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LATERAL DISTRIBUTION OF LIVE LOAD
in
PRESTRESSED CONCRETE I-BEAM BRIDGES

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ABSTRACT

This is the second report on the research investigation entitled "Development and Refinement of Load Distribution Provisions for Prestressed Concrete Beam-Slab Bridges" (PennDOT 72-4). The beam-slab bridges included in this study are of the I-beam type. Included are (1) a structural analysis, based on the finite element method, which describes superstructure response to design-vehicle loading, (2) a comparison of the structural analysis with results from the field tests of two in-service bridge superstructures, (3) the analysis of 219 superstructures ranging in length from 30 ft. to 135 ft. and in roadway width from 20 ft. to 78 ft., and (4) equations for evaluating live-load distribution factors for interior and exterior beams, based on the definition of traffic lanes set forth in the AASHTO "Standard Specifications for Highway Bridges - 1973" (1.2.6 - Traffic Lanes).
1. INTRODUCTION

1.1 General

Over the past eleven years, Lehigh University has conducted a major research program on the structural behavior of prestressed concrete beam-slab highway bridge superstructures subjected to design vehicle loading conditions. The superstructures basically consist of a number of longitudinal precast prestressed concrete beams, equally spaced and spread apart, along with a cast-in-place composite reinforced concrete deck slab. The research program has included: (1) field studies of eight in-service bridges, (2) laboratory studies of 1/16-scale model bridges, and (3) the development of a complex mathematical computer-based analysis.

The first part of the overall research program was devoted to a study of spread box-beam superstructures. Based on the results from the study, a new specification provision was proposed, covering lateral distribution of live loads. This provision was adopted by AASHTO in Fall, 1972, and now appears as Article 1.6.24 in the 1973 AASHTO Standard Specifications for Highway Bridges. Currently, the overall investigation is progressing under PennDOT Research Project No. 72-4, entitled "Development and Refinement of Load Distribution for Prestressed Concrete Beam-Slab Bridges". The primary objectives of the overall investigation are:

1. To develop a new provision for live load distribution
in prestressed concrete I-beam bridge superstructures, paralleling the already adopted provision for spread box-beam bridges.

2. To expand the live load distribution provisions for spread box-beam bridges (Article 1.6.24), and the proposed new provisions for I-beam bridges, to include provisions for the inclusion of the effects of skew.

3. To investigate the possibility of extending the analysis and specification development to cover: (a) the effects of interior-span diaphragms, (b) the effects of curb-parapet sections, and (c) continuous-span construction.

Currently, the AASHTO provisions for the distribution of live load in prestressed concrete I-beam superstructures are listed under Article 1.3.1 of the 1973 AASHTO Standard Specifications for Highway Bridges. Both field tests and preliminary analytical work have indicated the inadequacy of the current specifications. Under objective No. 1, two separate analyses have been conducted. The first analysis, reported herein, is based on the definition of traffic lanes as specified in Article 1.2.6 of the 1973 AASHTO Specifications. The second part, which will be presented in report No. 387.2B, is based on the current definition of traffic lanes as specified under the revised Article 1.2.6 set forth in the 1974 AASHTO Interim Specifications for Bridges. There will be two additional reports on this project, Nos. 387.3 and 387.4, which will cover objectives Nos. 2 and 3, respectively.
1.2 Objectives

The objective of the investigation reported herein is to develop a refined method for the evaluation of live-load distribution factors for right (no skew) beam-slab bridge superstructures of the prestressed concrete I-beam type, based on the definition of traffic lanes as specified in Article 1.2.6 of the 1973 AASHTO Specifications. The investigation is based on a structural analysis, developed at Lehigh University, which is a finite element stiffness formulation for eccentrically stiffened plate structures in the linear elastic range. A description of this formulation is presented in Chapter 2.

The analysis was first evaluated by comparison with the results from the field tests\textsuperscript{3,4,9,10} of two in-service bridges. Based on these comparisons, the analysis was refined to enable an accurate and efficient study of load distribution. This phase of the investigation is presented in Chapter 3.

Next, a plan was prepared to enable the systematic variation of parameters which would form the basis for the development of the equations for load distribution. Chapter 4 describes the plan, and the results are presented in Chapter 5.

Finally, the equations for evaluating live-load distribution factors for interior and exterior beams are presented in Chapter 6.
Load distribution in highway bridges has been studied for many years, both in this country and abroad. Though the previous work has resulted in a greater understanding of the behavior of bridges, a number of simplifying assumptions were made in each case in order to overcome the mathematical difficulties involved in the solution procedures. The methods used to study the behavior of bridges have been the grillage analysis, folded and orthotropic plate theories, the finite difference method, the finite strip method, and the finite element method. Of all of the methods, the finite element method requires the fewest simplifying assumptions in accounting for the greatest number of variables which govern the structural response of the bridge. Therefore, the technique chosen was a structural analysis for stiffened plate structures, developed at Lehigh University, which utilized the finite element displacement approach.

It is not the purpose of this report to provide a discussion of previous work. An up-to-date annotated bibliography containing references which are directly or indirectly applicable to the structural behavior, analysis, and design of beam-slab type highway bridges was presented in a previous report from this project.
2. ANALYSIS BY THE FINITE ELEMENT METHOD

2.1 Assumptions

The following assumptions were made in the finite element analysis of the bridge superstructures investigated as part of this research.

1. A small strain - small deflection theory was used.
2. Linearly elastic behavior of materials was assumed.
3. All superstructures were analyzed with simple supports.
   The effects of continuity were not included.
4. The longitudinal beams were prestressed concrete I-beams, either from Pennsylvania Standard\textsuperscript{[7]} or from AASHTO-PCI Standard cross-sections.
5. All loading conditions were static. No dynamic effects were considered.
6. The response of the slab was divided into out-of-plane and in-plane behavior. The out-of-plane behavior accounted for actions such as the normal stress associated with composite action of the beams and slab.
7. The in-plane and out-of-plane responses were superimposed.
8. The mid-plane of the deck slab was taken as the reference plane for the analysis technique.
9. The deck slab was assumed to have a constant thickness.
   Haunching for grade or camber was not included, nor was
the presence of permanent metal deck forms or the concrete below the top surface of the deck form. These are conservative assumptions.

10. Local stresses produced by the individual wheel loads were considered to have a negligible effect on the live load distribution factors, and were not considered in the analysis.

11. Beams and slabs were assumed to act in a completely composite manner. Thus, the strain compatibility between the deck slab and the beam was maintained.

