. The uniaxial bending results in uniaxial stresses in the concrete slabs. In this case the stress-strain curve of Fig. 2.2a is applicable and the concrete is assumed to have a maximum strength of $0.85f'_c$. This result is used in Ch. 5 for part of the maximum strength analysis of composite one-story assemblages.

The small tensile strength of concrete as shown in Fig. 2.2a is significant because it will lead to cracking at very early stages thereby completely losing its tensile strength. As a result the tensile strength of concrete is often neglected as will be done in this dissertation. The implication is that the stiffness of concrete members such as the reinforced concrete slabs of a composite frame will be underestimated at low loads.

2.2.2 Biaxial Behavior

Figure 2.2b shows the biaxial behavior of plain concrete. (2.3)
The stresses in two perpendicular directions are denoted by \( \sigma_1 \) and \( \sigma_2 \). Under biaxial compression the concrete strength can increase to a maximum value of \( 1.27f'_c \) in the presence of a stress ratio \( \sigma_2/\sigma_1 \) of 0.7. (2.3) The maximum strength under biaxial tension is essentially the same as under uniaxial tension.

A region of biaxial compression exists in the concrete slabs of composite frames at the beam-to-column connections. Under the action of lateral loads on the composite frame the columns press against the slabs thereby applying nearly concentrated loads to the slabs. Because of the continuity of the slab the region near the column is not free to expand sideways. This lateral confinement results in biaxial compression in the slab at a column. At the column face the concrete can reach its maximum biaxial compressive strength of \( 1.27f'_c \). (2.4, 2.5)

The effect of gravity loads on a composite frame is essentially to cause biaxial bending in the slabs. Biaxial bending creates biaxial stresses in the slab. These biaxial stresses are dominant in the upper stories of the building where the uniaxial stresses caused by the lateral loads (Art. 2.2.1) are comparatively small. In the lower stories the uniaxial stresses in the slabs are dominant.

2.2.3 Triaxial Behavior of Plain Concrete.

The triaxial behavior of concrete was studied in Refs. 2.6 and 2.7. Tests results have shown that both the maximum strength and ductility of concrete increase greatly under triaxial compression. Maximum strengths of 2 to 3 times \( f'_c \) have been obtained.
In composite frames subjected to lateral loads triaxial compression exists in the lower part of the slab at a beam-to-column connection. The lower part of the slab in addition to being confined by the column and adjacent concrete is also confined from below by the top flange of the frame beam. The concrete in this region is therefore under triaxial compression and will reach a strength higher than $1.27f'_c$ which can be obtained under biaxial compression (Art. 2.2.2). This result is also used in Art. 2.5.2 where the maximum strength of composite beam-to-column connections is discussed.

2.3 **Cracking of Reinforced Concrete Slabs**

2.3.1 **Schematic Model**

As noted in Art. 2.2.1 the role of lateral loads on a multi-story building is essentially to produce uniaxial stresses in the floor slabs. Of importance here is the uniaxial tension that is produced in the reinforced concrete slabs by the lateral loads. Because of the low tensile strength of the concrete the slabs will crack at relatively small loads. With increasing lateral loads more cracks form until the slab eventually consists of many cracks closely spaced. Additional cracking ceases when the reinforcing starts to yield at the cracks.

Figure 2.3a shows a reinforced concrete slab that is cracked under uniaxial tension. Under the action of tensile forces in the $x$-direction a series of cracks has formed in the $y$-direction. In subsequent work it will be assumed that the concrete has no ten-
sile strength. The reinforcement in the x-direction must therefore resist all the tensile forces.

Figure 2.3b shows the schematic model for a reinforced concrete slab with cracks. The slab is modeled as an orthotropic plate with different Young's moduli and Poisson ratios in the x- and y-directions. (2.8, 2.9) \( E_x \) and \( E_y \) are the Young's Moduli in the x- and y-directions respectively and \( \nu_{xy} \) and \( \nu_{yx} \) are the corresponding Poisson ratios.

2.3.2 Stress-Strain Relationship

Assuming that the slab is in a state of plane stress then the stress-strain relation for an orthotropic material is given by (2.8)

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
1 & \nu_{xy} & 0 \\
\nu_{xy} & n & 0 \\
0 & 0 & \frac{n-\nu_{xy}^2}{2(n+\nu_{xy})}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

where

\[
n = \frac{E_y}{E_x} = \frac{\nu_{xy}}{\nu_{yx}}
\]

For the cracked slab of Fig. 2.3a the material parameters are assumed as follows:

\( E_y = E_c \) = Young's modulus for plain concrete
\( \nu_{xy} = \nu_{c} \) = Poisson ratio for plain concrete
\( E_x = E_e \) = Equivalent Young's modulus for a cracked section
\( \nu_{yx} = \nu_{e} \) = Equivalent Poisson ratio for a cracked section.
Reference 2.10 presents equations to calculate \( E_c \). The value of \( v_c \) can be taken as 0.15. With concrete assumed to have no tensile strength the value of \( E_e \) is given by

\[
E_e = \rho_\lambda E
\]

where

\[
\rho_\lambda = \text{area of longitudinal reinforcement per unit cross-sectional area of the slab.}
\]

\[
E = \text{Young's modulus for steel reinforcement.}
\]

Using Eq. 2.2 the value of \( v_e \) is given by

\[
v_e = \frac{v_c}{E_c} \rho_\lambda E
\]

2.3.3 Effect on the Behavior of Composite Frames.

The equivalent modulus \( E_e \) as given by Eq. 2.3 is much smaller than the Young's modulus \( E_c \) for concrete for practical values of \( \rho_\lambda \). Consequently the axial stiffness of a cracked slab is much smaller than that of an uncracked slab. The bending stiffness of a composite steel-concrete section will therefore also be decreased by cracking although not by the same degree. The effect of cracking on the stiffness of a composite frame will even be less since only certain regions of the floor slabs are in tension.

2.4 Interaction Between the Slab and the Steel Beams.

2.4.1 Load-Slip Relationship of the Shear Connectors.

The interaction between the slab and the steel beams depends on the load-slip relationship of the shear connectors. In Ref. 2.11
the load-slip relationship of shear connectors is determined from tests on pushout specimens. Two empirical formulas are given for the load-slip relationship. For continuously loaded specimens the relationship is

\[ q = q_u (1 - e^{-18\Delta})^{2/5} \]  

where

- \( q \) = shear force in a shear connector (kip)
- \( q_u \) = maximum shear strength of a shear connector (kip)
- \( \Delta \) = relative slip between the slab and the steel beam (in)

For reloaded specimens the relationship is

\[ q = q_u \frac{80\Delta}{1 + 80\Delta} \]  

Equation 2.5 has a vertical tangent at zero load implying rigid shear connection between the slab and the steel beam. The rigid shear connection is due to the natural bond between the concrete slab and the steel beam. At a certain value of the applied load the natural bond between the slab and the steel beam is destroyed. The load-slip relationship for all subsequent load applications will then be that of Eq. 2.6 implying a flexible shear connection between the slab and the steel beam.

The effect of a flexible shear connection between the slab and the steel beam is to reduce the bending stiffness of a composite section. The stiffness of composite floors and consequently of a composite frame will likewise be affected. The effect of a flexible shear connection on the stiffness of a steel beam element is treated in Ch. 3.
A flexible shear connection however has no effect on the maximum strength of a composite section. Both Eqs. 2.5 and 2.6 lead to the same maximum shear strength $q_u$ at large values of the relative slip $\Delta$. The maximum strength of a composite frame will therefore not be affected by a flexible shear connection between the slab and the steel beams.

Reference 2.11 also gives an empirical formula for the maximum shear strength $q_u$ of a shear connector. The formula is

$$q_u = 0.5 \frac{a_c}{f_c} E_c$$

where

$$a_c = \text{area of a shear connector (in}^2\text{)}.$$  

Equation 2.7 is valid for both normal and lightweight concrete slabs.

2.4.2 Friction

An important parameter which is not included in Eqs. 2.5 or 2.6 is friction between the slab and the steel beam. Friction unlike natural bond is always present and increases both the maximum strength and stiffness of the shear connection. Friction between the slab and the steel beam is dependent on the coefficient of friction and the compressive force between the slab and the steel beam.

Two types of shear connection are used in composite design namely full shear connection and partial shear connection. (1.3) For any composite section the strength of the shear connection for partial shear connection is always less than that for full shear connection. The effect of friction would be to increase the
strengths of both types of shear connection. However, only in the case of partial shear connection will friction increase the maximum strength of the composite section and consequently of the composite frame.

The effect of friction between the slabs and the steel beams on the stiffness of a multistory building was reported in Ref. 1.28 (Art. 1.5). Because the tests in Ref. 1.28 were conducted in the linear elastic range no quantitative information is available on the effect of friction on the maximum strength of a multistory building. This aspect requires future research.

2.5 Composite Beam-to-Column Connections.

2.5.1 Introduction

An important factor contributing to the maximum strength and stiffness of composite frames is the composite beam-to-column connections. When a composite frame is subjected to lateral loads the columns apply concentrated moments to the composite floors at the beam-to-column connections. As a result peak moments exist in the composite floor at these locations. The strength and stiffness of the composite beam-to-column connections will therefore have significant bearing on the behavior of a composite frame.

Reference 2.4 and 2.5 present the results of an extensive investigation into the behavior of composite beam-to-column connections. The connections were tested under positive bending moment, that is, the concrete at the column is in compression. Figure 2.4 shows the test set-up that was used for testing composite beam-to-
column connections. Figure 2.5 shows typical moment-rotation curves obtained from the test results. In this figure curve A1 represents a connection without a shrinkage gap between the slab and the column face and curve A2 a connection with a shrinkage gap.

To prevent a shrinkage gap between the slab and column face the slab reinforcement was welded to the column flange. This procedure gave satisfactory results as shown by the difference in the initial slopes of the curves in Fig. 2.5. For practical purposes however, this procedure may be uneconomical and alternative measures such as a heavy band of reinforcement around the column need be investigated.

Several variables were investigated to determine their effects on the maximum strength, initial stiffness and ductility of composite beam-to-column connections. These were 1) concrete strength 2) slab thickness 3) slab width 4) a shrinkage gap between the column face and the slab 5) shear connector spacing near the column face 6) frame beam depth 7) formed metal deck slabs 8) lateral beams framing into the column and 9) repeated loads.

2.5.2 Maximum Strength

The test results showed that the maximum strength of a composite beam-to-column connection is mainly affected by the concrete strength, slab thickness, column face width, yield stress and depth of the frame beams and type of slab construction, that is, solid or formed metal deck slabs. With solid slab construction the composite beam-to-column connections exceeded the bare steel connection by

-22-
65 to 87 percent. With formed metal deck slab construction the increase in strength was from 54 to 61 percent.

The maximum strength of the composite beam-to-column connections was theoretically predicted through the use of upper and lower bounds.\(^{(2.4,2.5)}\) For the upper bound a failure mechanism was assumed as shown in Fig. 2.6a. The force \(P\) was determined by minimization of the internal dissipation of energy. The horizontal part of curve 4 in Fig. 2.5 represents the upper bound value.

For the lower bound a stress field was assumed as shown in Fig. 2.6b. In this figure \(t\) is the slab thickness, \(d\) the frame beam depth and \(f_y\) the yield stress of the frame beam. A maximum stress of \(1.3f'_c\) was used for the concrete in contact with the column face. The value of \(1.3f'_c\) was decided upon after consideration of the biaxial (Art. 2.2.2) and the triaxial (Art. 2.2.3) behavior of the concrete at this location. The horizontal part of curve 3 in Fig. 2.5 represents the lower bound value. In Ch. 5 extensive use is made of this lower bound value.

2.5.3 Stiffness

The stiffness of composite beam-to-column connections is mainly affected by slab thickness, frame beam size and a shrinkage gap at the column face. The effect of a shrinkage gap is essentially to decrease the initial stiffness of a connection as can be seen by comparing the initial slopes of the curves in Fig. 2.5. It is expected that shrinkage gaps will have the same effect on the stiffness of a composite frame.
Predicting the stiffness of a composite beam-to-column connection was found to be inconclusive. It was apparent that shear connector stiffness had to be accounted for. In an actual composite frame the full panel width contributes to the stiffness of the structure instead of the limited slab width of the test specimens. The stiffness of composite frames including the effect of a flexible shear connection is studied in Ch. 6.

2.5.4 Ductility

The ductility of a structure is usually given in terms of the ductility factor which is defined as the peak displacement divided by the yield displacement. A ductility factor of from 4 to 6 is usually recommended for buildings in earthquake areas. The ductility factors of all the connections tested lay between these values and it was therefore concluded that composite beam-to-column connections possess adequate ductility.

An essential requirement for the plastic design of multistory frames is that all the members must have adequate rotation capacity. Rotation capacity for a composite beam-to-column connection is defined in Fig. 2.7. It was concluded in Ref. 2.5 that composite beam-to-column connections possess adequate rotation capacity for plastic design to be applicable to composite frames.
2.6 Discontinuities

2.6.1 Discontinuity in the Concrete Floor Slab

In Art. 2.5.1 reference was made to a shrinkage gap between the reinforced concrete slab and the column. Figure 2.8a shows that shrinkage gaps occur all around the column. These shrinkage gaps constitute discontinuities in the concrete slab. Because these discontinuities occur in regions of peak bending moments their effect on the behavior of a composite frame may be significant.

The effects of the discontinuities in the concrete slab are shown in Fig. 2.8b. On the windward side of the column the shrinkage gap never closes so that the slab has a free edge on that side of the column. The width of this free edge is $B_c$, where $B_c$ is the column flange width. On this free edge the normal stress always remains zero. The effect of a free edge in the slab on the windward side of a column is considered in Chapters 5 and 6.

On the leeward side of the column the shrinkage gap closes and the slab applies a normal pressure to the column flange. Under this pressure the column flange will tend to bend out-of-plane as shown in Fig. 2.8b. The effect of this flange bending is included in the results of Ch. 4. In the analysis of composite frames as presented in Ch. 6 this flange bending is neglected by assuming that the column has rigid flanges.

2.6.2 Discontinuity in the Frame Beam Flanges

Another area of discontinuity is that of the frame beam flanges at the beam-to-column connections as shown in Fig. 2.9a.
Because the frame beam flanges are not continuous between the column flanges local distortions of the column flange occurs. An exact analysis of these local distortions of the column flange is very complicated and lies beyond the scope of this dissertation.

It is possible to overcome the discontinuity in the frame beam flanges by providing horizontal stiffeners as shown in Fig. 2.9b. The horizontal stiffeners are welded to both column flanges and the column web. There are two main objections against horizontal stiffeners. Increased cost is involved and the higher restraint now present in the welded regions provide a greater possibility of lamellar tearing. \(2.14\) In this dissertation it will be assumed that the frame beam flanges are continuous.

2.7 Additional Factors

There are several additional factors which affect the maximum strength and stiffness of composite frames. All have been treated in connection with steel frames and will only be referred to. There are the \(P\Delta\) forces and their effect on frame stability, \(2.13, 2.15\) yielding of steel and the formation of plastic hinges, \(2.13, 2.16\) strainhardening of steel \(2.13\) and the decrease in bending stiffness of columns due to the axial forces. \(2.17\) Other factors which are not considered in this dissertation are fatigue and fracture, \(2.18\) lamellar tearing \(2.14\) and local and lateral torsional buckling of members. \(2.15, 3.2\)
3. STIFFNESS MATRIX FOR THE BEAMS OF COMPOSITE STEEL-CONCRETE FLOORS.

3.1 Introduction

Figure 3.1 shows a typical detail of a composite steel-concrete floor. The steel beam of depth \(d\) is attached to the reinforced concrete slab of thickness \(t\) with headed steel stud shear connectors. In this figure \(c\) is the height of the shear connectors and \(p\) is the centerline distance between rows of connectors. The steel beam may either be a floor beam or a frame beam (see Fig. 1.2b).

In this chapter the stiffness matrix of a steel beam element of length \(L\) such as \(AB\) in Fig. 3.1 is derived. The eccentricity of the steel beam with respect to the concrete slab and the flexibility of the shear connectors are considered in the derivation of the stiffness matrix.