12. The beams were modeled as eccentric stiffeners to the slab.

13. The action of each beam was satisfactorily represented by a normal force, a bending moment about one axis, and a torsional moment. Weak-axis bending was ignored because of the relative stiffnesses of I-beam sections, and because only vehicular loading was considered.

14. The St. Venant torsional stiffness of the beams was considered. Warping torsion was assumed to be small because of the shape of the I-beams (Ref. 11). Appropriate values of the St. Venant torsional stiffness coefficient were computed and reported in Ref. 6.

15. The cross-sections of the structures analyzed in this research were reasonably proportioned. That is, for a particular structure, the beam size and spacing were appropriate for the span length, and the slab thickness was appropriate for the beam spacing.
16. The effect of the curb-parapet section was considered, as discussed in Sec. 3.3.4.

17. Intra-span diaphragms were not included in this analysis, since past research\(^9,11\) has shown that while these diaphragms are effective in distributing the live load from a single vehicle, the effect becomes minimal when several lanes are loaded.

18. The number of loaded lanes conformed to Article 1.2.6 of Ref. 1, as discussed in Sec. 4.2.

19. AASHTO type HS20-44 loading was used throughout the entire study. For spans up to 150 ft., a single HS20-44 vehicle was used. For spans in excess of 150 ft. the truck train was used which was the predecessor of the current lane loading, as described in Appendix B of Ref. 1. In deciding on the truck train, comparisons were made of the effects of a single HS20-44 vehicle, the truck train, and the lane loading. It was found that the lateral load distribution was not materially affected by the type of loading. Generally, there was less than 2% difference between the maximum and minimum distribution percentages produced by the three types of loadings. Therefore, the truck train was used for spans in excess of 150 ft. because the corresponding input could be handled automatically within the computer program.
2.2 Finite Element Analysis

The finite element method has three basic phases:

1) Structural Idealization

2) Evaluation of element properties

3) Assembly and analysis of the structural system.

In the current analysis, the beams and slab were treated separately, and then combined in the third phase. This presentation will follow the same pattern by discussing first the analysis of deck slabs, then the analysis of beams, and finally the assembly of beam and slab elements. This analysis is based on the formulation by Wegmuller and Kostem.11,12

2.2.1 The Deck Slab

As mentioned in Sec. 2.1, the response of the deck slab was further divided into out-of-plane (bending) and in-plane (membrane) actions.

2.2.1.1 The Out-of-Plane Behavior of the Deck Slab

The deck slab was analyzed using thin plate theory. Hence, the following assumptions were made:

1. Sections which were plane and normal to the middle surface before deformation remained plane and normal after deformation.

2. Transverse displacements were small compared to the plate thickness.

-8-
3. Since stresses normal to the plane of the plate were negligible, shearing stresses in the transverse direction were neglected, and the transverse displacement of any point on the plate was essentially the displacement of the corresponding point on the middle surface of the plate.

The deck slab was discretized into rectangular plate bending elements. The element developed by Adini, Clough, and Melosh\textsuperscript{2} was used. The plate elements were connected at node points. A node point was common to all of the elements which surrounded it. The displacements at the node points were the basic unknowns of the finite element stiffness analysis. There were three out-of-plane displacements assigned to each plate element node point. These displacements were the transverse displacement, \( W \), and the bending rotations \( \theta_x \) and \( \theta_y \). These displacements occurred at the mid-plane of the plate. Thus, there were a total of twelve out-of-plane degrees of freedom (i.e., unknown displacements) associated with each plate bending element.

A polynomial displacement function was used to describe the displacements within the plate bending element.

\[
W = \alpha_1 + \alpha_2 X + \alpha_3 Y + \alpha_4 XY + \alpha_5 X^2 + \alpha_6 Y^2 + \alpha_7 XY^2 + \alpha_8 X^2 Y + \alpha_9 X^3 + \alpha_{10} Y^3 + \alpha_{11} X^3 Y + \alpha_{12} XY^3
\] (2.1)
The nodal rotations are given as derivatives of the transverse displacement, $W$.

\[
\theta_x = \frac{\partial w}{\partial y} \quad (2.2)
\]

\[
\theta_y = -\frac{\partial w}{\partial x} \quad (2.3)
\]

There are twelve unknown constants in Eq. 2.1 and twelve boundary conditions for each element: three displacements at each of four nodes. Substituting Eq. 2.1 into Eqs. 2.2 and 2.3, and then substituting the coordinates of the corners of the elements with respect to the element axes (shown in Fig. 1), the following equation is obtained:

\[
\{\delta^e\}_o = [C]_o \{a\} \quad (2.4)
\]

The subscript "o" indicates out-of-plane displacements. The constants \(\{a\}\) are evaluated by matrix inversion.

\[
\{a\} = [C]^{-1}_o \{\delta^e\}_o \quad (2.5)
\]

The strains within the element are related to the displacement field by the strain displacement equations. Within the context of the finite element method, strains and stresses are usually referred to as generalized strains and generalized stresses.

The generalized strains for out-of-plane behavior are the bending curvatures. Thus, it is possible to define the strains as:
Substitution of Eq. 2.1 into Eq. 2.6 results in the matrix equation:

\[
\{ \varepsilon \} = [Q] \{ \alpha \} \tag{2.7}
\]

Substitution of Eq. 2.5 into Eq. 2.7 relates the generalized strains to the unknown nodal equations:

\[
\{ \varepsilon \} = [Q] [C]^{-1} \{ \delta^e \} \tag{2.8}
\]

Stresses are related to strains by an elasticity matrix:

\[
\{ \sigma \} = [D] \{ \varepsilon \} \tag{2.9}
\]

The stresses corresponding to the strains given by Eq. 2.6 are the bending moments per unit distance: \( M_x, M_y, \) and \( M_{xy} \). Using the well-known equations of plate analysis (Ref. 8), the elasticity matrix is defined as:

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix} \begin{bmatrix}
\phi_x \\
\phi_y \\
\phi_{xy}
\end{bmatrix} \tag{2.10}
\]

where \( E \) is the modulus of elasticity of the plate, \( h \) is the plate thickness, and \( \nu \) is Poisson's Ratio. Once these matrices have been defined, the well-established procedures of the finite element
method lead to the following stiffness matrix (Ref. 14):

\[
[K]_0 = [C]_0^{-1} \int_A [Q]^T [D] [Q] \, dx \, dy \, [C]_0^{-1}
\]  

(2.11)

The out-of-plane stiffness matrix, \([K]_0\), is given explicitly in Refs. 5, 11, and 14.

2.2.1.2 The In-Plane Behavior of the Deck Slab

The in-plane behavior of the plate is analyzed as a plane-stress elasticity problem. The discretization remains the same as discussed of out-of-plane behavior. There are two in-plane displacements at each node. The displacement in the \(x\)-direction (Fig. 1) is called \(U\), the displacement in the \(y\)-direction is \(V\). There are a total of eight in-plane degrees of freedom. The polynomial displacement functions are given by Eqs. 2.12 and 2.13.

\[
U = \alpha_{13} + \alpha_{14} \, x + \alpha_{15} \, y + \alpha_{16} \, xy
\]  

(2.12)

\[
V = \alpha_{17} + \alpha_{18} \, x + \alpha_{19} \, y + \alpha_{20} \, xy
\]  

(2.13)

As in the out-of-plane case, the eight unknown constants in Eqs. 2.12 and 2.13 are evaluated using the eight nodal displacements:

\[
\{\delta^e\}_I = [C]_I \{\alpha\}
\]  

(2.14)

\[
\{\alpha\} = [C]_I \{\delta^e\}_I
\]  

(2.15)

The generalized strains are taken as:
\[
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(2.16)

Substitution of Eqs. 2.12 and 2.13 into 2.16 results in:

\[
\{\varepsilon\} = [Q] \{\alpha\}
\]

(2.17)

Substituting Eq. 2.15 into Eq. 2.19 results in the strain-displace-
ment relations:

\[
\{\varepsilon\} = [Q] \{\delta^e\}
\]

(2.18)

The stresses are chosen as the membrane stresses \(\sigma_x, \sigma_y\) and \(\tau_{xy}\).

The resulting elasticity matrix, based on the assumption of plane
stress, is given by:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \frac{E}{1-\nu^2}
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(2.19)

The basic matrices necessary to evaluate Eq. 2.13 are now known for
the in-plane case, and the in-plane stiffness matrix, \([K]_I\), can now
be evaluated. The in-plane stiffness matrix is also given explicit-
ly in Ref. 11.
2.2.1.3 Superposition of In-Plane and Out-of-Plane Behaviors

Since the analysis is based on a small deflection theory with linear material properties, as mentioned in Sec. 2.1, the in-plane and out-of-plane stiffness matrices may be superimposed as follows:

\[
\begin{bmatrix}
[F]_I \\
[F]_O
\end{bmatrix} =
\begin{bmatrix}
K_I & 0 \\
0 & K_O
\end{bmatrix}
\begin{bmatrix}
\delta e_I \\
\delta e_O
\end{bmatrix} \tag{2.20}
\]

\([F]_I\) and \([F]_O\) are the in-plane and out-of-plane nodal force vectors, respectively.

2.2.2 The Beams

Figure 2 shows a beam element, nodal points, coordinate axes, and degrees of freedom. The degrees of freedom consist of an in-plane axial displacement, \(U\), out-of-plane bending displacements, \(W\) and \(\theta_y\), and a torsional rotation, \(\theta_x\), at each node. Beam elements are positioned between plate nodes in the \(x\)-coordinate direction.

The in-plane and out-of-plane response of beam elements are considered simultaneously. The torsional response is treated separately.

2.2.2.1 The In-Plane and Out-of-Plane Behavior of Beams

The polynomial displacement functions for the response of beam element, not including the effects of torsion, are given by:
These displacements occur in the same reference plane that is used for calculation of the plate displacements (Fig. 2). In this formulation the reference plane was the mid-plane of the deck slab. It should be noted that Eqs. 2.21 and 2.22 have the same form as Eqs. 2.12 and 2.1 when the coordinate $y$ is equal to a constant. This fact, combined with a choice of beam eccentricity referenced to the mid-plane of the deck slab, provides strain compatibility between the deck slab and the beam. This is necessary to correctly model composite beam-slab bridges. The bending rotation, $\theta_y$, is defined by Eq. 2.3.

The six unknown constants in Eqs. 2.21 and 2.22 are evaluated using the six nodal displacements, three at each end of the beam:

$$\{\delta^e\}_B = [C]_B \{\alpha\}$$

$$\{\alpha\} = [C]_B^{-1} \{\delta^e\}_B$$

The generalized strains are taken as the bending curvature and axial strain.

$$\{\epsilon\} = \begin{bmatrix} \frac{dy}{dx} \\ -\frac{d^2w}{dx^2} \end{bmatrix}$$
The generalized stresses corresponding to these strains are the axial force and bending moment.

\[
\{\sigma\} = \begin{bmatrix} N \\ M \end{bmatrix} \quad (2.26)
\]

The strain in the beam can be related to Eq. 2.25 as shown in Fig. 3.

\[
\bar{\varepsilon} = \frac{du}{dx} - Z \frac{d^2w}{dx^2} \quad (2.27)
\]

The bar indicates that the strain is referred to the reference plane. The stress is equal to Young's modulus times the strain.

\[
\bar{\sigma} = E \bar{\varepsilon} = E \left[ \frac{du}{dx} - Z \frac{d^2w}{dx^2} \right] \quad (2.28)
\]

The generalized stresses are related to \(\bar{\sigma}\) by the integrals:

\[
N = \int_A E \bar{\varepsilon} \, dA = E \frac{du}{dx} \int_A \, dA - E \frac{d^2w}{dx^2} \int_A Z \, dA = E \frac{A}{A} \frac{du}{dx} - E \frac{S}{S} \frac{d^2w}{dx^2} \quad (2.29)
\]

\[
M = \int_A E Z \bar{\varepsilon} \, dA = E \frac{du}{dx} \int_A Z \, dA - E \frac{d^2w}{dx^2} \int_A Z^2 \, dA = E \frac{S}{S} \frac{du}{dx} - E \frac{I}{I} \frac{d^2w}{dx^2} \quad (2.30)
\]

The elasticity matrix is defined by using Eqs. 2.29 and 2.30:

\[
\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{S} \\ \bar{S} & \bar{I} \end{bmatrix} \begin{bmatrix} \frac{dy}{dx} \\ - \frac{d^2w}{dx^2} \end{bmatrix} \quad (2.31)
\]

The bars in Eq. 2.31 indicate that the appropriate quantities are referred to the reference plane, not necessarily to the centroidal.
axis of the beam.