3.2 Assumptions

The derivation of the stiffness matrix for the steel beam element \(AB\) shown in Fig. 3.1 is based on the following assumptions:

1) The steel beam is prismatic.
2) The shear connectors transmit all forces between the slab and the steel beam.
3) The steel beam and the shear connectors behave linearly elastic and are homogeneous and isotropic.
4) Plane sections in the slab and steel beam before deformation remain plain and parallel after deformation.
5) All deflections are small.

6) The weak axis bending stiffness of the steel beam is neglected.

7) The slab and steel beam remain in contact.

Assumption 1 precludes non-prismatic beams from the analysis. Although non-prismatic beams can be considered with substantial additional effort, such generalization is not required for typical rolled steel beams.

Assumption 2 neglects the effects of bond and friction between the slab and the steel beam (Art. 2.4.2) on the shear connection forces. The stiffness of the shear connection is therefore underestimated, resulting in an underestimation of the bending stiffness of the composite section.

Assumption 2 also implies that torsion between the slab and the steel beam is transmitted only by differential elongation of the shear connectors. Bearing between the top flange of the steel beam and the slab during torsion transfer is neglected. As a result the torsional stiffness of the shear connection and consequently of the composite section is underestimated.

Assumption 3 neglects yielding of the steel beam. In addition the nonlinear load-slip behavior of the shear connectors (Art. 2.4.1) is ignored. The effect of this assumption is to overestimate the stiffness of a composite section at high loads.

Assumption 4 implies that plane sections in the slab and steel beam which lie in the same vertical plane may undergo relative horizontal displacement. Relative slip between the slab and steel beam
is therefore permissible.

Assumption 4 also implies that the steel beam does not warp under torsion. For warping to occur the flanges of the steel beam must be able to bend in their own planes. (3.2) Since the top flange is connected to the slab by the shear connectors, in-plane bending of this flange is prevented. Warping of the steel beam is therefore to a large degree inhibited. (3.3)

Assumption 5 enables curvatures to be computed from second derivatives of deflections.

Assumption 6 can be justified on the basis that the bending stiffness of the steel beam about its weak axis is very small in comparison with the in-plane bending stiffness of the slab.

Assumption 7 implies that the slab and steel beam have everywhere the same deflection and curvature. Although the extensibility of the shear connectors will cause partial separation between the slab and the steel beam, this implication is considered to be true if at least one edge of the top flange remains in contact with the slab.

3.3 Schematic Models

To determine the stiffness matrix of steel beam element AB in Fig. 3.1 it is necessary to establish schematic models representing the actual beam and shear connection. Subject to the assumptions of Art. 3.2 the schematic models need only represent the axial stiffness, bending stiffness about the strong axis and St. Venant torsional stiffness of the steel beam.
3.3.1 Model having Axial and Bending Stiffness

Figure 3.2a shows a schematic model of the steel beam and shear connection. The beam is modeled by a large number of horizontal springs of which three only are shown. The springs are distributed throughout the depth d to represent the axial and bending stiffness of the steel beam. The combined axial stiffness of all these springs is $K_s$, a quantity which is determined later (Art. 3.8).

The shear connection is modeled by a single spring of stiffness $K_c$ as shown in Fig. 3.2a. The value of $K_c$ is determined later (Art. 3.8). The force in this spring is proportional to the relative horizontal displacement or slip between the two rigid links. These rigid links represent plane sections in the slab and steel beam and thus conform to assumption 4 (Art. 3.2). By the same assumption these links always remain parallel to each other.

In subsequent work all forces and displacements will be referred to a reference plane which is taken as the middle surface of the slab as shown in Fig. 3.2a. An orthogonal system of coordinate axes located in the reference plane will also be used. Positive x and z are as shown in Fig. 3.2a. The direction of positive y is 90° anticlockwise from positive x in the reference plane. Forces and displacements are positive when in the direction of positive coordinate axes. Rotations follow the right hand rule.
3.3.2 Model having St. Venant Torsional Stiffness

Figure 3.2b shows another schematic model of the steel beam and shear connection. The beam is modeled by a torsional spring of stiffness $J_s$ and the shear connectors by a torsional spring of stiffness $J_c$. Each spring has only St. Venant torsional stiffness and a length $\ell$ equal to that of element AB (Fig. 3.1). The values of $J_s$ and $J_c$ are determined later (Art. 3.9).

In Fig. 3.2b the torsional springs are placed in series. This complies with the actual situation, that is, the slab applies a torsional moment to the shear connectors which in turn applies the same torsional moment to the steel beam. The rigid links in Fig. 3.2b serve only to separate the torsional springs and clarify the model.

3.4 Displacements of the Steel Beam

The steel beam as modeled in Fig. 3.2a has three degrees of freedom namely displacements in the x- and z- directions and a rotation about the y-axis. These three displacements can be written in terms of the corresponding displacements $u$, $w$ and $\theta_y$ of the reference plane. By assumption 7 (Art. 3.2) the displacement of the beam in the z-direction is equal to $w$. By assumption 4 (Art. 3.2) the rotation of the beam about the y-axis is equal to $\theta_y$. To determine the displacement of the beam in the x-direction the contributions from $u$, $w$ and $\theta_y$ of the reference plane will be investigated.
3.4.1 Due to Displacement \( u \) of the Reference Plane

Figure 3.3a shows the displacements caused by a displacement \( u \) of the reference plane. In this figure \( \Delta \) is the relative horizontal displacement between the two rigid links. At any distance \( z \) below the reference plane the displacement in the \( x \)-direction is constant and equal to \( u_z \). In particular the value of \( u_z \) at the centroidal axis of the steel beam located a distance \( \bar{y} \) below the reference plane is \( \bar{u} \). The remaining symbols in the figure have previously been defined.

Referring to Fig. 3.3.a, the force \( Q \) in spring \( K_c \) is given by

\[
Q = K_c \Delta = K_c (u - u_z) \tag{3.1}
\]

The axial force in the steel beam is also equal to \( Q \) and is proportional to \( \bar{u} \), as follows:

\[
Q = K_s \bar{u} \tag{3.2}
\]

Since

\[
\bar{u} = u_z
\]

then

\[
Q = K_s u_z \tag{3.3}
\]

From Eqs. 3.1 and 3.3 the displacement \( u_z \) is given by

\[
u_z = \frac{K_c}{K_c + K_s} u \tag{3.4}
\]

Using the notation

\[
K' = \frac{K_c}{K_c + K_s} \tag{3.5}
\]

then Eq. 3.4 becomes

\[
u_z = K'u \tag{3.6}
\]
3.4.2 Due to Rotation $\Theta_y$ of the Reference Plane

Figure 3.3b shows the displacements caused by a rotation $\Theta_y$ of the reference plane. All symbols used in this figure have previously been defined.

The force $Q$ in spring $K_c$ is again given by

$$Q = K_c \Delta$$  \hspace{1cm} 3.7

The displacement $\bar{u}$ is equal to

$$\bar{u} = -\bar{y} \Theta_y + \Delta$$  \hspace{1cm} 3.8

Since the axial force in the steel beam is also equal to $Q$, this gives

$$Q = -K_s \bar{u} = K_s (\bar{y} \Theta_y - \Delta)$$  \hspace{1cm} 3.9

From Eqs. 3.7 and 3.9 the displacement $\Delta$ is given by

$$\Delta = \frac{K_s}{K_c + K_s} \bar{y} \Theta_y$$  \hspace{1cm} 3.10

Let

$$K'' = \frac{K_s}{K_c + K_s}$$  \hspace{1cm} 3.11

Then Eq. 3.10 becomes

$$\Delta = K'' \bar{y} \Theta_y$$  \hspace{1cm} 3.12

Note that from Eqs. 3.5 and 3.11

$$K' + K'' = 1$$  \hspace{1cm} 3.13

From Fig. 3.3b the displacement $u_z$ is given by

$$u_z = -z \Theta_y + \Delta$$

and with $\Delta$ from Eq. 3.12 gives

$$u_z = (-z + K'' \bar{y}) \Theta_y$$  \hspace{1cm} 3.14
3.4.3 Due to Displacement \( w \) of the Reference Plane

Figure 3.4a shows the displacements caused by a displacement \( w \) of the reference plane. Because of assumption 5 (Art 3.2) this displacement does not cause any significant displacement in the \( x \)-direction in the steel beam and requires no further consideration.

3.4.4 Total Displacement in the \( x \)-direction

The total displacement of the steel beam in the \( x \)-direction is obtained by adding \( u \) from Eqs. 3.6 and 3.14. This gives

\[
\begin{align*}
    u_z &= K'u + (-z + K''y) \theta_y \\
    & \quad \text{(3.15)}
\end{align*}
\]

The axial strain and stress at any level in the steel beam can be calculated from Eq. 3.15 (Art. 3.6).

3.5 Twisting of the Steel Beam

Figure 3.4b shows the rotation \( \Theta_{xS} \) of the steel beam caused by a rotation \( \Theta_x \) of the reference plane. It is assumed that the angle of twist per unit length \( \phi \) in each of the two torsional springs varies linearly. Consequently the angle of twist per unit length \( \phi_c \) in the shear connector spring \( J_c \) is equal to

\[
\phi_c = \frac{\Theta_x - \Theta_{xS}}{\lambda} \quad \text{(3.16)}
\]

Similarly the angle of twist per unit length \( \phi_s \) in the steel beam is given by

\[
\phi_s = \frac{\Theta_{xS}}{\lambda} \quad \text{(3.17)}
\]

-34-
The torsional moment $T$ in spring $J$ is therefore equal to
\[
T = J_c \phi_c = \frac{J_c}{\ell} (\Theta_x - \Theta_{xs}) \quad 3.18
\]
The torsional moment in the steel beam is also equal to $T$.

Therefore
\[
T = J_s \phi_s = \frac{J_s}{\ell} \Theta_{xs} \quad 3.19
\]

From Eqs. 3.18 and 3.19 the rotation $\Theta_{xs}$ is given by
\[
\Theta_{xs} = \frac{J_c}{J_c + J_s} \Theta_x \quad 3.20
\]

Let
\[
J' = \frac{J_c}{J_c + J_s} \quad 3.21
\]

Then Eq. 3.20 becomes
\[
\Theta_{xs} = J' \Theta_x \quad 3.22
\]

From Eqs. 3.19 and 3.22 the torsional moment $T$ is given by
\[
T = \frac{J'J_s}{\ell} \Theta_x \quad 3.23
\]

By using the notation
\[
J'_s = J'J_s \quad 3.24
\]

then Eq. 3.23 becomes
\[
T = \frac{J'_s}{\ell} \Theta_x \quad 3.25
\]

This equation for $T$ will be used directly in the stiffness matrix of the steel beam element (Art. 3.7).

3.6 Stress Resultants at the Reference Plane

The axial strain $\varepsilon$ in the steel beam is given by
\[
\varepsilon_x = \frac{\partial u_z}{\partial x}
\]
and taking \( u \) from Eq. 3.15 this becomes

\[
\varepsilon_x = K' \frac{\partial u}{\partial x} + (- z + K'' \overline{y}) \frac{\partial \theta}{\partial x} \tag{3.26}
\]

Because of assumption 5 (Art. 3.2) the rotation \( \theta \) can be written as \( \theta = \frac{\partial w}{\partial x} \). Equation 3.26 now becomes

\[
\varepsilon_x = K' \frac{\partial u}{\partial x} + (- z + K'' \overline{y}) \frac{\partial^2 w}{\partial x^2} \tag{3.27}
\]

The axial stress \( \sigma_x \) in the steel beam is equal to \( \sigma_x = \varepsilon_x E \)

where \( E \) is the Young's modulus of steel. Therefore

\[
\sigma_x = E K' \frac{\partial u}{\partial x} + E (- z + K'' \overline{y}) \frac{\partial^2 w}{\partial x^2} \tag{3.28}
\]

3.6.1 Force \( N_s \) in the \( x \)-direction

The axial force \( N_s \) in the steel beam referred to the reference plane is

\[
N_s = \int_{t/2}^{t/2 + d} \sigma_x dA_s \tag{3.29}
\]

where

\[
A_s = \text{area of the steel beam}
\]

Substituting Eq. 3.28 into 3.29 gives

\[
N_s = \int_{t/2}^{t/2 + d} \left[ E K' \frac{\partial u}{\partial x} + E (- z + K'' \overline{y}) \frac{\partial^2 w}{\partial x^2} \right] dA_s
\]

or

\[
N_s = E K' A_s \frac{\partial u}{\partial x} + E (- S_x + K'' S_x) \frac{\partial^2 w}{\partial x^2} \tag{3.30}
\]

where

\[
S_x = \text{statical moment of the steel beam area about the reference plane}
\]

From Eq. 3.13 \( K'' = 1 - K' \) and using the notations
\[ A'_s = K' A_s \] \hspace{1cm} 3.31

\[ S'_x = K' S_x \] \hspace{1cm} 3.32

then Eq. 3.30 can be written as

\[ N_s = E A'_s \left( \frac{\partial u}{\partial x} - E S'_x \frac{\partial^2 w}{\partial x^2} \right) \] \hspace{1cm} 3.33

Except for the primes this equation is the same as Eq. 8a of Ref. 1.27.

### 3.6.2 Moment \( M \) about the \( y \)-axis

The moment \( M_s \) about the \( y \)-axis in the reference plane as caused by the axial stress \( \sigma_x \) in the steel beam is given by

\[ M_s = \int_{t/2}^{t/2 + d} \sigma_x z \, dA_s \] \hspace{1cm} 3.34

Using \( \sigma_x \) from Eq. 3.28 this gives

\[ M_s = \int_{t/2}^{t/2 + d} \left[ E K'_x \left( \frac{\partial u}{\partial x} + E \left( -z + K'' y \right) z \right) \frac{\partial^2 w}{\partial x^2} \right] dA_s \]

that is

\[ M_s = E K'_x S_x \frac{\partial u}{\partial x} + E \left( -I_x + K'' \overline{y} \right) S_x \frac{\partial^2 w}{\partial x^2} \] \hspace{1cm} 3.35

where

\[ I_x = \text{moment of inertia of the steel beam area about the reference plane} \]

The value of \( I_x \) is given by

\[ I_x = I_o + A_s \overline{y}^2 \] \hspace{1cm} 3.36

where

\[ I_o = \text{moment of inertia of the steel beam about its centroidal axis} \]
Also
\[ S = A \bar{y} \quad 3.37 \]

With the aid of Eqs. 3.13, 3.31, 3.32, 3.36 and 3.37, Eq. 3.35 may be rewritten as
\[ M_s = E S' \frac{\partial u}{\partial x} - E (I_o + A' \bar{y}) \frac{\partial^2 w}{\partial x^2} \quad 3.38 \]

Using the notation
\[ I'_x = I_o + A' \bar{y} \quad 3.39 \]

then Eq. 3.38 becomes
\[ M_s = E S' \frac{\partial u}{\partial x} - E I'_x \frac{\partial^2 w}{\partial x^2} \quad 3.40 \]

Except for the primes this equation is the same as Eq. 8b of Ref. 1.27.

3.7 Stiffness Matrix

Further development to obtain the stiffness matrix of the steel beam element AB follows exactly the procedure set out in Ref. 1.27 using Eqs. 3.33 and 3.40. In its final form the stiffness matrix is given by Eq. 3.41 with the nodal forces and displacements as shown in Fig. 3.5. The only parameter not previously defined is
\[ \gamma = \frac{1}{E} \quad 3.42 \]

In the case of complete composite action, that is, when \( K_c \) and \( J_c \) become infinite, then from Eqs. 3.5 and 3.21 \( K'=J'=1 \). In this case all the primed quantities in Eq. 3.41 reach their full values and the stiffness matrix is the same as that of Ref. 1.27.