Substituting Eqs. 2.21 and 2.22 into Eq. 2.25 leads to the
definition of [Q]. Once this is done, all of the matrices are defin-
ed to evaluate the nontorsional stiffness matrix of the eccentric
beam element:

\[ [K] = [c]_B^{-1} \int \frac{[Q]^{T} [D] [Q]}{x} \, dx \, [c]_B^{-1} \]  \hspace{1cm} (2.32)

The beam stiffness matrix above is given explicitly in Ref. 11.

2.2.2.2 The Torsional Behavior of the Beams

The St. Venant torsional stiffness of the prestressed
concrete I-beams is included in the analysis. The warping torsion
effects are neglected. The St. Venant torsional moment can be re-
lated to the unit angle of twist by:

\[ T_{sv} = G K_T \phi' \] \hspace{1cm} (2.33)

The unit angle of twist can be related to the axial rotation of the
beam by:

\[ \{e\} = \phi' = \frac{3}{\alpha x} \theta_x = \frac{3}{\alpha x} \frac{\partial w}{\partial y} \] \hspace{1cm} (2.34)

Substitution of the displacement function for the plate (Eq. 2.1) in-
to Eq. 2.34 results in the assumed displacement function for \( \theta_x \) along
a line defined by a constant y coordinate.

\[ \theta_x = \frac{\partial w}{\partial y} = a_{27} + a_{28} x \] \hspace{1cm} (2.35)

The elemental displacement vector consists of values of
\( \theta_x \) at each end of the beam. Thus, a connection matrix analogous to
Eqs. 2.4, 2.14, 2.15, and 2.23 can be defined.
The generalized stress and strain are the torsional bending moment and the unit angle of twist, respectively. Thus, an elasticity matrix is defined as shown above.

\[
\{\delta_e\}_T = [C]_T \{\alpha\} \tag{2.36}
\]

\[
\{\alpha\} = [C]^{-1}_T \{\delta_e\}_T \tag{2.37}
\]

The matrix \([Q]\) is again defined by substituting the displacement functions given by Eq. 2.35 into the definition of strain given by Eq. 2.34. When this is done, all of the matrices needed to define the stiffness matrix are known, and evaluation may proceed. An explicit torsional stiffness matrix is given in Ref. 11.

2.3 Assembly of Elements

The assembly of elements in the finite element method is analogous to the assembly of member-stiffness matrices in conventional matrix structural analysis. The slab element stiffness matrix relates a force at one node to the displacements of the remaining nodes in that element. Each node may be surrounded by as many as four slab elements which join that node. Thus, a force at one node may be related to the displacements of all the nodes in four elements. This means that, including the fact that some nodes will be common to the adjoining elements, a total of 9 nodes having forty-five degrees of freedom could be related to the single force component. The process of relating the force to all of the adjoining ele-
ments and their degrees of freedom is called assembly of the global stiffness matrix. The problem of finding the appropriate node points related to a given node point is a matter of specifying structural topology to the computer program which actually performs the arithmetic operations, and will not be discussed in this report.

The superposition of beam stiffness components is accomplished by straight-forward addition of corresponding beam and slab element stiffness components. This includes isolating the nodes to which beam elements are attached. The force at a node having a beam element is related to the beam displacements at the adjacent nodes in the x-direction. This is also a matter of topology which is specified as input to the computer program, and will not be discussed in this report.

2.4 Solution and Back Substitution

The assembly of the element stiffness matrices results in a set of simultaneous equations relating nodal forces to nodal displacements. These equations are solved for the nodal displacements after the boundary conditions are enforced. Once the displacements are known, it is possible to back-substitute them into appropriate equations to compute the generalized stresses. Thus, substitution of nodal displacements into the beam stiffness matrix results in the normal force, bending moment, and torsional moment at the beam node points. These forces act at the plane of reference, i.e., the mid-plane of the plate. This fact is important in evaluating the lateral load distribution in bridges.
Substitution of the appropriate nodal displacements into Eq. 2.8, followed by substitution of the results into Eq. 2.9, enables the evaluation of the unit bending moments $M_x, M_y, and M_{xy}$ at the node points. The inplane stresses (or forces) can be evaluated in a similar manner.

2.5 Computation of Moment Percentages

A moment percentage is defined as the bending moment carried by one beam, where the beam can be considered as the total composite cross-section, divided by the total of the moments carried by all the beams, and multiplied by 100. The moment carried by one composite cross-section is given by:

$$M_c = \int_{\text{beam}} \sigma_x Z \, dA + \int_{\text{slab}} \sigma_x Z \, dA$$  \hspace{1cm} (2.39)

where $Z$ is a coordinate from any reference plane. If the reference plane is chosen as the mid-plane of the plate, Eq. 2.39 may be re-written as:

$$M_c = M_{x_{\text{beam}}} + b_{\text{eff}} \int_{o}^{b_{\text{eff}}} (M_{x_{\text{slab}}}) \, dl$$  \hspace{1cm} (2.40)

in which $b_{\text{eff}}$ is the effective width of the slab. It was noted in Sec. 2.2 that provisions were made to reference the beam moment to any arbitrary reference plane, including the mid-plane of the plate. It is this moment which is found by back-substitution, as discussed in Sec. 2.2.4.
The problem of finding the effective flange width is simplified by the relative sizes of the unit slab bending moment, \( M_{x_{\text{slab}}} \), and the beam bending moment about the mid-plane of the plate. The total slab moment across the bridge width is only a small percentage of the total of the composite beam moments. Sample calculations indicate that for beam-slab bridges, the total slab moment is generally \(< 5\%\) of the total. Therefore, the effect of a small error in the effective flange width is an insignificant difference in the moment percentages as calculated in this research. As a result, the following approximate effective flange widths were used in lieu of more exact calculations:

1. For interior beams, the actual beam spacing was used.
2. For exterior beams, one half of the spacing, plus the over-hang was used.

Having the effective flange width and choosing the slab moment at the node over the beam as representative width of the superstructure, Eq. 2.40 reduces to:

\[
M_c = M_{\text{beam}} + (M_{x_{\text{slab}}}) (b_{\text{eff}}) \tag{2.41}
\]

The moment percentage of one beam is then calculated as:

\[
M_{\text{p}} = \frac{M_{c_i}}{\sum_{i=1}^{n} M_{c_i}} \tag{2.42}
\]
in which i denotes the beam in question and n is the total number of beams. These moment percentages were used to produce influence lines for a given bridge. These influence lines were then loaded to determine the maximum distribution factor for a given bridge.
3. ANALYTIC MODELING STUDY

3.1 Purpose of Analytic Modeling Study

The finite element technique described in Chapter 2 of this report was used in the study of lateral load distribution in I-beam bridges. A preliminary study was undertaken to investigate different methods of analytically modeling the I-beam bridges so as to use the finite element method effectively and efficiently. In this study the analytic models were compared to the field test results of two in-service I-beam bridges located near Lehighton and Bartonsville, Pennsylvania.

The results of the analytic modeling study were threefold. First, important design parameters of a bridge were isolated, described, and analyzed using analytic approximations. Thus, the influence of these design parameters such as the curb-parapet section and permanent metal deck forms were taken into account. Second, the analysis was verified by comparison with the results from the field tests. Third, the analytic bridge model was refined, to enable an accurate and efficient study of lateral load distribution.

3.2 Description of Field Test Bridges

The field testing of the Lehighton and Bartonsville bridges analyzed in this investigation is described in detail by Chen and VanHorn and Wegmuller and VanHorn. Initially, only the field
test results of the Lehighton bridge were used in comparison with
different analytic models. The reason for the emphasis on the
Lehighton bridge was two-fold. First, the Lehighton bridge was test-
ed both with and without midspan diaphragms between beams. Second,
there was only one curb-parapet section on the Lehighton bridge,
which allowed the effect of the curb-parapet section on load distri-
bution to be seen more readily. The Bartonsville bridge test results
were then compared to an analytic model which included all of the
features of modeling discussed in this chapter which are appropriate
to the Bartonsville bridge.

The cross-section of the Lehighton Bridge is shown in
Fig. 4. The main supporting members were six identical PennDOT 24/45
prestressed concrete I-beams spaced 6 feet 9 inches center-to-center.
The slab was cast-in-place over a permanent metal deck form, with a
nominal thickness of 7-1/2 inches. With a curb and parapet section
on only one side of the superstructure, the roadway width was 35 feet
11-1/2 inches. The span length was 71 feet 6 inches, center-to-center
of bearings.

The cross-section of the Bartonsville Bridge is shown in
Fig. 5. The main supporting members were five identical AASHTO-PCI
Type III prestressed concrete I-beams spaced 8 feet center-to-center.
The slab was cast-in-place with a nominal thickness of 7-1/2 inches.
The roadway width was 32 feet. The span length was 68 feet 6 inches,
center-to-center of bearings.
3.3 Analytic Modeling

3.3.1 Discretization of the Superstructure

Using the finite element technique, the actual bridges were modeled by a discretized bridge containing a suitable number of finite elements. Figure 6 shows the cross-section of the Bartonsville test bridge. Also shown is the plan view of the bridge, with the discretization indicated. The lines indicate boundaries between elements, and the intersections of those lines are nodal points. The beams were also discretized into beam elements, connected at the appropriate nodal points. In the discretization shown in Fig. 6, there are two plate elements between the beams. In the analytic modeling study, the discretization was varied according to the requirements of a particular analytic model.

In comparing the analytic and field test results, the moments at a cross-section called the maximum moment section of the bridge were used. The maximum moment section, shown as section M in Fig. 6, is the section at which the absolute maximum moment would occur in a simple beam of the same span as the bridge, when loaded with the test vehicle. The test vehicle, which closely approximated the AASHTO HS20-44 design vehicle, is shown in Fig. 7.

Comparisons of different analytic models were made using moment percentage diagrams. The definition of moment percentage for a particular beam is defined in Section 2.5 of this report.
3.3.2 Refinement of Slab Discretization

Figure 8 shows a typical segment of the cross-section of the test bridge. The figure shows that portions of the slab are supported by the relatively stiff flange of the I-beams. Because of the support provided by the flanges, the first investigation undertaken was the analytic modeling of the effective bending span of the slab between the beams.

Two different models were used to model the effective bending span of the slab. The first model was a mathematical approximation that was an accurate and efficient modeling technique. The second model was a theoretically better approximation, but was a far less efficient model. Though this second model would not be used in an extensive study, it was used here to verify the first modeling technique.

The first model, shown in Fig. 8, consisted of nodes positioned above the center of the beams and midway between the beams. This discretization, which consisted of two slab elements between beams was designated the 2 PL mesh. Using this discretization, the effective bending span was approximated by introducing an orthotropy factor (Dy) in the analysis. This factor was defined as the ratio of the transverse-to-longitudinal stiffness of a unit area of slab. The orthotropy factor was calculated as the square of the ratio of the center-to-center beam spacing to the flange-to-flange spacing.
As shown in Fig. 8, the orthotropy factor calculated for the Lehighton Bridge was 1.69.

The moment percentage diagram (Section 2.5) shown in Fig. 9 is a comparison of two analytic models with the field test results. One model included the orthotropy factor in the analysis, while the other did not. As shown in Fig. 9, the test vehicle is located between the third and fourth beams, as indicated by the wheels and axle. Comparison of the analytic models with the field test results showed that a closer correlation to the field test results was obtained when the orthotropy factor was included in the analysis.

To verify that this method was an effective way of modeling the bending span of the slab, a comparison was made with another theoretical model. The discretization for the latter model is shown in Fig. 10. There are four slab elements between the beams, with two elements over the flange of each beam, and two elements between the flanges of the beams. This discretization was designated the 4 PL mesh. The slab elements over the flanges of the beams were assigned an orthotropy factor of 100.0. This orthotropy factor defined the stiffness of the slab elements, above the beam flanges, in the transverse direction to be 100 times greater than stiffness in the longitudinal direction. In effect, the slab elements above the flanges were allowed to deform in the longitudinal direction, while essentially remaining rigid in the transverse direction. This prevented relative deformation of the slab with respect to the beam flange in...
the transverse direction. The elements between the beams were assigned an orthotropy factor of 1.00, therefore those elements would deform in an isotropic manner.

In Fig. 11, the results from use of the 4 PL mesh are compared with those from the 2 PL mesh. The position of the test vehicle is indicated. It is seen in this comparison that both models yielded virtually the same results. Thus, the methods of modeling the appropriate bending span were verified. Based on the comparison, the 2 PL mesh was selected for the remainder of the study because it was as equally effective as, and more efficient than, the 4 PL model in representing the bending span of the slab.