In the case of non-composite action, that is, when \( K_c \) and \( J_c \)
\[
\begin{bmatrix}
N_1 \\
V_1 \\
T_1 \\
M_1 \\
N_j \\
V_j \\
T_j \\
M_j
\end{bmatrix}
= E
\begin{bmatrix}
\frac{A_s'}{\ell} & 0 & 0 & \frac{-S_x'}{\ell} & \frac{-A_s'}{\ell} & 0 & 0 & \frac{S_x'}{\ell} \\
0 & \frac{12I_x'}{\ell^3} & 0 & \frac{6I_x'}{\ell^2} & 0 & -\frac{12I_x'}{\ell^3} & 0 & \frac{6I_x'}{\ell^2} \\
0 & 0 & \frac{-\gamma J_y'}{\ell} & 0 & 0 & 0 & -\frac{\gamma J_y'}{\ell} & 0 \\
-\frac{S_x'}{\ell} & \frac{6I_x'}{\ell^2} & 0 & \frac{4I_x'}{\ell} & \frac{S_x'}{\ell} & -\frac{6I_x'}{\ell^2} & 0 & \frac{2I_x'}{\ell} \\
-\frac{A_s'}{\ell} & 0 & 0 & \frac{S_x'}{\ell} & \frac{A_s'}{\ell} & 0 & 0 & -\frac{S_x'}{\ell} \\
0 & \frac{-12I_x'}{\ell^3} & 0 & \frac{-6I_x'}{\ell^2} & 0 & \frac{12I_x'}{\ell^3} & 0 & \frac{-6I_x'}{\ell^2} \\
0 & 0 & \frac{-\gamma J_y'}{\ell} & 0 & 0 & 0 & \frac{\gamma J_y'}{\ell} & 0 \\
\frac{S_x'}{\ell} & \frac{6I_x'}{\ell^2} & 0 & \frac{2I_x'}{\ell} & \frac{-S_x'}{\ell} & \frac{-6I_x'}{\ell^2} & 0 & \frac{4I_x'}{\ell}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
w_1 \\
\theta_{xi} \\
\theta_{yi} \\
u_j \\
w_j \\
\theta_{xj} \\
\theta_{yj}
\end{bmatrix}
\]

3.41
are both equal to zero, then \( K' = J' = 0 \). In this case the stiffness matrix of Eq. 3.41 reduces to that of a concentric beam element under bending moments and shear forces only.

3.8 Evaluation of the Stiffnesses \( K_s \) and \( K_c \)

The stiffnesses \( K_s \) and \( K_c \) in Fig. 3.2a are not constant but depend on which section of element AB in Fig. 3.1 is under consideration. These values therefore vary along the length of the beam element. It will be assumed that the values of \( K_s \) and \( K_c \) can be determined on the basis of the full length of the element. Consequently the axial stiffness \( K_s \) is given by

\[
K_s = \frac{AE}{L} \tag{3.43}
\]

The shear stiffness \( K_c \) can be determined by referring to Fig. 3.6a. This figure shows a relative horizontal slip \( \Delta \) between the slab and the steel beam. It will be assumed that \( \Delta \) varies linearly along the length of the beam element. An even spacing of the shear connectors will also be assumed. The total shear force \( Q \) between the slab and the steel beam element is equal to

\[
Q = N \frac{k_c \Delta}{2} \tag{3.44}
\]

where

\[
N = \text{number of shear connectors on the steel beam element}
\]

\[
k_c = \text{initial shear stiffness of a shear connector.}
\]

The value of \( k_c \) can be obtained from Eq. 2.6 (Art. 2.4.1) by
taking the derivative of \( q \) with respect to \( \Delta \) and then setting \( \Delta \) equal to zero. This gives

\[
k_c = 80 \frac{q_u}{\text{kip/in}}
\]

(3.45)

Using the value of \( q_u \) from Eq. 2.7 (Art. 2.4.1) then Eq. 3.44 becomes

\[
Q = \left( 20 N_a \frac{1}{c} \sqrt{\frac{f'}{E}} \right) \Delta
\]

(3.46)

Since by Eq. 3.1 (Art. 3.4.1) \( Q = K_c \Delta \), therefore

\[
k_c = 20 N_a \frac{1}{c} \sqrt{\frac{f'}{E}}
\]

(3.47)

All units must be in kips and inches.

3.9 **Evaluation of the Stiffnesses \( J_s \) and \( J_c \)**

The torsional stiffness \( J_s \) of the steel beam is given by

\[
J_s = GJ
\]

(3.48)

where

- \( G \) = shear modulus of steel
- \( J \) = St. Venant torsion constant of the steel beam

The torsional stiffness \( J_c \) of the shear connectors can be determined by referring to Fig. 3.6b. This figure shows a relative rotation \( \theta \) between the slab and the steel beam. By assumption 2 (Art. 3.2) rotation is considered to occur about point \( Q \) midway between the rows of shear connectors and the torsional moment is proportional to the relative elongation of the shear connectors.
From Fig. 3.6b the relative elongation $\delta c$ of each shear connector is

$$\delta c = \frac{P}{2c} \theta_x$$

Assuming that this elongation occurs over the whole length of a shear connector, then the corresponding force $F$ in each connector is equal to

$$F = \frac{c E p}{2c} \theta_x$$

For an average rotation of $\frac{1}{2} \theta_x$ the value of $F$ becomes

$$F = \frac{a c E p}{4c} \theta_x$$

The torsional moment $T_o$ due to each pair of shear connectors is equal to

$$T_o = F p = \frac{a c E p^2}{4c} \theta_x$$

If there are $N_p$ pairs of shear connectors then the total torsional moment $T$ is given by

$$T = \frac{a c E N_p}{4c} \theta_x$$

Equation 3.49 may be rewritten as

$$T = \left(\frac{a c E N_p}{4c}\right) \theta_x$$

or

$$T = \left(\frac{a c E N_p}{4c}\right) \phi_c$$

where

$$\phi_c = \text{angle of twist per unit length in the shear connectors}$$

From Eq. 3.18 (Art. 3.5)

$$T = J_{c} \phi_c$$

therefore
In Ref. 3.1 rotation in Fig. 3.6b was assumed to occur about point R resulting in a point reaction at that point. Since the contact stress at point R would be infinite it would require the assumption of a rigid slab and rigid steel beam flange. Such an approach over-estimates the torsional stiffness of the shear connection and was consequently not used in this dissertation.

3.10 The Effect of Flexible Shear Connection

The effect of flexible shear connection on the elements of the stiffness matrix of Eq. 3.41 is embodied in the nondimensional variables $K'$ and $J'$ as given by Eqs. 3.5 and 3.21. This can be seen from the values of $J'_s$, $A'_s$, $S'_x$ and $I'_x$ as given by Eqs. 3.24, 3.31, 3.32 and 3.39 respectively.

Table 3.1 shows the effect of flexible shear connection on the values of $A'_s$, $S'_x$, $J'_s$ and $I'_x$. These values have been nondimensionalized with respect to the corresponding values for a rigid shear connection. The variables investigated are beam size, beam element length, concrete strength, number of shear connectors, distance between rows of shear connectors and connector length. It is apparent that flexible shear connection has the greatest effect on $A'_s$ and $S'_x$ while $J'_s$ is the least effected.

As far as composite action is concerned it is probably the value of $I'_x$ which will have the greatest effect. From Table 3.1
it can be seen that $I'_x$ can be as small as 40% of the full composite value. The reduction in the bending stiffness of a composite section due to flexible shear connection may therefore be substantial.
4. STIFFNESS ANALYSIS OF COMPOSITE ONE-STORY ASSEMBLAGES

4.1 Introduction.

Figure 4.1 shows a symmetrical composite one-story assemblage. This structure is obtained by making two horizontal cuts through the composite frame of Fig. 1.2b just above each of two consecutive floors. In Fig. 4.1 h is the story height, L₁ and L₂ are the span lengths, B is the panel width and Bₖ is the average column flange width in the story. Any number of bays and any distribution of floor beams are permissible but the assemblage is assumed to be symmetrical about a vertical plane along the column centerline. The directions of the x-, y- and z-axes are as shown in the figure.

Figure 4.1 also shows the vertical forces P₁, P₂ and P₃, the horizontal forces Q₁, Q₂ and Q₃ and the moments M₁, M₂ and M₃ which act on the composite assemblage. These forces correspond to the axial force, shear force and bending moment in each column at the top of the slab caused by the combined gravity and wind loads on the multistory building. All other loads on the assemblage such as the self weight of the floor and superimposed live loads on the floor can be included.

In this chapter a method is described to perform a structural analysis of the composite assemblage of Fig. 4.1. Attention will focus on the horizontal deflection (lateral drift) of the assemblage caused by the forces shown in Fig. 4.1.

4.2 Assumptions.

The structural analysis of the composite assemblage of Fig. 4.1
is based on the following assumptions in addition to those of Art. 3.2:

1) Beam and column lengths are measured from center line to center line.

2) Steel beam and column flanges are fully continuous at the beam-to-column connections.

3) The reinforced concrete slab is idealized as a thin orthotropic plate having linear elastic behavior.

4) The steel columns are idealized as two dimensional members lying in the y-z plane, having linear elastic behavior and all the stiffness properties of the actual columns.

Assumption 1 has the effect of assigning lengths to the members which are greater than those of the actual structure. The stiffness of the members and consequently of the composite assemblage are therefore underestimated.

Assumption 2 is made to preclude the effect of discontinuities in the beam and column flanges (Art. 2.6.2). This assumption is valid if the columns are continuous and horizontal stiffeners are used between the column flanges at the level of the beam flanges (Fig. 2.9b).

Assumption 3 enables classical thin plate theory to be used for the analysis of the reinforced concrete slab. (4.1) This assumption also ignores the effects of cracking and crushing of the concrete and yielding of the reinforcement on the bending and axial stiffness of the slab.

By assumption 4 the column length and flange width but not
the depth of the section will be considered in the analysis. Neglecting the column depth is not expected to have a significant bearing on the results.

4.3 Analytical Method for the Floor Slab.

4.3.1 Governing Differential Equations.

Under the action of the forces in Fig. 4.1 the floor slab of the composite assemblage is forced into bending. Because of the eccentricity of the steel beams with respect to the slab the latter is also subjected to in-plane stresses. On the basis of the assumption of small deflections (Art. 3.2) the bending and in-plane behavior of the slab can be assumed to be uncoupled. In this case the governing differential equation for orthotropic plate bending in the absence of gravity loads is (4.1).

\[
D_x \frac{\partial^4 w}{\partial x^4} + 2(D_1 + 2 D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = 0 \tag{4.1}
\]

where \( w \) is the vertical deflection of the plate and

\[
\begin{align*}
D_x &= \frac{E_x t^3}{12} \\
D_y &= \frac{E_y t^3}{12} \\
D_1 &= \frac{\nu_{xy} E_y t^3}{(n-\nu_{xy})^2 12} \\
D_{xy} &= \frac{G_{xy} t^3}{12} \\
n &= \frac{E_x}{E_y} = \frac{\nu_{xy}}{\nu_{yx}}
\end{align*}
\]

\( t = \) plate thickness
$E_x$, $E_y$, $\nu_{xy}$ and $\nu_{yx}$ are the Young's Moduli and Poisson ratios in the $x$- and $y$- directions. The governing differential equation for in-plane behavior of the slab is given by

$$\frac{1}{E_x} \frac{\partial^4 \phi}{\partial y^4} + \left(\frac{1}{E_y} - \frac{2\nu_{xy}}{G_{xy}}\right) \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{1}{E_y} \frac{\partial^4 \phi}{\partial x^4} = 0 \quad 4.2$$

where

$$\phi = \text{Airy stress function.} (4.2)$$

Equation 4.2 is derived in the Appendix.

Equations 4.1 and 4.2 must be solved simultaneously. Because of the eccentric floor and frame beams, of the flexible shear connection between the slab and of the steel beams and of discontinuities in the slab (Art. 2.6) a closed form solution of Eqs. 4.1 and 4.2 is practically impossible. It is therefore necessary to resort to numerical methods.

4.3.2 Finite Difference Method.

The finite difference method is still widely used to solve differential equations.\(^{(4.3, 4.4, 4.5)}\) However the presence of discontinuities in the slab (Art. 2.6) and flexible shear connection between the slab and steel beams provides considerable difficulties for this method. Irregular meshes as will be extensively used in this study create further complications.\(^{(4.6)}\) It has also been shown that the finite element method gives a much better approximation of the original continuum system than the finite difference method.\(^{(4.7)}\) For these reasons the finite difference method will not be used in this study.
4.3.3 **Finite Strip Method.**

The finite strip method is a modified version of the finite element method.\(^{(4.10,4.11,4.12)}\) It is more efficient than the finite element method for a certain class of problems. However it cannot be applied when local discontinuities in the slab (Art. 2.6) are present. The plate must also be homogeneous since the same moment-curvature relationship is assumed to hold throughout the strip. The finite strip method is consequently not as versatile as the finite element method and is therefore not used in this study.

4.3.4 **Finite Element Method.**

The finite element method is already well documented.\(^{(4.8,4.9)}\) It is a versatile method and can be applied with ease to a wide variety of problems. Any type of boundary condition can be satisfied by simply selecting a suitable finite element. Also the problems mentioned in connection with the finite difference method do not occur when the finite element method is used. For these reasons the finite element method was selected for the numerical solution of Eqs. 4.1 and 4.2.

4.4 **Selection of a Finite Element for the Slab.**

4.4.1 **Factors to be Considered.**

Because of the variety of finite elements available the selection of a finite element should be made with care. The following factors should be considered:
1) **Required Accuracy.**

Certain structures require much greater accuracy of the analytical results than others. Some elements provide better accuracy than others.

2) **Representation of the Actual Continuum.**

In the actual continuum displacements and slopes are everywhere continuous. Certain elements are non-conforming, that is, along the interelement boundaries transverse slopes are not continuous.

3) **Boundary Conditions.**

At the boundaries of a structure strains or displacements may be specified. Some elements can satisfy only displacement boundary conditions. Others, called higher order elements can also satisfy boundary conditions on strain.

4) **Computational Effort.**

A relative measure of the time required to solve the set of simultaneous equations is the computational effort. This is defined as

\[
\text{Computational Effort} = N \times B^2
\]

where

- \(N\) = size of the stiffness matrix
- \(B\) = semi-bandwidth of the stiffness matrix

The higher order elements often lead to larger values of \(B\) but usually cut down on the magnitude of \(N\).

4.4.2 **Finite Element Methods.**

There are essentially four methods for deriving the stiffness
matrix of a finite element: \((4.9, 4.14, 4.15)\)

1) **Displacement Method.**

A displacement field is assumed within the element and the element stiffness matrix is derived from the principle of minimum potential energy. If the displacement field results in compatible slopes and displacements at the interelement boundaries then monotonic convergence to the correct solution from below is ensured. In this case the stiffness of the actual structure is always overestimated. The displacement method is thoroughly treated in Ref. 4.8.

2) **Equilibrium Method.**

A stress field is assumed within the element which satisfies the equations of equilibrium. The element stiffness matrix is then derived from the principle of minimum complementary energy. If the stress field also satisfies the boundary tractions then monotonic convergence to the correct solution occurs from above. In this case the stiffness of the actual structure is always underestimated. References 4.9 and 4.16 to 4.18 show the derivation of element stiffness matrices using the equilibrium method.

3) **Hybrid Method.**

A stress field satisfying the equations of equilibrium is assumed within the element in addition to a separate displacement field along the interelement boundaries or vice versa. The stiffness matrix is then derived from a modified version of the complementary energy principle. References 4.9, 4.19 and 4.20 show the derivation of element stiffness matrices using the hybrid method.
4) **Mixed Method.**

Assuming both an equilibrium stress field and displacement field separately within each element the element stiffness matrix is derived from Reissner's variational principle. References 4.9 and 4.21 to 4.24 show the derivation of stiffness matrices using the mixed method.

A survey of the literature indicated that the displacement method is used most often. The use of polynomials or interpolation functions to specify displacement fields makes the displacement method relatively easy. Furthermore, the other methods usually give greater values for \( N \) and \( B \) thereby increasing the computational effort.\(^{(4.9, 4.24)}\) It was therefore decided to use the displacement method for the finite element analysis of the slab. The displacement method however may sometimes give slightly less accurate results than the other methods.\(^{(4.9, 4.14, 4.19, 4.24)}\)

4.4.3 **Element Nodal Parameters.**

The required element nodal parameters corresponding to the displacement method must be determined for both bending and in-plane behavior.