A further investigation was then performed to determine the effect of a different slab discretization on the analysis. The discretization in Fig. 12(a) is the 2 PL mesh, described earlier in this section, while the discretization in Fig. 12(b) has one slab element between the beams, and will be designated the 1 PL mesh. Both of these models contain the appropriate orthotropy factors and results from their use are compared in Figs. 13 & 14. Two different truck positions are indicated. These figures both show that there was no perceptible difference between either of the modeling techniques.
3.3.3 Permanent Metal Deck Form

The concrete slab of the test bridge was placed over a permanent metal deck form which had ribs running in the transverse direction (Fig. 15). The effects of the deck form on lateral load distribution were modeled by introducing another orthotropy factor \( D_y \). As indicated in Fig. 16 the orthotropy factor was calculated as the ratio of moments of inertia \( I'/I \), where \( I' \) was defined as the moment of inertia of the transformed concrete section and the metal deck form in the transverse direction, and \( I \) was the moment of inertia of the concrete slab of nominal thickness in the longitudinal direction. For the Lehighton test bridge, the orthotropy factor was calculated as 1.48. The effect of including this factor in the analysis is shown in Fig. 17. When the permanent metal deck form was included in the analysis, the agreement between analytic and field test results was improved.

3.3.4 Curb-Parapet Section

In order to verify that the analytic model accurately represented the actual superstructure behavior, it was also necessary to make an investigation to assess the effect of the single curb-parapet section, shown on the right side of the cross-section in Fig. 4. The curb-parapet section was considered as a beam element in the analysis. Two different models of the section were studied: (1) The section, shown in Fig. 18, was considered to be
fully effective. (2) The section was considered to be partially effective. That is, only the cross-sectional properties up to the dashed line were considered, as indicated in Fig. 18. In the actual bridge, the curb-parapet section was interrupted by deflection joints one inch in width at intervals of approximately 14 feet along the span length. The joints were filled with a pre-molded joint filler in the portion of the section between the top of the slab and the dotted line. Therefore, the two models represented the upper and lower bounds of effectiveness.

Both modeling techniques are compared to the field test results in Figs. 19, 20, and 21. Each figure corresponds to a different truck position. It is seen in Fig. 19 that there is very little difference between results from the two models. This was expected for a truck position which was as far as possible from the curb-parapet section. In this case the bending moments in the beams in the vicinity of the curb-parapet are negligible, and therefore, the influence of the curb-parapet would be small. In Fig. 20 the test vehicle is placed between the third and fourth beams of the bridge. For this load case, there was a noticeable difference between the fully effective and partially effective models. Use of the partially effective section produced results which correlated better with the field test than those obtained with the fully effective section. In Fig. 21 the truck is positioned as close as possible to the curb-parapet section.
With this position of the truck, use of the fully effective curb-parapet section, resulted in an over-estimation of the moment carried by the exterior beam under the curb-parapet section, while use of the partially effective curb-parapet section, yielded very good correlation with the field test results. Thus, it was concluded that the effect of the curb-parapet section on lateral load distribution increases as the load approaches that section. These studies have also indicated that the partially effective section is a more realistic model of the curb-parapet than a fully effective section.

3.4 Summary

A study of different analytic modeling techniques has been presented. In this study, an accurate and efficient model was developed for use in the study of lateral load distribution. Figures 22 and 23 show the correlation between analytic and field test results for two additional load cases on the Lehighton Bridge. Figures 24 and 25 compare analytic and field test results for two load cases on the Bartonsville Bridge. The difference between the analytic and field test results is no greater than 6% for any load case.

Based on this study, the following conclusions are drawn:

1) The permanent metal deck form and the top flanges of the beams stiffen the slab in the transverse direction. This stiffening effect can be accounted for by using an orthotropy
factor. Suggested methods of computing these orthotropy factors are presented in Sec. 3.3.2 and Sec. 3.3.3.

2) The number of elements between beams can be reduced with a considerable increase in efficiency, but without a significant loss in accuracy.

3) The curb-parapet section affects the distribution of live load. The results from this preliminary study indicate that a partially effective curb-parapet model yields more realistic results than a fully effective model.
4. DESIGN OF ANALYTIC EXPERIMENT

4.1 General

To obtain a general method for the evaluation of distribution factors that will be reliable for all bridges over a range of different dimensions, many bridges were considered in the investigation. Although field tests were important in establishing the validity of analytical techniques, an investigation of the size required in this study eliminates the possibility of sufficient field testing to provide the basis for a general specification provision. Therefore, an analytic experiment was designed to yield information which would form the basis for development of new design provisions for live-load distribution factors. In this analytic experiment, 219 bridges were designed and analyzed. The experiment was a computer based analytic simulation. The analytic simulation was accomplished by using the theoretical technique described in Chapter 2, which incorporated the analytic model developed in Chapter 3.

4.2 Type of Superstructure and Loading Configuration

The bridges that were considered in the analytic experiment were all simple-span, without skew. The bridges consist of a reinforced concrete deck slab supported longitudinally by equally spaced prestressed concrete I-beams. The effects of the curb-parapet section and the intra-span diaphragms were neglected. All
bridges were designed using the provisions of the 1973 AASHTO Specification, and the PennDOT Standards for Bridge Design, BD-2017, and AASHTO HS20-44 truck loadings were used.

4.3 Bridge Dimensions and Variation of Parameters

The following bridge design parameters were varied in the analytic experiment. A representative range of bridge roadway widths were chosen, using Art. 1.2.6 of the 1973 AASHTO Specification as a guide. The roadway widths used were 20, 30, 42, 54, 66, and 78 ft. For each bridge width, the number of beams was varied, which provided a range in beam spacing. The beam spacings varied from 4 ft. to 10 ft. - 6 in. For each beam spacing, the length of the bridge was varied from approximately 30 ft. to approximately 150 ft. The slab thickness used for each case was the thickness appropriate for the beam spacing and length, as specified in PennDOT BD-2017. The beams for each bridge were selected as the straight strand beams, having the smallest cross-sectional area, which would meet all current design requirements. The consideration of draped-strand beams would have yielded smaller beams in many cases. The use of larger beams in the analysis yielded distribution factors which were slightly larger, and therefore, were on the conservative side. Both PennDOT and AASHTO-PCI prestressed I-beam shapes were used.

Tables 1, 2, and 3 give an overall scope of the range of the analytic experiment. Table 1 indicates the range for bridges with roadway widths of 20 ft. and 30 ft., and Tables 2 and 3 show the range
for bridges with roadway widths of 42, 54, 66, and 78 ft., respectively. For each bridge width, the tables indicate the range of the number of beams, the beam spacing in feet, the minimum and maximum lengths and the number of bridges actually analyzed for a given bridge width.