1) **Plate Bending.**

The plate behavior described by Eq. 4.1 involves the displacement \( w \), the rotations \( \theta_x \) and \( \theta_y \) about the \( x \)- and \( y \)-axes, the curvatures \( \phi_x \) and \( \phi_y \) about the \( x \)- and \( y \)-axes and the twist \( \phi_{xy} \).\(^{(4.1)}\)

The displacement \( w \) is a necessary nodal parameter. To ensure continuity of \( w \) along the interelement boundaries \( \theta_x \) and \( \theta_y \) must also...
be present as nodal parameters. \textsuperscript{(4.8)} Elements which have only these three nodal parameters are described in Refs. 4.8, 4.9 and 4.25 to 4.28. Elements with additional nodal parameters or with midside nodes, that is, higher order elements are described in Refs. 4.29 to 4.31.

The higher order elements give improved accuracy when few elements are used. One reason is that these elements can better satisfy the boundary conditions. The higher order elements have mainly two disadvantages. More time is required to generate the element stiffness matrices and they can not be used where curvatures are discontinuous, for example, where abrupt changes in plate thickness occur. Because of these disadvantages it was decided not to use higher order elements for the bending analysis of the slab.

2) In-plane Behavior.

The in-plane behavior described by Eq. 4.2 involves the displacements $u$ and $v$ in the $x$- and $y$- directions, the normal strains $\varepsilon_x$ and $\varepsilon_y$ in the $x$- and $y$- directions and the shear strain $\gamma_{xy}$. Elements having only $u$ and $v$ as nodal parameters are described in Refs. 4.9, 4.30 and 4.32. Higher order elements are described in Refs. 4.33 to 4.35. For the same reasons as were mentioned with regard to plate bending elements, higher order elements will not be used for the in-plane analysis of the slab.

The finite element to be used for both bending and in-plane behavior of the slab will therefore have five nodal parameters namely $u, v, w, \theta_x$ and $\theta_y$. 
4.4.4 Element Type.

Several types of elements are available for the analysis of both plate bending and in-plane behavior.

1) Plate versus Bar Elements.

In contrast to the conventional plate elements Refs. 4.26 and 4.28 present bar or truss elements. These elements comprise individual bars to simulate the continuum. This is also the reason why these elements are not expected to yield the same accuracy as the conventional plate elements and will therefore not be used.

2) Triangular versus Quadilateral Elements.

Comparisons of the accuracy of triangular and quadrilateral elements in Refs. 4.13, 4.25 and 4.36 to 4.38 show that quadrilateral or rectangular elements give better accuracy. There are two reasons for this behavior. The assumed displacement function for the triangular element is often not a complete polynomial and therefore creates preferential directions. \( \text{(4.15, 4.25)} \) The fewer nodes of the triangular element impose a greater restriction on the displacements within the element and therefore create a stiffer element. This is clearly shown in Ref. 4.25. Triangular elements will consequently not be used for the analysis of the slab.

3) Conforming Versus Non-conforming Elements.

The process of elimination has reduced the number of plate bending elements to essentially two, namely the Adini, Clough, Melosh (ACM) rectangular element \( \text{(4.8, 4.25)} \) and the Q-19 quadrilateral element. \( \text{(4.27)} \) The ACM element is non-conforming whereas the Q-19
element is conforming. Comparisons of the accuracy of the two elements show that the Q-19 element is undoubtedly superior.\(^{(4.8, \, 4.13)}\)

Because the Q-19 element is conforming it always approaches the correct displacement from below, that is, it always overestimates the stiffness of a slab. The ACM element being non-conforming always give displacements greater than that of the Q-19 element and may even exceed the correct displacement.\(^{(4.8, \, 4.13)}\) In the latter case the ACM element underestimates the stiffness of a slab. Since it is preferable to underestimate rather than overestimate the stiffness of a slab the ACM element was selected for the analysis of the slab.

The 12x12 flexural stiffness matrix of the ACM element is given in Ch. 10 of Ref. 4.8. The associated 8x8 in-plane stiffness matrix is given in Appendix IV of Ref. 4.30. These two stiffness matrices are combined to give the complete 20x20 uncoupled stiffness matrix of the rectangular plate element which is used for the analysis of the slab. This element and the degrees of freedom at each node is shown in Fig. 4.2a.

4.5 Beam and Column Elements.

The floor and frame beams in Fig. 4.1 are also divided into smaller elements. For each beam element in the x-direction the stiffness matrix given by Eq. 3.41 (Art. 3.7) will be used. For beam elements in the y-direction a coordinate transformation of the stiffness matrix of Eq. 3.41 is performed.\(^{(1.4)}\)

Each column is represented by one element. The stiffness matrix for this element is that of a prismatic member under bending
moments and concentric axial force. (1.4) Because of the symmetry of geometry and loading of the composite assemblage of Fig. 4.1 the columns do not twist. Torsion need therefore not be included in the stiffness matrix of a column. Figure 4.2b shows a column element and the degrees of freedom at each node.

4.6 Boundary Conditions.

Because of the symmetric geometry and loading of the composite assemblage of Fig. 4.1 only half of the structure need be considered. Figure 4.3 shows the boundary conditions of half of the composite assemblage. Along the boundaries AC and BD the displacement $v$ and rotation $\theta_x$ are equal to zero. The boundaries AB and CD are free edges so that the strain $\varepsilon_x$ and curvature $\phi_y$ are zero along these edges. Because the bottom ends of the columns are fixed the displacements $u$ and $w$ and the rotation $\theta_y$ are zero at these points.

The plate element of Fig. 4.2a can satisfy the boundary conditions along AC and BD in Fig. 4.3 but not along AB and CD. This will have a small effect on the vertical displacements of the slab but the effect on the horizontal displacement (lateral drift) of the composite assemblage is expected to be negligible. Since the emphasis of this study is on the horizontal displacement of the composite assemblage this factor will have no significant bearing on the results.

4.7 Finite Element Discretization.

Figure 4.4 shows the finite element discretization of the composite one-story assemblage of Fig. 4.1. The slab is divided into
a graded mesh of rectangular elements. Smaller elements are used near the columns where high stress gradients can be expected in the slab. Grid lines defining the slab elements are selected to coincide with the locations of the floor or frame beams. The lengths of the beam elements are determined by the spacing of the grid lines. Nodal points A, E and C indicate the centroids of the columns. Each column is represented by one column element.

The accuracy of the solution depends on the fineness of the finite element discretization. Increased discretization of the structure improves the accuracy but also increases the computational effort (Art. 4.4.1). The problem of what mesh is both sufficiently accurate and economical is treated in Art. 4.9.

4.8 Description of Program COMPFRAME.

A computer program called COMPFRAME was developed to perform a finite element analysis of the composite one-story assemblage as discretized in Fig. 4.4. The program generates all the element stiffness matrices, assembles them to form a total stiffness matrix for the structure, imposes the boundary conditions and then solves the set of simultaneous equations using Cholesky decomposition. All nodal displacements are then printed. Only the half bandwidth is stored in the computer.

The column forces of Fig. 4.1 are applied at nodes A, E and C in Fig. 4.4. Because the program performs a first-order linear-elastic analysis the column axial forces P have no significant effect on the horizontal drift and were consequently omitted from the
analysis. Furthermore, the horizontal drift due to unsymmetrical floor loads will usually be small in comparison with that caused by the column forces M and Q (Fig. 4.1). This is especially the case in the lower stories of a multistory frame. Floor loads were consequently also omitted from the analysis.

To minimize input data a mesh generation program was written. By providing such data as the number of elements in the x- and y-directions, number of columns and positions of beams and columns, the program generates the necessary data for all the elements. Provision was made to change the stiffnesses of any number of elements at any location in any desired manner. Discontinuities (Art. 2.6) or cracking (Art. 2.3) of the slab can therefore be included in the analysis.

4.9 A Study of Graded Meshes.

As mentioned in Art. 4.7 the steep stress gradients in the slab near the columns require a graded mesh for the finite element lay-out. A study was consequently made to determine what the graded mesh should be for any composite one-story assemblage and any desired degree of accuracy.

4.9.1 Analysis of a One-bay Composite Assemblage.

It is conceivable that the stress gradients in the slab of a one-bay assemblage would be more severe than in the case of a multi-bay assemblage. It was therefore decided to analyze the one-bay composite one-story assemblage of Fig. 4.5 to study graded
meshes. Because of symmetry only the half structure is shown. The member sizes correspond to those of the lower stories of a 30 story composite frame. The column loads are arbitrary but are deliberately chosen unsymmetric so as not to influence the results.

Figure 4.6a shows a typical finite element lay-out of the slab. The columns are located at nodal points A and C. Starting at these points the distances between the first three grid lines in the x- and y- directions were kept the same. For grid lines in the x- direction this distance was \( B_c / 2 \) to later investigate the effect of column flange rigidity. For grid lines in the y- direction this distance was either \( q_1 \) or \( q_2 \). The distances \( q_1 \) and \( q_2 \) are defined in Fig. 4.6b. This definition of \( q_1 \) and \( q_2 \) is for the purpose of incorporating the effect of discontinuities in the slab in Ch. 6.

The distances between subsequent grid lines were then increased according to predetermined ratios called mesh grades. The mesh grades in the x- and y- directions were always the same. Figure 4.7a shows the finite element lay-out for a mesh grade of 1:2.5 and Fig. 4.7b for a mesh grade of 1:1.1.

The forces of Fig. 4.5 were then applied to the structure at nodes A and C and the horizontal deflection at node C in Fig. 4.6a determined for several mesh grades. Figure 4.8 shows a typical plot of horizontal deflection versus mesh grade. The horizontal deflection for a mesh grade of 1:1.0 was always determined by linear extrapolation as shown in Fig. 4.8. A check on the particular problem of Fig. 4.8 showed that this procedure is satisfactory as indicated by comparing the calculated and extrapolated values as shown in the figure.

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4.9.2 **Effect of Four Variables on the Mesh Grade.**

For the purpose of determining a mesh grade which gives sufficient accuracy and is also economical for any composite one-story assemblage, it was decided to investigate the effect of four variables on the mesh grade. These are 1) the span length over panel width (L/B) ratio 2) slab thickness \( t \) 3) flexibility of the column flanges and 4) frame beam size. To investigate the effect of flexibility of the column flanges additional shear elements were placed between nodes A and F and C and G in Fig. 4.6a. These elements had arbitrary stiffnesses. To simulate rigid flanges the rigidity of each shear element was made very large.

For each variable a plot similar to that of Fig. 4.8 was obtained. The deflections were then nondimensionalized with respect to the deflection for a uniform mesh and then replotted as a percentage. This procedure was adopted because a uniform mesh gives the best values as reported in Refs. 4.41 to 4.43. The results of this study is shown in Fig. 4.9.

The following conclusions may be drawn from Fig. 4.9:

1) Monotonic convergence to the best value is obtained for all the variables investigated.

2) All the deflections are less than the best value. This implies that the stiffness of the structure is always overestimated.

3) Even with a very coarse mesh of 1:2.5 deflections are still within approximately 6 percent of the best value.

4) Increasing the L/B ratio, slab thickness or beam size de-
creases the accuracy of the results.

4.9.3 Recommended Mesh Grade.

From Fig. 4.9 it can be seen that a 3 percent accuracy can be obtained with a mesh grade of 1: 1.5. Similarly a mesh grade of 1: 2.0 gives better than 5 percent accuracy. Since an accuracy of better than 5 percent is satisfactory for all the problems that will subsequently be studied, a mesh grade of between 1: 1.5 and 1: 2.0 may be used. The computer results showed that an analysis using a mesh grade of 1: 2.0 may cost as little as one tenth of that of a uniform mesh.

The 3 or 5 percent accuracy mentioned above is not the absolute accuracy that will be obtained. The absolute accuracy may either be smaller or greater and is determined by the aspect ratio of the elements in contact with each column. (4.41-4.43) Square elements give the best results.
5. STRENGTH ANALYSIS OF COMPOSITE ONE-STORY ASSEMBLAGES

5.1 Introduction

Figure 5.1a shows a composite one-story assemblage under combined gravity and wind loads. All symbols in this figure have previously been defined. The direction parallel to the wind direction will be referred to as the longitudinal direction and the direction perpendicular to the wind direction as the transverse direction.

To determine the maximum strength of the composite assemblage of Fig. 5.1a the plastic moment of any transverse cross-section of the composite floor must be known. Since the plastic moment of a composite steel-concrete section is dependent on the sign of the bending moment at the section, the bending moment diagram for the composite floor must first be established.

Figure 5.1b shows a typical bending moment diagram for the composite floor at maximum load. On the windward side of each column a negative bending moment (bottom flange of frame beam in compression) exists. On the leeward side of each column a positive bending moment (bottom flange of frame beam in tension) exists.

In this chapter the plastic moment of any transverse cross-section of the composite floor in Fig. 5.1a subject to the bending moment diagram of Fig. 5.1b is determined. The forces resisted by the longitudinal shear strength of the reinforced concrete slab are studied. The number of shear connectors required on the frame beam of each span is also determined.
5.2 Assumptions

The plastic moment of a transverse cross-section of the composite floor is calculated on the basis of the following assumptions:

1) The composite floor of the one-story assemblage is treated as a continuous steel-concrete composite beam in the longitudinal direction.

2) Only the composite slab and frame beams contribute to the plastic moment of a transverse cross-section of the floor.

3) The full panel width of slab is effective in resisting compressive or tensile forces in the slab.

4) Only the steel reinforcement is effective in resisting tensile forces in the slab.

5) The maximum stress in all steel members is the yield stress of the material.

6) Only the leeward sides of the columns are in contact with the slab.

Assumption 1 implies that transverse bending of the slab is not considered. Transverse bending will cause longitudinal cracks in the slab but this is not expected to affect the plastic moment of a transverse cross-section of the floor.

Assumption 2 neglects the contribution of any floor beam to the plastic moment of a transverse cross-section of the floor. Since the floor beams are usually much smaller than the frame beams the plastic moment will only be slightly underestimated.
In Ref. 1.3 the effective width of a slab is used for calculating the maximum compressive force in the slab of a composite beam. The effective width of a slab is calculated from linear elastic theory and was not intended for maximum strength design. The use of the full panel width for maximum strength design as permitted by assumption 3 is more rational provided the concrete has sufficient ductility. This aspect requires future research on very wide slabs.

Assumption 4 is a good approximation of the actual condition in a reinforced concrete slab at maximum load. Assumption 5 ignores the effect of strain hardening in the steel members and slab reinforcement. The plastic moment of a transverse cross-section of the floor is therefore slightly underestimated.

Assumption 6 implies that the windward side of each column remains separated from the slab since there is no positive anchorage between the slab and the column (Art. 2.5.1). The slab therefore has a free edge of width $B_c$ on the windward side of a column.

5.3 Maximum Strength of the Slab

5.3.1 Maximum Compressive Strength in the Span

Reference 1.3 permits a maximum compressive stress of $0.85f_c'$ in the slab of a composite beam. This maximum stress is applicable to those regions of the slab where uniaxial compression exists.
These regions occur practically everywhere in the slab except near the columns (Art. 2.2.1). Using the full panel width as permitted by assumption 3 (Art. 5.2) and neglecting the longitudinal reinforcement in the slab the maximum compressive strength $C$ in the slab away from the columns is therefore given by

$$C = 0.85f'_c B_t$$  \hspace{1cm} 5.1

5.3.2 Maximum Compressive Strength at the Columns

The maximum compressive strength of the slab at the beam-to-column connections was treated in Art. 2.5.2. It was shown that a maximum stress of $1.3f'_c$ may be used in the slab at that location. The maximum compressive strength $C$ in the slab at the beam-to-column connections is therefore equal to

$$C = 1.3f'_c B_t$$  \hspace{1cm} 5.2

5.3.3 Maximum Tensile Strength in the Span

On the basis of assumptions 3, 4 and 5 (Art. 5.2) the maximum tensile strength $T$ in the slab away from the columns is equal to

$$T = A_{sr} f_{yr} = \rho_{\lambda} B_t f_{yr}$$  \hspace{1cm} 5.3

where

$A_{sr} = \text{total area of longitudinal reinforcement in the slab}$

$f_{yr} = \text{yield stress of the steel reinforcement}$

$\rho_{\lambda} = \text{area of longitudinal reinforcement per unit cross-sectional area of the slab}.$
5.3.4 Maximum Tensile Strength at the Columns

Because of assumption 6 (Art. 5.2) the slab has a free edge of width $B_c$ on the windward side of each column. The maximum tensile strength $T$ in the slab at the columns is therefore obtained by modifying Eq. 5.3 to

$$T = \rho_l (B - B_c) f_{yr}$$

5.4 Maximum Tensile Force $V_i$ at the Columns

5.4.1 Description of the Force $V_i$

Figure 5.2a shows an interior composite beam-to-column connection obtained from the composite one-story assemblage of Fig. 5.1a. The slab width of the connection is equal to the panel width $B$. Because of assumptions 1 and 2 (Art. 5.2) the concrete slab and frame beam constitute a continuous steel-concrete composite beam. The bending moment diagram of this composite beam at the beam-to-column connection can be obtained from Fig. 5.1b and is shown in Fig. 5.2b.

If a transverse cut is made through the slab on both sides of the column in Fig. 5.2a then the maximum possible forces in the slab will be as shown in Fig. 5.3. Between sections $i$ and $i-1$ of the slab at column $i$ a tensile force $V_i$ with direction as shown exists in the slab. Between the leeward column flange and the slab a maximum compressive force of $1.3 f'_c B t$ as given by Eq. 5.2 (Art. 5.3.2) acts. By assumption 6 (Art. 5.2) no force exists between the windward column flange and the slab. In Fig. 5.3 the area of the
frame beam at section \(i-1\) is denoted by \(A_s(i-1)\) and that at section \(i\) by \(A_s(i)\).

5.4.2 Maximum Value of the Force \(V_i\)

The maximum value of the force \(V_i\) in Fig. 5.3 can be determined by using composite beam design as described in Ref. 2.13. By this method the maximum force in the slab of a composite beam may not exceed the lesser of the maximum strength of the slab or the yield force of the steel beam. Referring to section \(i-1\) in Fig. 5.3 the maximum tensile strength of the slab is given by Eq. 5.4 (Art. 5.3.4). The yield force \(F_y(i-1)\) of the steel beam is equal to

\[
F_y(i-1) = A_{s(i-1)} f_y
\]

5.5

The maximum value of \(V_i\) as determined by section \(i-1\) is therefore equal to

\[
V_i = \text{Min} \left[ \rho \ell (B-B_c) t f_{yr}, A_{s(i-1)} f_y \right] \quad 5.6
\]

Equation 5.6 implies that \(V_i\) should be assigned the smaller of the two values in brackets.

The maximum possible forces in the slab and steel beam at section \(i\) in Fig. 5.3 are shown in Fig. 5.4. In this figure \(M_i\) and \(M_r\) are the applied and resistance moments respectively at section \(i\). Other symbols have previously been defined. The maximum force in the slab must be less than or equal to the yield force of the steel beam, \(2.13\). This implies that

\[
V_i - 1.3 f'c B_c t \leq A_s(i) f_y
\]
or

\[ V_i \leq 1.3 f'_c B_c t + A_s(i) f_y \]  \tag{5.7}

The resistance moment \( M_r \) must be greater than or equal to zero. This gives

\[ M_r = -(V_i - 1.3 f'_c B_c t) \frac{t+d}{2} \geq 0 \]

that is

\[ V_i - 1.3 f'_c B_c t \leq 0 \]

or

\[ V_i \leq 1.3 f'_c B_c t \]  \tag{5.8}

Equations 5.6, 5.7 and 5.8 can be combined to give

\[ V_i = \min \left[ \rho \alpha (B-B_c) t f_{yr}, A_s(i-1) f_y, 1.3 f'_c B_c t \right] \]  \tag{5.9}

Equation 5.9 gives the maximum value of the slab force \( V_i \). This equation can be used to determine the value of \( V_i \) at every column \( i \).

### 5.5 Transition Lengths

Figure 5.5 shows the slab forces \( V_1 \) and \( V_2 \) at columns 1 and 2 of an interior panel of a composite one-story assemblage. In this figure \( L' \) is the clear span between the columns. Also shown are two lengths \( a' \) and \( a'' \) measured from the faces of columns 1 and 2 respectively. The longitudinal shear strength of the slab associated with the length \( a' \) is denoted by \( Q' \) and that of the length \( a'' \) by \( Q'' \) as shown in the figure.

The lengths \( a' \) and \( a'' \) are chosen such that the longitudinal shear strengths \( Q' \) and \( Q'' \) equal the slab forces \( \frac{V_1}{2} \) and \( \frac{V_2}{2} \), that is
\[ Q' = \frac{V_1}{2} \]

and

\[ Q'' = \frac{V_2}{2} \]

The shear strengths \( Q' \) and \( Q'' \) can be written in terms of the mean ultimate longitudinal shear stress \( \nu_u \) of the slab as follows:

\[ Q' = \nu_u t a' \]

and

\[ Q'' = \nu_u t a'' \]

From Eqs. 5.10 and 5.12 the value of \( a' \) is given by

\[ a' = \frac{V_1}{2 \nu_u t} \]

The value of \( a'' \) is obtained from Eqs. 5.11 and 5.13 as

\[ a'' = \frac{V_2}{2 \nu_u t} \]

The lengths \( a' \) and \( a'' \) will be referred to as the transition lengths of the panel.

5.6 Maximum Compressive Force \( V_c \) in the Span

5.6.1 Description of the Force \( V_c \)

Figure 5.6 shows the location of any transverse cross-section C in an interior panel of a composite one-story assemblage. Section C is located a distance \( x \) from section 1 at the leeward face of Column 1 and a distance \( L'-x \) from section 2 at the windward face of Column 2. The maximum compressive force \( V_c \) in the slab at section C will be determined.
The force \( V_c \) is a function of the slab forces \( V_1 \) and \( V_2 \) at columns 1 and 2 respectively. It is also a function of the number of shear connectors between section \( C \) and sections 1 and 2. \( V_c \) is further affected by the longitudinal shear strength of the slab. Therefore, to determine the maximum value of \( V_c \) conditions to the left and right of section \( C \) must be investigated. The maximum value of \( V_c \) as determined by conditions to the left of section \( C \) will be referred to as \( V_{c1} \). Similarly \( V_{c2} \) is based on conditions to the right of section \( C \).

### 5.6.2 Maximum Compressive Force \( V_{c1} \)

Figure 5.7a shows the slab force \( V_{c1} \) in equilibrium with the shear connector force \( V' \) and the resultant compressive force \( 1.3 f'_c B_c t - V_1 \) in the slab at section 1. \( V' \) is the total shear strength of all the shear connectors between sections \( C \) and 1.

From horizontal equilibrium of forces the value of \( V_{c1} \) is given by

\[
V_{c1} = V' + 1.3 f'_c B_c t - V_1 \tag{5.16}
\]

Figure 5.7b shows that the force \( V_{c1} \) can be considered as the sum of two components \( V'_{c1} \) and \( V''_{c1} \). \( V'_{c1} \) is the component acting over a width \( B_c \) of the slab in line with the column. \( V''_{c1} \) is the component acting over the rest of the slab width. The slab force \( V_{c1} \) is therefore given in terms of its components as

\[
V_{c1} = V'_{c1} + V''_{c1} \tag{5.17}
\]

It is evident that the maximum value of the force \( V'_{c1} \) is \( 1.3 f'_c B_c t \) since the concrete stress can at most reach \( 1.3 f'_c \).
in a laterally confined region (Art. 2.2.2). It is also clear from Fig. 5.7b that the shear strength $Q'$ of the slab plays no role in transmitting $V'_{cl}$ to the shear connectors. The value of $V'_{cl}$ is therefore independent of the transition length $a'$. The maximum value of $V'_{cl}$ is consequently given by

$$V'_{cl} = 1.3 f' B_t$$

for all values of $x$.

As shown in Fig. 5.7b the forces $\frac{V''_{cl}}{2}$ are transmitted via the shear strength $Q'$ of the slab to the shear connectors. The value of $V''_{cl}$ is therefore a function of the transition length $a'$. Within the transition length $V''_{cl}$ is equal to zero since the shear strength of the slab is just sufficient to resist the slab force $V_l$. Consequently

$$V''_{cl} = 0$$

for $x < a'$

Outside the transition length $V''_{cl}$ is equal to the excess shear strength of the slab, that is

$$V''_{cl} = 2(x - a') v_u$$

for $x \geq a'$

Equations 5.15 to 5.20 can be combined to give the maximum value of $V_{cl}$ as follows:

$$V_{cl} = 1.3 f' B_t$$

for $x < a'$ and

$$V_{cl} = 1.3 f' B_t + 2 (x - a') v_u$$

for $x \geq a'$.  

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5.6.3 Maximum Compressive Force $V_{c2}$

Figure 5.8a shows the slab force $V_{c2}$ in equilibrium with the shear connector force $V''$ and the slab force $V_2$ at section 2. $V''$ is the total shear strength of all the shear connectors between sections C and 2. From equilibrium of forces the value of $V_{c2}$ is given by

$$V_{c2} = V'' - V_2$$

5.23

Figure 5.8b shows that the force $V_{c2}$ can be considered as the sum of two components $V_{c2}'$ and $V_{c2}''$. $V_{c2}'$ is the component acting over a width $p$ of the slab where $p$ is the distance between the outer rows of connectors. $V_{c2}''$ is the component acting over the rest of the slab width. The slab force $V_{c2}$ is therefore given in terms of its components as

$$V_{c2} = V_{c2}' + V_{c2}''$$

5.24

Using the same argument as for $V_{c1}'$ (Art. 5.6.2) the maximum value of $V_{c2}'$ is equal to $1.3 f_c' p t$. It is also clear from Fig. 5.8b that $V_{c2}'$ is solely resisted by the shear connectors with the shear strength $Q''$ of the slab playing no role in this respect. Consequently $V_{c2}'$ is not a function of the transition length $a''$. Therefore

$$V_{c2}' = 1.3 f_c' p t$$

5.25

for all values of $L'-x$.

As shown in Fig. 5.8b the forces $\frac{V_{c2}''}{2}$ are transmitted via the shear strength $Q''$ of the slab to the shear connectors. The value of $V_{c2}''$ is therefore a function of the transition length $a''$. Using
the same argument as for $V''_{cl}$ (Art. 5.6.2) the value of $V''_{c2}$ is equal
to zero within the transition length and equal to the excess shear
strength of the slab outside this region. Therefore,

$$V''_{c2} = 0$$

5.26

for $L' - x < a''$ and

$$V''_{c2} = 2(L' - x - a'') t v_u$$

5.27

for $L' - x > a''$.

Combining Eqs. 5.24 to 5.27 gives the maximum value of the
slab force $V_{c2}$ as follows:

$$V_{c2} = 1.3 f_c' p t$$

5.28

for $L' - x < a''$ and

$$V_{c2} = 1.3 f_c' p t + 2(L' - x - a'') t v_u$$

5.29

for $L' - x > a''$

5.6.4 Maximum Value of $V_c$

The force $V_c$ in the slab at section C must be less than or
equal to the smaller of the forces $V_{cl}$ and $V_{c2}$ as determined in
Arts. 5.6.2 and 5.6.3. It is now assumed that the number of shear
connectors between sections C and 1 is sufficient to make $V_{cl}$ as
given by Eq. 5.16 equal to $V_{c1}$ as given by Eqs. 5.21 and 5.22.
Similarly the value of $V_{c2}$ as given by Eq. 5.23 will be assumed
equal to $V_{c2}$ as given by Eqs. 5.28 and 5.29. The number of shear
connectors required on the frame beam to satisfy these conditions
will be determined in Art. 5.9.

The force $V_c$ must also be less than or equal to the smaller
of the yield force $A_s(i)f_y$ of the frame beam and the compressive strength $C$ of the slab as given by Eq. 5.1 (Art. 5.3.1). The maximum value of $V_c$ is therefore given by

$$V_c = \text{Min} \left[ 0.85 f_c B t, V_{cl}, V_{c2}, A_s(i) f_y \right]$$

where $V_{cl}$ is calculated from Eqs. 5.21 and 5.22 (Art. 5.6.2) and $V_{c2}$ from Eqs. 5.28 and 5.29 (Art. 5.6.3).

5.7 Plastic Moments.

With the maximum slab forces known at the columns and in the span the plastic moments of those sections can be determined. Figure 5.9 shows the stress diagrams which should be used for calculating the plastic moments at the windward and leeward side of column $i$ and at any point in the span. The forces $T_a$ and $T_b$ are the stress resultants in the steel beam above and below the plastic centroid of the composite section.

From Fig. 5.9 the plastic moment $M_p$ at the windward side of column $i$ is given by

$$M_p = V_i e_1 + T_a e_2$$

where the eccentricities $e_1$ and $e_2$ are determined from horizontal equilibrium of forces. The force $T_a$ in Eq. 5.31 can be shown to be equal to

$$T_a = \frac{F_y (1-1) - V_i}{2}$$

The plastic moment at the leeward side of column $i$ is equal to

$$M_p = (1.3 f_c B t - V_i) e_1 + T_a e_2$$
with the force $T_a$ given by

$$T_a = \frac{F_y(i) - 1.3 f' c t + V_i}{2}$$

5.34

The eccentricities $e_1$ and $e_2$ are again determined from horizontal equilibrium of forces.

The plastic moment in the span is equal to

$$M_p = V_c e_1 + T_a e_2$$

5.35

with $T_a$ given by

$$T_a = \frac{F_y(i) - V_c}{2}$$

5.36

5.8 Longitudinal Shear Strength of the Reinforced Concrete Slab

The mean ultimate longitudinal shear stress $\nu_u$ of the concrete slab (Art. 5.5) must still be determined. In Ref. 5.4 the longitudinal shear strength of the concrete slab of a composite steel-concrete beam was studied. The study pertains to normal density concrete slabs of composite beams not subjected to fatigue loading or positive transverse bending moments of the slab (tension in the bottom of the slab). It applies to positive and negative moment regions of continuous composite beams with or without negative transverse bending of the slab (compression in bottom of the slab).

The study showed that all transverse reinforcement contributed to the longitudinal shear strength of the slab irrespective of its level in the slab and of the magnitude of the negative transverse bending moment (bottom of slab in compression). No account need be taken of the longitudinal bending moments in the composite
beam in determining the longitudinal shear strength of the slab.

Two sets of equations are given in Ref. 5.4 which must be satisfied simultaneously. These are

\[
\rho_t f_{yr} > 1.26 \nu_u - 3.8 \sqrt{f'_c} \tag{5.37a}
\]

\[
\rho_t f_{yr} > 80 \tag{5.37b}
\]

and

\[
\rho_{tb} f_{yr} > 0.63 \nu_u - 1.9 \sqrt{f'_c} \tag{5.38a}
\]

\[
\rho_{tb} f_{yr} > 40 \tag{5.38b}
\]

where

\[
\rho_t = \text{area of transverse reinforcement per unit cross-sectional area of the slab}
\]

\[
\rho_{tb} = \rho_t \text{ for the bottom layer of reinforcement}
\]

\[
\nu_u = \text{the mean ultimate longitudinal shear stress of the concrete slab}
\]

All units must be in psi. The amount of reinforcement given by Eqs. 5.37 and 5.38 will prevent splitting of the slab along a line of shear connectors as well as longitudinal shear failure in the slab.

It is now assumed that the amount of reinforcement in the top and bottom layers of the slab are the same. In this case Eqs. 5.37 and 5.38 become the same equation. Using the equality condition as a limiting case in Eq. 5.37a then that equation becomes

\[
\rho_t f_{yr} = 1.26 \nu_u - 3.8 \sqrt{f'_c}
\]

or

\[
\nu_u = 0.79 \rho_t f_{yr} + 3.0 \sqrt{f'_c} \tag{5.39}
\]
5.9 **Required Number of Shear Connectors**

With the maximum slab force $V_c$ known from Eq. 5.30 the number of shear connectors required on the frame beam of each panel can be determined. From Eq. 5.16 (Art. 5.6.2) the value of $V'$ is given by

$$V' = V_c - 1.3 f'_c B_c t + V_1$$

5.40

where $V_c$ is substituted for $V_{c1}$.