Table 4 demonstrates the scheme used to vary the bridge parameters in the analytic experiment. The table provides a detailed outline of the experiment for all bridges having a roadway width of 20 ft. Each X represents a bridge that was designed and analyzed. Across the top of the table, the number of beams is varied from 3 to 6. On the left hand side of the table, the S/L ratio is indicated. The quantity S/L is the ratio of beam spacing to span length. Thus, for a 3-beam bridge with a beam spacing of 10 ft. and a span length of 30 ft. the S/L ratio is 1/3. For the same beam spacing, if the span length is increased to 150 ft., the S/L ratio is 1/15. The S/L ratios were varied from about 1/3 to 1/30 for each particular beam spacing. As shown in Tables 5-9, this technique was used for other bridge widths included in the analytic experiment. The results of the experiment are presented in Chapter 5.
5. RESULTS OF THE EXPERIMENT

5.1 General

The design of an extensive analytic experiment to study lateral load distribution was presented in Chapter 4. This chapter presents the method in which the results of the bridge analyses were utilized to arrive at a new equation for determination of lateral load distribution in simple span right (no skew) prestressed concrete I-beam highway bridges.

The following is a brief outline of the steps involved in the determination of the lateral load distribution developed in this research.

1) Analyze the bridges listed in Chapter 4.
2) Obtain influence lines for each beam of each bridge.
3) Calculate the maximum distribution factor for each bridge for a number of loaded lanes from one to the number as set forth in Article 1.2.6 of Ref. 1.
4) Plot maximum distribution factors versus the S/L ratio.
5) Determine a new lateral load distribution equation by fitting the data plotted in step 4 with an appropriate equation.

5.2 Analysis of Bridges and Resulting Influence Lines

The finite element method described in Chapter 2 was the
method used to analyze the bridges in the experiment. A single HS20-44 vehicle was placed in a number of positions across the width of the bridge, and an analysis was performed for each position. The longitudinal position of the vehicle was always the one that would produce an absolute maximum moment in an analogous single beam of length equal to the span length of the bridge. The bridge was discretized in such a way that the maximum moment was obtained directly in the analysis.

For each position of the vehicle, a moment percentage diagram was obtained, similar to the diagrams used in Chapter 3. The moment percentage diagrams were then used to produce the influence lines for each beam. Each influence line was plotted using approximately ten vehicle positions across the width of the bridge. These influence lines were then used to produce the distribution factors for each beam.

The technique of obtaining the influence lines for beams can be illustrated by using one of the 219 bridges that were analyzed in the experiment. This bridge was 42 ft. in width and 105 ft. in length. There were 7 beams spaced at 7 ft. Influence lines for the exterior beam and center beam are shown in Fig. 26 and 27, respectively. The lines were developed using eleven vehicle positions.
5.3 Determination and Plotting of Maximum Distribution Factors

This section explains how the maximum distribution factor for each bridge was determined, using the influence lines. As explained in Chapter 4, each bridge width that was included in the experiment, except the 20 foot wide bridge, was considered as two design lane configurations. For example, the 42-foot wide bridge was considered as a three and four lane structure, as set forth in Art. 1.2.6 of Ref. 1. Shown in Figs. 26 and 27 are the positions of the design traffic lanes when the example bridge was considered as a three-lane and then four-lane structure. Thus, two maximum distribution factors were developed from the analysis of every bridge.

Considering the example bridge as a three-lane structure, the following method was used to calculate the maximum distribution factor for the center beam (Fig. 27). A vehicle was placed in each of the three lanes. The vehicles were positioned within their individual traffic lanes so as to produce the maximum moment percentage in each lane. These values were then summed to produce the maximum summation of moment percentages for the center beam. The summation was then multiplied by two to convert the vehicle axle load to wheel loads. This calculation produced the maximum distribution factor for the center beam of the bridge when the structure was considered as a three-lane bridge.
To obtain the maximum distribution factor for the interior beams of the bridge, this process was repeated for the remainder of the interior beams. The calculated distribution factors were then compared to determine the maximum distribution factor for the interior beams of the example bridge using a three-lane configuration. The example bridge was then considered as a four-lane bridge and the complete process was repeated. The calculations for the example bridge yielded a maximum distribution factor of 1.16 for the three-lane case and 1.38 for the four-lane case.

Figure 26 shows the influence line for the exterior beam of the example bridge. The distribution factors for the exterior beam were calculated using the same technique as used for the interior beams. The maximum exterior beam distribution factors were obtained by again calculating distribution factors for the three-lane and four-lane cases.

Though the calculations for the interior and exterior beams were similar, the influence line for the exterior beams serves as a good example to demonstrate the care required in calculation of the maximum distribution factor. As shown in Fig. 26 for the three-lane case, one of the three lanes is positioned where negative moment is produced. If this negative moment percentage was included in the summation, the maximum distribution factor would not be obtained. Therefore, this negative value was excluded from the summation. The case in which two of the three lanes were loaded was
more critical for this case, and was the loading used in calculating the maximum distribution factor for the beam.

This process for calculating the maximum distribution factor was repeated for all of the 42-foot wide, 7-beam bridges listed in Table 6, resulting in the list in Table 10. The bridge lengths ranged from 42 feet to 105 feet. The distribution factors ranged from 1.38 to 1.42 for the four-lane case. The results in Table 10 are shown in graphical form in Fig. 28. The maximum distribution factor for each bridge is plotted against the beam spacing-to-span length ratio of the bridges. Figure 28 shows only the results of the 42-foot wide, 7-beam bridges. In Fig. 29, the maximum distribution factors for all of the 42-ft. wide bridges listed in Table 7 are shown. Plots of maximum distribution factors were obtained for all 219 bridges studied. It was from these plots that the new method for calculating the distribution factor for interior and exterior beams was obtained.

5.4 Distribution Factors

Separate provisions currently exist for the calculation of distribution factors for interior and exterior beams in Ref. 1. The results for interior and exterior beams obtained in this research are also presented separately. Figures 30 to 40 are plots of the maximum distribution factors for interior beams. Figures
41 to 51 are plots of the maximum distribution factors for exterior beams. The plots, which include results for 219 bridges, are grouped by bridge widths and number of design lanes considered. They include the complete range of beam spacings considered for each width. The solid lines represent the computer analysis. The dashed lines represent the analytic expression that approximates the computer results.