But

$$V' = N_{c1} q_u$$

5.41

where

$$N_{c1} = \text{number of shear connectors between sections C and 1}$$

The maximum shear strength $q_u$ of a shear connector is given by

Eq. 2.7 (Art. 2.4.1). From Eqs. 5.40 and 5.41 the value of $N_{c1}$ is given by

$$N_{c1} = \frac{V_c - 1.3 f'_c B_c t + V_1}{q_u}$$

5.42

From Eq. 5.23 (Art. 5.6.3) the value of $V''$ is given by

$$V'' = V_c + V_2$$

5.43

where $V_c$ is substituted for $V_{c2}$.

But

$$V'' = N_{c2} q_u$$

5.44

where

$$N_{c2} = \text{number of shear connectors between sections C and 2}$$

The value of $N_{c2}$ is obtained from Eqs. 5.43 and 5.44 as

$$N_{c2} = \frac{V_c + V_2}{q_u}$$

5.45

The values of $N_{c1}$ and $N_{c2}$ as obtained from Eqs. 5.42 and 5.45 are the number of shear connectors required on the frame beam to
the left and right of section C respectively. The maximum slab forces $V_1$ and $V_2$ in Eqs. 5.42 and 5.45 respectively are computed from Eq. 5.9 (Art. 5.4.2) by letting $i = 1, 2$. 
6. METHOD OF ANALYSIS FOR COMPOSITE FRAMES

6.1 Introduction

All the material developed in Chapters 2 to 5 can now be combined to form an analytical method for analyzing composite frames. The proposed method is a two-step procedure. Each floor of the composite frame is first subjected to a finite element analysis as described in Ch. 4. The results of these analyses are then used in a second-order elastic-plastic analysis to obtain the complete load-drift curve of the composite frame.

The analytical method allows for the effects of a flexible shear connection between the floor slabs and steel beams, of discontinuities in the floor slabs (Art. 2.6) and of cracking of the concrete slabs (Art. 2.3) to be included. It is also shown that the load-drift curve of the composite frame is obtained at much lower cost than if conventional methods of analysis had been used.

6.2 Assumptions

The analytical method developed in this chapter is based on the following assumptions in addition to those of the previous chapters:

1) All members behave elastic-perfectly plastic.

2) Out-of-plane instability due to local or lateral-torsional buckling of the members is prevented.

3) The column bases are assumed to be fixed.
Assumption 1 ignores the effect of residual stresses in the steel members. The stiffness of the composite frame at high loads will therefore be slightly overestimated. This assumption however has no effect on the maximum strength of a composite frame. (2.13)

Assumption 2 is valid if adequate b/t ratios and lateral bracing are used for the steel members. (2.13) Because of the stiffening effect of the steel beams the possibility of slab buckling is remote.

Assumption 3 is inherent to the computer program used in this dissertation for a nonlinear analysis. (4.39) This restriction can however be removed for more general cases.

6.3 Description of the Analytical Method

Considering the composite frame of Fig. 1.2a each story is subjected to a finite element analysis as described in Ch. 4. The purpose is to determine an equivalent floor slab for each composite one-story assemblage. This equivalent slab when used in the absence of the actual floor slab and floor beams but rigidly attached to the original frame beams, gives the same horizontal deflection as the actual composite one-story assemblage.

The equivalent slab for each floor has the same thickness as the original slab and its width is uniform throughout the story. When determining the width of the equivalent slab the effects of a flexible shear connection between the original slab and the steel beams (Ch. 3), of discontinuities (Art. 2.6) and of cracking of the original concrete slab (Art. 2.3) are included.

-80-
The composite frame is now replaced by an equivalent plane frame consisting of the original steel columns and frame beams but with the original slabs and floor beams replaced by the equivalent slabs. A rigid shear connection is assumed between the equivalent slabs and the frame beams. The gravity loads on the equivalent plane frame are the reactions of the floor beams in the composite frame.

A second-order elastic-plastic analysis is now performed on the equivalent plane frame using a program called SOCOFRANDIN (Art. 6.5). In this program the gravity loads are applied first and then held constant. The frame is then subjected to increments of lateral deflection to give the complete load-drift curve of the composite frame. The plastic moments of the beams of the equivalent plane frame are those of the composite floors as determined in Ch. 5.

6.4 Equivalent Slab Widths

6.4.1 Method of Calculation

The width of the equivalent slab for each composite one-story assemblage is calculated in program COMPFRAME (Art. 4.8). An arbitrary set of horizontal forces and moments is applied to the composite assemblage according to the portal method. (6.1) The horizontal deflection of the composite assemblage is then determined by the finite element analysis as described in Ch. 4.

A substitute composite assemblage is then formed consisting of the original columns and frame beams but with the original slab and floor beams removed and replaced by a concrete slab which is
rigidly attached to the frame beams. The thickness of this slab is

the same as that of the original slab and its width is uniform

throughout the length of the composite assemblage.

The horizontal deflection of the substitute composite assem-

blage is then determined for the same set of loads as used on the

original composite assemblage for various values of slab width.

Using the False-position method \(^{(4.40)}\) that slab width which gives

a horizontal deflection within tolerance of that of the original com-

posite assemblage is obtained. This width is the equivalent slab

width for the composite one-story assemblage.

Although the portal method was used for proportioning the

loads on the composite assemblage the cantilever method could also

have been used. \(^{(6.1)}\) Both methods give reasonable results for

buildings up to approximately 25 stories and moderate height-width

ratios. \(^{(6.1)}\) For the problem under consideration the portal method

turned out to be faster and was consequently used.

As mentioned in Art. 4.8 gravity loads are not included in

the analysis of a composite assemblage and consequently in the
determination of the equivalent slab width. Although gravity loads
affect the cracking in the concrete slabs and consequently the

equivalent slab width, the effect of cracking is accounted for

separately in an approximate manner \(\text{(Art. 6.4.3)}\). However, the

second-order effects of the gravity loads are included in the

elastic-plastic analysis of the equivalent plane frame. \(^{(1.5)}\)
6.4.2 Effect of Discontinuities in the Concrete Slabs

Because of the discontinuities in the concrete slabs of a composite frame a free edge exists in the slab on the windward side of each column as shown in Fig. 2.8b (Art. 2.6). On this edge the stress \( \sigma_x \) in the slab in the \( x \)-direction is equal to zero.

Figure 6.1a shows the finite element representation of discontinuities in the slab. The columns are located at nodes A, E and C. For each element marked "a" located between the column centerline and the first row of shear connectors on the windward side of a column the Young's modulus \( E_x \) in the \( x \)-direction is set equal to zero. As a result \( \sigma_x \) will be equal to zero for the whole element.

Because of shear lag parts of the elements marked "a" in Fig. 6.1a will have some stress in the \( x \)-direction. The procedure outlined above consequently makes ample provision for discontinuities in the slab.

6.4.3 Effect of Cracking of the Reinforced Concrete Slabs

Figure 5.1b (Art. 5.1) shows the assumed bending moment diagram for the floors of a composite one-story assemblage at maximum load. Within the negative moment regions the reinforced concrete slab is assumed to be in tension throughout its depth. Because concrete is assumed to have no tensile strength (Art. 2.2.1) the slab in the negative moment region is completely cracked. The stress-strain relationship derived in Art. 2.3.2 for a cracked slab is therefore applicable in this region.
It is now assumed that the length of each negative moment region is one quarter of the span length. This assumption overestimates the negative moment region in the upper stories of a composite frame and underestimates the region in the lower stories. (6.2) It is therefore considered as a good approximation of the average length of the negative moment regions in a composite frame.

For the purpose of determining the equivalent slab width, the finite element representation of cracking of the concrete slabs is as shown in Fig. 6.1b. All the elements marked "b" lie in the assumed negative moment region and for these elements the Young's modulus $E_x$ in the $x$-direction is assigned a value $E_e$ (Art. 2.3.2). The elements marked "a" were treated in Art. 6.4.2.

The reinforced concrete slabs at service load are most likely cracked only part-through and not completely as assumed above. The equivalent slab width as determined above consequently underestimates the stiffness of the composite frame at service load. This situation is satisfactory because of the importance of occupational comfort at the service load level.

6.5 Description of Program SOCOFRANDIN

Program SOCOFRANDIN (4.39) performs the nonlinear analysis of the equivalent plane frame (Art. 6.3). Input for the program includes the equivalent slab width for each floor, the panel width $B$, concrete strength $f'_c$, slab thickness $t$ and longitudinal and transverse reinforcement ratios $\rho_L$ and $\rho_T$. The output of the program is the lateral load and corresponding drift index at various values of the lateral
load. The drift index is the horizontal deflection (drift) at the top of the frame divided by the total height of the frame.

After reading the input data the program calculates the areas and moments of inertia of the beams of the equivalent plane frame. In this process the equivalent slab and the frame beams are reduced to the transformed steel sections. The transformed sections then remain constant throughout the nonlinear analysis.

After each increment of lateral drift the total bending moments in the members of the equivalent plane frame are calculated. These moments are then checked against the plastic moments of the members. The plastic moments in the beams are those of the composite floors as determined in Ch. 5. If the bending moment at any section equals or exceeds the plastic moment then a real hinge is inserted at that location.

6.6 Comparison with Conventional Methods of Analysis

To compare the computational effort of the analytical method developed in this chapter with that of a conventional method of analysis, consider the 2-bay composite frame of Fig. 1.2a (Art. 1.2). The semibandwidth of the stiffness matrix of the complete composite frame can easily be 20 times greater than that of only one floor. The computational effort to analyze the composite frame using a conventional method of analysis would therefore be approximately \((20)^2\) or 400 times greater than that of the analytical method developed herein. It is therefore concluded that the analytical method developed in this study is much less expensive than conventional methods of analysis.
7. DESIGN EXAMPLES

7.1 Design Example 1

7.1.1 Description of Steel Frame No. 7.1

Figure 7.1 shows a detail of steel frame No. 7.1. This two-bay three-story frame is taken from a three-dimensional building frame with 20 ft. by 20 ft. floor panels. The story height is 12 ft. Each floor has W16x40 frame beams connected to W8x31 columns. The W12x31 floor beams are spaced at 10 ft. centers. All the steel members have a yield stress of 36 ksi.

The floor loads on the frame include a service live load of 80 lb. per square ft. and a dead load of 100 lb. per square ft. Provision is made for a 5 in. reinforced concrete floor slab in determining the dead load. The loads on all three floors are assumed equal.

All the members are designed according to the AISC code (Ref. 1.3). The steel beams are required to carry the floor loads without composite action with the floor slabs. However, the floor slabs provide lateral support for the steel beams and this is taken into account in determining allowable stresses.

7.1.2 Effect of the Floor System

Figure 7.2 shows several load-drift curves for steel frame No. 7.1. On the horizontal axis the drift index $\Delta/H$ is plotted which is defined as the maximum drift $\Delta$ at the top of the frame
divided by the total height $H$ of the frame. On the vertical axis the wind load factor is plotted. The wind load factor is the actual wind load per story on the frame divided by the service wind load per story. The service wind load is based on a wind pressure of 20 lb. per square ft. on the vertical face of the building.

The dotted lines in Fig. 7.2 are the load-drift curves for the bare steel frame for gravity load factors (GLF) of 1.0 and 1.3. A gravity load factor is the ratio of the actual floor load on a building divided by the service floor load. The drift index of the frame at service load is therefore obtained from the load-drift curve for a gravity load factor of 1.0. A gravity load factor of 1.3 is the maximum value specified for a frame that is subjected to combined gravity and wind loads. The maximum strength of a frame is therefore obtained from the load-drift curve for a gravity load factor of 1.3.

Figure 7.2 shows that the steel frame obtained a wind load factor of 1.75 for a gravity load factor of 1.3. The maximum required wind load factor for this condition is 1.3 so that the steel frame is slightly stronger than necessary. For a gravity load factor of 1.0 the drift index of the frame at a wind load factor of 1.0 is 0.0015 which is also satisfactory. The steel frame therefore represents a satisfactory design both from a maximum strength and stiffness point of view.

Figure 7.2 also shows the load-drift curves when the floor system interacts with the steel frame assuming an essentially rigid shear connection between the slabs and frame beams. This composite
frame is also numbered 7.1. The equivalent slab width for each floor is 32.0 in. and was obtained using a mesh grade of 1:1.5 (Art. 4.9). It can be seen that the effect of the floor system is to increase the maximum strength of the steel frame by approximately 14%. Furthermore, the drift index at a wind load factor of 1.0 is decreased by approximately 20%.

The plastic hinge pattern at ultimate load for the composite frame and the steel frame for a gravity load factor of 1.3 is also shown in Fig. 7.2. It is clear that ultimate load coincided with a panel mechanism in the bottom story.

7.1.3 Effect of Shear Connector Spacing

Figure 7.3 shows the effect of several variables on the service load drift of composite frame No. 7.1. Curve 1 is for a very small shear connector spacing implying an essentially rigid shear connection. Curve 2 is for the AISC minimum shear connector spacing of 1 in. concrete cover between adjacent shear connectors. Curve 3 is for a connector spacing of 6 in. which is the connector spacing necessary to obtain full composite action between the slabs and the frame beams. (1.3)

By comparing curves 1, 2 and 3 in Fig. 7.3 it is evident that increasing the connector spacing significantly increases the service load drift of a composite frame. This is also evident from Table 7.1 where the nondimensionalized equivalent slab widths for the various curves are given. Increasing the connector spacing caused a substantial decrease in the equivalent slab width.
The maximum strength of the composite frame for the three connector spacings of curves 1, 2 and 3 was also obtained. The results showed that connector spacing has no significant effect on the maximum strength of a composite frame.

7.1.4 Effect of Discontinuities in the Slab

Curve 4 in Fig. 7.3 represents the effect of a discontinuity between the slab and the windward face of each column (Art. 2.6.1). The shear connector spacing for curve 4 is the same as for curve 3. As shown in Table 7.1 the equivalent slab widths for curves 4 and 3 are the same leading to the same load-drift curve. It is therefore concluded that a discontinuity in the slab at the windward face of a column has no effect on the service load drift of a composite frame.

As for the maximum strength of a composite frame the effect of a discontinuity in the slab at the windward face of each column is included in the maximum strength analysis (see Ch. 5).

7.1.5 Effect of Cracking of the Concrete Slabs

Curve 5 in Fig. 7.3 represents the effect of cracking of the concrete slabs in the negative moment regions. It is evident that cracking of the slabs increases the service load drift of the composite frame. However, as explained in Art. 6.4.3 the method of including the effect of cracking greatly overestimates this effect at service loads. Cracking of the concrete slabs is therefore not
expected to have a large influence on the service load drift of a composite frame.

Regarding the maximum strength of the composite frame the method of analysis in Ch. 5 includes the effect of cracking of the concrete slabs.

7.1.6 Saving of Steel Through Composite Action

The previous results showed that the maximum strength of the steel frame is significantly increased by the presence of the floor system. It would therefore be possible to design an alternative composite frame which has the same maximum strength as steel frame No. 7.1. If in addition the service load drift of the alternative composite frame is equal to or less than that of steel frame No. 7.1 then considerable saving in steel may be achieved.

Figure 7.4 shows detail of composite frame No. 7.2. The only difference between this composite frame and steel frame No. 7.1 is that the frame beams have been decreased from W16x40 to W14x26. Figure 7.5 shows the load-drift curves of composite frame No. 7.2 and steel frame No. 7.1. The composite frame attains the same maximum strength as the steel frame with a gravity load factor of 1.3. Furthermore, the composite frame exhibits a slightly smaller drift at the service load level.

Reducing the frame beams from W16x40 to W14x26 resulted in a saving of 35 percent in the weight of the frame beams. However, a certain amount of this saving will be off-set by the additional shear connectors necessary to achieve composite action between the
floor slabs and the frame beams.

7.1.7 Comparison with Experimental Results

The load-drift curves of Fig. 7.3 shows that at a wind load factor of 1.0 the drift of the composite frame assuming rigid shear connection (curve 1) was 20 percent less than that of the steel frame. This amounts to an increase of 25 percent in the stiffness of the steel frame. The 25 percent increase in stiffness is greater than the 15 percent reported in Ref. 1.29 but considerably less than the 67 percent of Ref. 1.28.

The increase in stiffness of the steel frame due to the floor system is greatly affected by the relative magnitude of the bending stiffnesses of the frame beams and columns. For steel frame No. 7.1 the frame beams have a bending stiffness five times that of the columns. The stiffness of this steel frame is therefore essentially determined by the columns. Adding the floor system to the frame will consequently not have a large effect on the stiffness of the steel frame. A different situation exists in the next design example.

7.2 Design Example 2

7.2.1 Description of Steel Frame No. 7.3

Figure 7.6 shows a detail of steel frame No. 7.3. This three-bay ten-story frame is taken from a three-dimensional build-
ing frame with 20 ft. by 20 ft. floor panels. The story height is 9'-6". Sizes of the columns and frame beams are as shown. All the floor beams are W10x15 at 10 ft. centers. All the steel members have a yield stress level of 36 ksi.

The floor at roof level is subjected to a service live load of 30 psf and a dead load of 40 psf. All other floors are subjected to a service live load of 40 psf and a dead load of 55 psf. Provision is made in the dead load for a 4 in. reinforced concrete floor slab. The columns are further loaded by point loads representing exterior and interior walls. The service wind load on the building is taken as 20 psf.

The steel beams were designed to resist the floor loads without consideration of composite action with the floor slabs. All the members were designed according to the AISC Code (Ref. 1.3) for both the gravity and combined gravity and wind load conditions. An effective length factor of 1 was assumed for the columns.

7.2.2 Effect of the Floor System

The load-drift curves for steel frame No. 7.3 for gravity load factors of 1.0 and 1.3 are shown in Fig. 7.7. The maximum load on the steel frame was reached when the third column from the left in the bottom story reached its maximum load capacity. This accounts for the abrupt ending of the load-drift curves.

Figure 7.7 also shows the load-drift curves when the floor system interacts with the steel frame assuming an essentially rigid shear connection between the slabs and the frame beams. This
composite frame is also numbered 7.3. The equivalent slab widths for this composite frame were determined for floor levels 4 and 8 (see Fig. 7.6) as shown in Table 7.2. A mesh grade of 1:2.0 was used in the finite element analysis (Art. 4.9). The equivalent slab widths for the other floor levels were then assumed as shown in Table 7.2.

Figure 7.7 shows that the effect of the floor system is to significantly increase the maximum strength and stiffness of the steel frame. With a gravity load factor of 1.3 the maximum strength of the composite frame is 18 percent greater than that of the steel frame. With a gravity load factor of 1.0 the drift index of the composite frame at a wind load factor of 1.0 is 41 percent less than that of the steel frame. This implies that the composite frame is 70% stiffer than the steel frame.

Figure 7.7 also shows the hinge pattern in the composite frame at maximum load. A panel mechanism is close to being formed in the bottom story.

7.2.3 Saving in Steel Through Composite Action

As for the steel frame No. 7.1 it is possible to design an alternative composite frame which has the same maximum strength as steel frame No. 7.3. This composite frame, numbered 7.4, is shown in Fig. 7.8. The only difference between composite frame No. 7.4 and steel frame No. 7.3 is that the frame beams have been decreased as shown in the figure. The 4 in. reinforced concrete slabs are assumed to be rigidly connected to the frame beams.
Figure 7.9 shows the load-drift curves for steel frame No. 7.3 and composite frame No. 7.4. For a gravity load factor of 1.3 both frames reached essentially the same maximum load. For a gravity load factor of 1.0 and a wind load factor of 1.0 the composite frame has a drift index 24 percent less than that of the steel frame. The stiffness of the composite frame is consequently 32 percent greater than that of the steel frame.

Comparing the weight of the frame beams indicates that composite frame No. 7.4 has 29 percent less steel than steel frame No. 7.3. Therefore, by considering interaction between the floor system and the steel frame a significant saving in steel is achieved. In addition a substantial increase in stiffness is obtained as noted above.

7.2.4 Comparison with Experimental Results

As noted in Art. 7.2.2 composite frame No. 7.3 had a stiffness 70 percent greater than that of steel frame No. 7.3. This increase is slightly greater than the 67 percent reported in Ref. 1.28. As mentioned in Art. 7.1.7 the increase in stiffness is greatly dependent on the ratio of the bending stiffnesses of the frame beams to that of the columns. The bending stiffnesses of the frame beams of steel frame No. 7.3 are approximately the same as that of the columns. The increase in stiffness of steel frame No. 7.3 due to the floor system is therefore expected to be substantial.
7.3 Comparison of Equivalent Slab Widths

It is of interest to compare the equivalent slab widths of this study with those reported in Refs. 1.20 and 1.21. This comparison is shown in Table 7.3. The values reported in the references were obtained in the absence of floor and frame beams. This factor must be considered when comparing the results.

It is apparent from Table 7.3 that the equivalent slab widths obtained in this study are considerably smaller than those reported in Refs. 1.20 and 1.21. One reason for this difference is the absence of all beams in the problems treated in the references. Table 7.4 shows the equivalent slab widths for all the composite frames of this study. Decreasing the frame beams caused a significant increase in the equivalent slab width. The equivalent slab widths reported in the references are therefore expected to be greater than those obtained in this study.

A second reason may lie in the modeling of the columns. In this study the columns were modeled as having finite width but no depth. In Refs. 1.20 and 1.21 the columns were also given finite depth.

The equivalent slab widths obtained in this study gave stiffnesses for the composite frames which compared favorably with the available experimental results (Arts. 7.1.7 and 7.2.4). Considerable confidence can therefore be placed on the results of this study. However, because of the wide scatter of the values in Table 7.3 more work need be done to confirm the effect of beams on the equivalent slab width.

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8. RECOMMENDATIONS FOR FUTURE WORK

8.1 Formed Metal Deck Slabs

Although only solid floor slabs are considered in this dissertation the basic theory also applies to formed metal deck slabs. When determining the stiffness of composite one-story assemblages the effect of a formed metal deck slab can be included by using orthotropic plate theory. The maximum strength analysis of composite one-story assemblages with formed metal deck construction presents no problem.

8.2 Discontinuity on the Leeward Sides of the Columns

The load-deflection curves presented in this dissertation include the effect of a discontinuity in the concrete floor slabs on the windward sides of the columns but not on the leeward sides. To include a discontinuity on the leeward sides of the columns requires an extension of the nonlinear analysis presented on this dissertation.

8.3 Friction between the Slab and the Steel Beams

The experimental results presented in Ref. 1.28 showed that friction between the slab and the steel beams has a significant effect on the stiffness of a composite frame. The effect of friction can be included by suitably modifying the load-slip relationship of the shear connectors. It will be affected by the magnitude of the gravity loads on the floors, by the friction coefficient of concrete on steel and by the lateral spacing of the unbraced steel frames.
8.4 Schematic Model of a Composite Slab-Beam System

In the schematic model of a composite slab-beam system presented in this dissertation the total interface shear force is applied as a concentrated force at the end of the steel beam element. This ignores the fact that the shear force is in fact, distributed along the length of the steel beam element. It is conceivable that the distributed nature of the interface shear force can be included by suitably modifying the axial stiffness of the steel beam element.

8.5 Inelastic Analysis of Composite One-Story Assemblages

The analysis of composite one-story assemblages presented in this dissertation is linearly elastic. This analysis can be extended to a nonlinear analysis for incorporation in the sway-subassemblage method.

8.6 Ductility Requirements for the Concrete Slabs

The maximum strength analysis of composite one-story assemblages presented herein assumes that concrete has sufficient ductility in compression to reach and maintain the maximum strength. The limited ductility on concrete is well known and further theoretical and experimental work is necessary in this respect.

8.7 Analysis of Tubes, Tube-in-Tubes and Framed Tubed Structures

In this dissertation the composite behavior of floor systems and unbraced steel frames were considered. This study can be extended to also include other structural steel systems such as the tube, tube-in-
tube and the framed tube. Combined frame and shear wall systems can also be considered by extending the analytical procedures presented in this dissertation.

8.8 Simplification of the Finite Element Analysis

For the stiffness analysis of composite one-story assemblages a twenty degree of freedom finite element was used. A considerable saving in computer execution time could be attained if a finite element of fewer degrees of freedom could be used. This would be possible if it could be shown that either the in-plane or bending behavior of the floor slabs can be neglected without significantly affecting the results.

8.9 Nonsymmetrical Buildings

In this dissertation only symmetrical multistory buildings subjected to symmetrical loads were considered. The extension of nonsymmetrical buildings requires considerable additional development but would be a significant contribution.

8.10 Experimental Studies

There is clearly a lack of experimental studies against which the results presented in this dissertation can be evaluated. The two experimental studies of Refs. 1.28 and 1.29 gave results which are too far apart to form a significant conclusion regarding the theoretical results of this dissertation. Furthermore the two experimental studies
provide no information on the maximum strength of composite frames. Additional experimental work is therefore necessary.

8.11 Finite Element Representation of the Columns

In the finite element analysis of a composite one-story assembly the length and flange width of the columns were considered but not the depth of the column section. The depth of the column section may have an effect on the equivalent slab width and this aspect requires further investigation.

8.12 Dynamic Behavior of Composite Frames

Only the static behavior of composite frames was studied in this dissertation. The extension of the study to dynamic behavior is important from an earthquake point of view.
9. **SUMMARY AND CONCLUSIONS**

This study concerns the effect of floor systems on the maximum strength and stiffness of three-dimensional multistory steel frames. The three-dimensional steel frame consists of a series of parallel unbraced frames forming the main gravity and lateral load resisting elements. The unbraced frames have symmetric geometry, are evenly spaced and are subjected to symmetric distributions of floor loads as viewed in a direction parallel to the unbraced frames.

The floor system consists of solid reinforced concrete slabs attached with headed steel study shear connectors to the supporting floor and frame beams. Since the floor slabs are also attached to the frame beams, the floor system interacts with the unbraced frames in resisting the applied gravity and lateral loads.

The symmetric geometry and loading of the three-dimensional frame allow any one of the unbraced frames with its panel wide floor system to be studied in order to obtain the load-drift behavior of the complete frame. Such a reduced frame is called a composite frame in this dissertation. The study of composite frames forms the major part of this dissertation.

The composite frame is further reduced to an equivalent plane frame consisting of the unbraced steel frame but with the floor system at each level replaced by an equivalent slab. The equivalent slab has the same thickness as the actual floor slab and its width is uniform along the length of the frame. A rigid shear connection is assumed between the equivalent slab and the frame beam.
The width of the equivalent slab is obtained from a finite element analysis of the composite one-story assemblage of each floor level. This composite assemblage consists of the floor slab, floor and frame beams and the columns of the story below. Using the portal method a set of loads is applied to the composite assemblage and the horizontal deflection determined. The equivalent slab width is obtained as that width which gives the same horizontal deflection as the actual floor system.

The equivalent slab width includes the effect of a flexible shear connection between the floor slabs and the steel beams. Discontinuities in the floor slabs such as shrinkage gaps at the columns or holes are also accounted for in the equivalent slab width. Cracking of the concrete slabs is considered in the determination of the equivalent slab width.

The equivalent plane frame is then subjected to a second-order elastic-plastic analysis to yield the load-drift curve of the composite frame. The plastic moments of the frame beams are replaced by the composite plastic moments of the floor system. Equations are derived to calculate the plastic moments for any transverse cross-section of the floor.

The composite plastic moment of any transverse cross-section of the floor is determined by considering the floor slab and frame beam as constituting a continuous composite beam in the longitudinal direction. Using standard composite beam theory the plastic moment at any section can be determined. The longitudinal shear strength of the reinforced concrete slab is one of the factors governing the maximum

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force in the slab.

To include the effect of flexibility of the shear connectors a new element stiffness matrix is developed for the steel beams. The stiffness matrix includes the effect of relative horizontal slip between the slab and the steel beam. Eccentricity of the steel beam with respect to the floor slab is also accounted for.

Two design examples are presented. One is a two-bay three-story composite frame and the other is a three-bay ten-story composite frame. Load-drift curves are presented for both the composite frames and the associated unbraced steel frames for gravity load factors of 1.0 and 1.3. The effect of the floor system on the maximum strength and stiffness of the steel frame is demonstrated. It is also shown how flexible shear connection, shrinkage gaps at the columns and cracking of the concrete slabs affect the load-drift curve of a composite frame.

From this study several conclusions may be drawn:

1) The method of analyzing three-dimensional multistory frames as presented in this dissertation is approximate but much less expensive than conventional methods of analysis.

2) The analytical method gives results which compare favorably with available experimental results.

3) The floor system can increase the stiffness of the steel frame at the service load level by as much as 70 percent.

4) The floor system can increase the maximum strength
of the steel frame by 14 to 18 percent.

5) A saving of 35 percent on the weight of the frame beams can be achieved by considering interaction between the floor system and the steel frame.

6) The effect of the floor system is more pronounced when the bending stiffnesses of the frame beams are not much greater than those of the columns.

7) A flexible shear connection between the floor slabs and the frame beams significantly decreases the stiffness of a composite frame at the service load level but has no effect on the maximum strength.

8) Shrinkage gaps between the floor slabs and the windward faces of the columns have no appreciable effect on either the maximum strength or stiffness of a composite frame.

9) Cracking of the reinforced concrete slabs has a small effect on the service load drift of a composite frame.
10. Nomenclature

\( A_s \) = area of the steel beam

\( A'_s \) = \( K'A_s \)

\( A_{sr} \) = area of the slab reinforcement

\( B \) = panel width of a floor, semi-band width of a stiffness matrix

\( B_c \) = average column flange width in a story

\( C \) = maximum compressive force in a slab

\( D \) = elastic bending stiffness of a slab

\( E \) = Young's modulus of steel

\( E_c \) = Young's modulus for concrete

\( F \) = axial force in a shear connector

\( G \) = shear modulus

\( H \) = horizontal load on a building, total height of frame

\( I_x \) = moment of inertia of the steel beam area about the reference plane

\( I_o \) = moment of inertia of the steel beam about centroidal axis

\( I'_x \) = \( I_o + A'_s \bar{y}^2 \)

\( J_c \) = torsional stiffness of a group of shear connectors

\( J_s \) = St. Venant torsional stiffness of the steel beam

\( J' \) = \( \frac{J_c}{J_c + J_s} \)

\( J'_s \) = \( J' J_s \)

\( K_c \) = total stiffness of all the shear connectors on a beam element

\( K_s \) = axial stiffness of the steel beam element
\[ K' = \frac{K_c}{K_c + K_s} \]
\[ K'' = \frac{K_s}{K_c + K_s} \]
\[ L = \text{span length} \]
\[ L' = \text{clear span length} \]
\[ M = \text{bending moment (general)} \]
\[ M_P = \text{plastic moment} \]
\[ M_r = \text{moment of resistance of a composite section} \]
\[ N = \text{size of the stiffness matrix} \]
\[ N_c = \text{number of shear connectors on a beam element} \]
\[ N_p = \text{number of pairs of shear connectors on a beam element} \]
\[ N_s = \text{axial force in the steel beam} \]
\[ P = \text{column force} \]
\[ Q = \text{total shear force in all the connectors on a beam element, shear force in a column} \]
\[ Q' = \text{horizontal shear strength of the reinforced concrete slab} \]
\[ S_x = \text{statical moment of the steel beam area about the reference plane} \]
\[ S'' = \frac{K'' S_x}{K_c + K_s} \]
\[ T = \text{torque, maximum tensile force in the slab, beam force} \]
\[ T_o = \text{torque due to a pair of connectors} \]
\[ V = \text{shear force (general), maximum permissible force in the slab} \]
\[ V' = \text{shear strength of a group of connectors} \]
\[ W = \text{gravity load on a floor} \]
\( a_c \) = area of a shear connector
\( a' \) = transition length
\( b \) = flange width (general)
\( c \) = height of the shear connectors
\( d \) = depth of the steel beam
\( e \) = eccentricity
\( f'_c \) = unconfined compressive strength of concrete
\( f_t \) = tensile strength of concrete
\( f_y \) = yield stress of steel (general)
\( f_{yr} \) = yield stress of slab reinforcement
\( h \) = story height
\( k_c \) = elastic stiffness of a shear connector
\( \lambda \) = length of a beam element
\( n \) = ratio of Young's moduli \( \frac{E_y}{E_x} \)
\( p \) = distance between rows of connectors
\( q \) = distance from column centerline to first shear connector
\( q_u \) = maximum shear strength of a shear connector
\( t \) = flange thickness, slab thickness
\( u \) = displacement in the x-direction of the reference plane
\( v \) = displacement in the y-direction of the reference plane
\( v_u \) = mean ultimate longitudinal shear stress in the slab
\( w \) = displacement in the z-direction of the reference plane
\( y \) = distance from reference plane to centroidal axis of steel beam
\( \alpha \) = angle
\( \gamma \) = shear strain, \( 1/E \)
$\Delta$ = horizontal deflection (drift) of a building, relative horizontal slip between the slab and the steel beam

$\varepsilon$ = normal strain

$\theta$ = rotation of the reference plane

$\phi$ = Airy stress function, curvature

$\sigma$ = normal stress

$\rho$ = area of longitudinal reinforcement per unit cross-sectional area of the slab

$\tau$ = shear stress

$\nu$ = Poisson ratio
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BEAM SIZE</th>
<th>( \ell ) in.</th>
<th>( f'_c ) ksi</th>
<th>( N_c )</th>
<th>( p ) in.</th>
<th>( c ) in.</th>
<th>( \frac{A'_s}{A_s} )</th>
<th>( \frac{J_s'}{J_s} )</th>
<th>( \frac{I'_x}{I_x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEAM SIZE</td>
<td>W16x40</td>
<td>30</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.27</td>
<td>.98</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>W30x116</td>
<td>30</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.11</td>
<td>.88</td>
<td>.40</td>
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<tr>
<td>BEAM ELEMENT LENGTH (( \ell ))</td>
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<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.27</td>
<td>.98</td>
<td>.48</td>
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<td>3</td>
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<td>.27</td>
<td>.98</td>
<td>.48</td>
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<tr>
<td></td>
<td>W16x40</td>
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<td>6</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.38</td>
<td>.98</td>
<td>.56</td>
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<tr>
<td>NUMBER OF SHEAR CONNECTORS (( N_c ))</td>
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<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.27</td>
<td>.98</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>W16x40</td>
<td>30</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>.16</td>
<td>.97</td>
<td>.40</td>
</tr>
<tr>
<td>DISTANCE BETWEEN ROWS OF CONNECTORS (( p ))</td>
<td>W16x40</td>
<td>30</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>.27</td>
<td>.98</td>
<td>.48</td>
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<td></td>
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<td>4</td>
<td>.27</td>
<td>1.00</td>
<td>.48</td>
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<tr>
<td>CONNECTOR LENGTH (( c ))</td>
<td>W16x40</td>
<td>30</td>
<td>3</td>
<td>6</td>
<td>3</td>
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<tr>
<td></td>
<td>W16x40</td>
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<td>3</td>
<td>2</td>
<td>.27</td>
<td>.99</td>
<td>.48</td>
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</table>

DIAMETER OF CONNECTORS = 3/4 in.
SLAB THICKNESS = 5 in.

Table 3.1: Effect of Flexible Shear Connection on the Transformed Properties of the Steel Beam
### Table 7.1: Equivalent Slab Widths for Composite Frame No. 7.1

<table>
<thead>
<tr>
<th>CURVE NUMBER</th>
<th>CONNECTOR SPACING ON FRAME BEAMS</th>
<th>DISCONTINUITY IN THE SLABS</th>
<th>CONCRETE CRACKING</th>
<th>EQUIVALENT SLAB WIDTH PANEL WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VERY SMALL</td>
<td>AISC MIN.</td>
<td>NORMAL (6&quot;)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td>X</td>
<td>X</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
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</table>

### Table 7.2: Equivalent Slab Widths for Composite Frame No. 7.3

<table>
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<tr>
<th>FLOOR LEVEL</th>
<th>EQUIVALENT SLAB WIDTH PANEL WIDTH</th>
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<tbody>
<tr>
<td></td>
<td>CALCULATED</td>
</tr>
<tr>
<td>1</td>
<td>-----</td>
</tr>
<tr>
<td>2</td>
<td>-----</td>
</tr>
<tr>
<td>3</td>
<td>-----</td>
</tr>
<tr>
<td>4</td>
<td>.132</td>
</tr>
<tr>
<td>5</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>-----</td>
</tr>
<tr>
<td>7</td>
<td>-----</td>
</tr>
<tr>
<td>8</td>
<td>.133</td>
</tr>
<tr>
<td>9</td>
<td>-----</td>
</tr>
<tr>
<td>10</td>
<td>-----</td>
</tr>
</tbody>
</table>
Table 7.3: Comparison of Equivalent Slab Widths

<table>
<thead>
<tr>
<th>COMPOSITE FRAME NUMBER</th>
<th>EQUIVALENT SLAB WIDTH PANEL WIDTH</th>
<th>WITH FRAME BEAMS</th>
<th>WITH FRAME BEAMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>REF. 1.20</td>
<td>REF. 1.21</td>
<td>DISSERTATION</td>
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<tr>
<td>7.1</td>
<td>.32</td>
<td>.24</td>
<td>.133</td>
</tr>
<tr>
<td>7.3</td>
<td>.32</td>
<td>.24</td>
<td>.132</td>
</tr>
</tbody>
</table>

Table 7.4: Equivalent Slab Widths for all Composite Frames

<table>
<thead>
<tr>
<th>COMPOSITE FRAME NUMBER</th>
<th>FRAME BEAM SIZE</th>
<th>EQUIVALENT SLAB WIDTH PANEL WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>W16x40</td>
<td>.133</td>
</tr>
<tr>
<td>7.2</td>
<td>W14x26</td>
<td>.161</td>
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<tr>
<td>7.3</td>
<td>W14x22</td>
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<td>7.4</td>
<td>W12x16.5</td>
<td>.145</td>
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Fig. 1.1: MULTISTORY FRAME WITH SYMMETRIC GEOMETRY AND LOADING
Fig. 1.2a: COMPOSITE FRAME

Fig. 1.2b: STRUCTURAL DETAIL OF A COMPOSITE FRAME
Fig. 2.1: ASSUMED LOAD–DRIFT CURVE FOR A COMPOSITE FRAME
Fig. 2.2a: UNIAXIAL BEHAVIOR OF PLAIN CONCRETE

Fig. 2.2b: BIAXIAL BEHAVIOR OF PLAIN CONCRETE
Fig. 2.3a: REINFORCED CONCRETE SLAB CRACKED UNDER UNIAXIAL TENSION

Fig. 2.3b: SCHEMATIC MODEL FOR A REINFORCED CONCRETE SLAB WITH CRACKS
Fig. 2.4: TEST SET-UP FOR COMPOSITE BEAM-TO-COLUMN CONNECTIONS
Fig. 2.3: TYPICAL MOMENT-ROTATION CURVES FOR COMPOSITE BEAM-TO-COLUMN CONNECTIONS
Fig. 2.6a: UPPER BOUND FAILURE MECHANISM

Fig. 2.6b: LOWER BOUND STRESS FIELD
Fig. 2.7: DEFINITION OF ROTATION CAPACITY
Fig. 2.8a: SHRINKAGE GAPS BETWEEN THE SLAB AND THE COLUMN

Fig. 2.8b: EFFECTS OF DISCONTINUITIES IN THE SLAB
Fig. 2.9a: DISCONTINUITY OF THE FRAME BEAM FLANGES

Fig. 2.9b: CONTINUITY THROUGH PRESENCE OF HORIZONTAL STIFFENERS
Reinforced Concrete Slab

Steel Beam

Steel Stud Shear Connectors

Fig. 3.1: TYPICAL DETAIL OF A COMPOSITE STEEL-CONCRETE FLOOR
Fig. 3.2a: SCHEMATIC MODEL HAVING AXIAL AND BENDING STIFFNESS

Fig. 3.2b: SCHEMATIC MODEL HAVING ST. VENANT TORSIONAL STIFFNESS
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Fig. 3.3b: DISPLACEMENTS CAUSED BY A ROTATION $\theta_y$ OF THE REFERENCE PLANE
Fig. 3.4a: DISPLACEMENTS CAUSED BY A DISPLACEMENT W OF THE REFERENCE PLANE

Fig. 3.4b: ROTATION $\theta_{xs}$ CAUSED BY A ROTATION $\delta_x$ OF THE REFERENCE PLANE
Fig. 3.5: NODAL FORCES AND DISPLACEMENTS
Fig. 3.6a: RELATIVE HORIZONTAL SLIP BETWEEN THE SLAB AND THE STEEL BEAM

Fig. 3.6b: RELATIVE ROTATION BETWEEN THE SLAB AND THE STEEL BEAM
Fig. 4.1: SYMMETRIC COMPOSITE ONE-STORY ASSEMBLAGE
**Fig. 4.2a:** RECTANGULAR PLATE ELEMENT AND DEGREES OF FREEDOM AT EACH NODE

**Fig. 4.2b:** COLUMN ELEMENT AND DEGREES OF FREEDOM AT EACH NODE
Fig. 4.3: BOUNDARY CONDITIONS
Fig. 4.4: FINITE ELEMENT DISCRETIZATION
Fig. 4.5: ONE-BAY COMPOSITE ONE-STORY ASSEMBLAGE

Diameter of Connectors = \( \frac{3}{4} \)"

Spacing of Connectors = 9"

Height of Connectors = 4"

C of Panel

Beam Under

C of Columns

120,000 k.in.

2000k

5" Concrete Slab

240,000 k.in.

4000k

W12 x 27

W14 x 370

W30 x 116

W14 x 760

15'

13'

30'
Fig. 4.6a: TYPICAL FINITE ELEMENT LAYOUT OF THE SLAB

Fig. 4.6b: DEFINITION OF THE DIMENSIONS $q_1$ AND $q_2$
Fig. 4.7a: FINITE ELEMENT LAY-OUT FOR MESH GRADE OF 1:2.5

Fig. 4.7b: FINITE ELEMENT LAY-OUT FOR MESH GRADE OF 1:1.1
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Fig. 4.9: EFFECT OF FOUR VARIABLES ON THE MESH GRADE
Fig. 5.1a: COMPOSITE ONE-STORY ASSEMBLAGE UNDER COMBINED GRAVITY AND WIND LOADS

Fig. 5.1b: TYPICAL BENDING MOMENT DIAGRAM AT MAXIMUM LOAD
Fig. 5.2a: INTERIOR COMPOSITE BEAM-TO-COLUMN CONNECTION

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Fig. 5.3: MAXIMUM POSSIBLE FORCES IN THE SLAB AT THE COLUMN

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Fig. 5.5: DEFINITION OF TRANSITION LENGTHS
Fig. 5.6: INTERIOR PANEL
Fig. 5.7a: SLAB FORCE $V_{cl}$

Fig. 5.7b: COMPONENTS OF $V_{cl}$
Fig. 5.8a: SLAB FORCE $V_{c2}$

Fig. 5.8b: COMPONENTS OF $V_{c2}$
Fig. 5.9: STRESS DIAGRAMS FOR DETERMINING PLASTIC MOMENTS
Fig. 6.1a: FINITE ELEMENT REPRESENTATION OF DISCONTINUITIES IN THE SLAB

\[ E_x = 0 \]

Fig. 6.1b: FINITE ELEMENT REPRESENTATION OF CRACKING OF THE CONCRETE SLABS

\[ E_x = E_e \]
Fig. 7.1: DETAIL OF STEEL FRAME NO. 7.1

\( f_y = 36 \text{ ksi} \)
Fig. 7.2: LOAD-DRIFT CURVES OF STEEL FRAME NO. 7.1 AND COMPOSITE FRAME NO. 7.1
Fig. 7.3: EFFECT OF SEVERAL VARIABLES ON THE SERVICE LOAD DRIFT OF COMPOSITE FRAME NO. 7.1
Concrete: $f'_c = 3$ ksi
Steel: $f_y = 36$ ksi

Fig. 7.4: DETAIL OF COMPOSITE FRAME NO. 7.2
Fig. 7.5: LOAD-DRIFT CURVES OF COMPOSITE FRAME NO. 7.2
AND STEEL FRAME NO. 7.1
Fig. 7.6: DETAIL OF STEEL FRAME NO. 7.3

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Fig. 7.7: LOAD-DRIFT CURVES OF STEEL FRAME NO. 7.3 AND COMPOSITE FRAME NO. 7.3
Fig. 7.8: DETAIL OF COMPOSITE FRAME NO. 7.4
Fig. 7.9: LOAD-DRIFT CURVES FOR STEEL FRAME NO. 7.3 AND COMPOSITE FRAME NO. 7.4
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CHAPTER 6

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14. APPENDIX

DIFFERENTIAL EQUATION FOR IN-PLANE BEHAVIOR OF AN ORTHOTROPIC PLATE

The strain-stress relationship for an orthotropic material in-plane stress is

\[ \varepsilon_x = \frac{\sigma_x}{E_x} - \nu \frac{\sigma_y}{E_y} \]  \hspace{1cm} (10.1a)

\[ \varepsilon_y = -\nu \frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} \]  \hspace{1cm} (10.1b)

\[ \gamma_{xy} = \frac{\tau_{xy}}{G_{xy}} \]  \hspace{1cm} (10.1c)

Symmetry of Eqs. 10.1a and 10.1b requires that

\[ \frac{\nu_y x}{E_x} = \frac{\nu_y x}{E_y} \]  \hspace{1cm} (10.2)

In Art. 2.3.2 the value for \( G_{xy} \) as given in Ref. 2.8 was used. That value for \( G_{xy} \) was obtained by considering a layered material. In this appendix the value for \( G_{xy} \) will not be specified in order to keep the formulation general.

The stresses can be written in terms of the Airy stress function \( \phi \), that is,

\[ \sigma_x = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \]  \hspace{1cm} (10.3)

Substitution of Eq. 10.3 in 10.1 gives

\[ \varepsilon_x = \frac{1}{E_x} \frac{\partial^2 \phi}{\partial y^2} - \nu \frac{\partial^2 \phi}{E_y} \frac{\partial^2 \phi}{\partial x^2} \]  \hspace{1cm} (10.4a)
The compatibility condition in two dimensions is given by (4.2)

\[ \frac{\partial^2 \varepsilon_y}{\partial y^2} + \frac{\partial^2 \varepsilon_x}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]  

Substitution of Eqs. 10.4 in Eq. 10.5 gives

\[ \frac{1}{E_x} \frac{\partial^4 \phi}{\partial y^4} - \frac{\nu_{yx}}{E_y} \frac{\partial^4 \phi}{\partial x^2 \partial y^2} - \frac{\nu_{yx}}{E_x} \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{1}{E_y} \frac{\partial^4 \phi}{\partial x^4} = - \frac{1}{G_{xy}} \frac{\partial^4 \phi}{\partial x \partial y^2} \]

that is

\[ \frac{1}{E_x} \frac{\partial^4 \phi}{\partial y^4} + \left( \frac{1}{G_{xy}} \frac{\nu_{yx}}{E_y} - \frac{\nu_{yx}}{E_x} \right) \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{1}{E_y} \frac{\partial^4 \phi}{\partial x^4} = 0 \]  

Using Eq. 10.2 then Eq. 10.6 becomes

\[ \frac{1}{E_x} \frac{\partial^4 \phi}{\partial y^4} + \left[ \frac{1}{G_{xy}} - \frac{2 \nu_{yx}}{E_y} \right] \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{1}{E_y} \frac{\partial^4 \phi}{\partial x^4} = 0 \]  

Equation 10.7 is the governing differential equation for in-plane behavior of an orthotropic plate.