5.5 Summary

In this chapter, the method of obtaining the maximum distribution factors for the bridges studied was presented. The final plots presented in section 5.4 were obtained after 219 bridge analyses were performed, which included a total of approximately 1500 vehicle load cases. From these analyses, approximately 1200 influence lines were studied under many lane load configurations to determine new lateral load distribution equations for the interior and exterior beams. These equations are presented in chapter 6.
6. DISTRIBUTION FACTORS

6.1 Interior Beams

The analytic expression which was developed to calculate the live-load distribution factor for interior beams is presented in this section.

Figures 30 to 40 are plots of maximum distribution factors versus S/L (ratio of beam spacing-to-span length ratio) for the interior beams. The solid lines represent the computer analysis results, while the dashed lines represent the analytic expression that approximates the computer results. The figures show the results obtained for the complete range of bridge widths studied. The 42-ft. wide bridge distribution factors will serve as a representative sample of the trends that are apparent in the figures.

Figure 33 is the plot of maximum distribution factors for bridges that are 42-ft. wide and with three design lanes, while Fig. 34 is the plot of maximum distribution factors for the same bridges except that the bridges have four design lanes. As expected, the following trends are apparent when the figures are compared.

1) As the length of the bridge increases, the distribution factor decreases.

2) As the number of beams increase, the distribution factor decreases.
3) The distribution factors for the four-design lane case are higher than those for the bridges considered as having three-design lanes.

The analytic expression for evaluating distribution factors contains provisions accounting for the above referred trends. Of the many equations studied to approximate the experimental distribution factors for interior beams, the following equation produced consistent correlation with the experimental results:

$$D.F. = (W_c + \frac{W_c}{N_B} - \gamma) \frac{1}{\beta} - 0.45 (0.25 - \frac{S}{L})$$

$$\gamma = 0.3 (W_c - W_{\text{min}})$$

$$\beta = 4.7 N_B$$

where

- $N_L$ = number of design traffic lanes (as defined in Article 1.2.6 of "Standard Specifications for Highway Bridges - 1973")
- $N_B$ = number of beams ($3 \leq N_B \leq 17$)
- $S$ = beam spacing, in feet ($4.00 \leq S \leq 10.83$)
- $L$ = span length, in feet ($30 \leq L \leq 150$)
- $W_c$ = roadway width between curbs, in feet ($20 \leq W_c \leq 78$)
- $W_{\text{min}}$ = minimum curb-to-curb width which qualifies as an $N_L$ design lane bridge, in feet

The distribution factors that are calculated using this equation are shown in Figs. 30-40 by the dashed lines. A comparison
of the results of the computer analysis (D.F.\textsubscript{comp.}) and the analytic expression (D.F.\textsubscript{anal.exp.}) is made using the ratio D.F.\textsubscript{anal.exp.}/D.F.\textsubscript{comp.}.

Using this ratio, a mean of 1.04 was calculated. That is, the analytic expression is, on the mean, 4\% higher than the computer analysis. The standard deviation is 4\%. Thus, there is a 95\% probability that the results using the equation will be between 96\% and 112\% of the experimental results.

6.2 Exterior Beams

The maximum distribution factors for the exterior beams are plotted in Figs. 41-51. For the exterior beams, the maximum distribution factors are plotted versus span length. The solid lines represent the computer analysis results, while the dashed lines represent the analytical expression approximating the computer results. As expected, the following trends, similar to those for the interior beams, became apparent when the figures are compared.

1) The distribution factor increases as the length of the bridge increases.

2) The distribution factor decreases as the number of beams increases.

3) As the number of lanes increases, for a given width, the distribution factor increases.

The following equation approximates the computer analysis results:
\[
\text{D.F.} = \frac{S}{10} + \frac{L}{750} + 0.1
\]

where

- \( S \) = beam spacing, in ft. \((4.00 < S < 10.83)\)
- \( L \) = span length, in ft. \((30 < L < 150)\)

The distribution factors that are calculated using this equation are shown by the dashed lines in Figs. 41-51.

Finally, it should be noted that the provisions of Art. 1.2.9 - 'Reduction in Load Intensity' were not applied in the development of the expressions for the two distribution factors.
7. SUMMARY AND CONCLUSIONS

7.1 Summary

A method of analysis based on the finite element method is presented in Chapter 2. A review of the assumptions and limitations of the previously developed analysis technique is discussed, and the analysis technique is then described.

In Chapter 3 comparisons are made between results from the theoretical analysis technique, and values yielded from the field testing of two in-service bridges. Different methods of analytically modeling the bridges were used in comparison with the field test results. Through these comparisons, the validity of the theoretical analysis technique was verified. Also, by refining the analytic bridge model, the accuracy and efficiency of the study of live load distribution was increased.

An analytic experiment to study live load distribution is presented in Chapter 4. A total of 219 different bridges were designed and analyzed under AASHTO HS20-44 design loading. Chapter 5 shows how the results of the bridge analyses which constituted the analytic experiment were utilized to arrive at new equations to describe the lateral load distribution.

In Chapter 6 a design recommendation for the determination of lateral live-load distribution is presented. Separate procedures
are given for the interior and exterior beams.

7.2 Conclusions

Very good agreement was obtained between the theoretical analysis technique and the field test results. Through an analytic modeling study, the analytic bridge model was refined to obtain optimum accuracy and efficiency.

Based on the results of the analyses of 219 bridges the following conclusions can be made.

1. The lateral live-load distribution in prestressed concrete I-beam bridges can be accurately described by the equations presented in Chapter 6. The behavior of interior and exterior beams is described by separate equations.

2. The span length of the bridge, the beam spacing, and the number of design traffic lanes are very important factors in determining the live-load distribution factors.

3. The effect of the curbs and parapets were not considered in the development of the equations to describe lateral live-load distribution. However, based on the results of the analytic modeling study, it was found that the curbs and parapets do have an influence on the distribution of live-load. Therefore, it is felt the design procedures should be modified to permit the effect of curbs and parapets to be considered.
8. **ACKNOWLEDGMENTS**

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The following members of the staff at Lehigh University made significant contributions: E. S. deCastro and W. S. Peterson in the programming and execution of the computer analysis, and S. Tumminelli in the review of this report.

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9. TABLES
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<td>5'-0&quot;</td>
<td>125'</td>
<td>50'</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>4'-3&quot;</td>
<td>127'-6&quot;</td>
<td>51'</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Tot. = 30</strong></td>
</tr>
</tbody>
</table>

b)

TABLE 1  RANGE OF BRIDGE DESIGN PARAMETERS
### Table 2 Range of Bridge Design Parameters

<table>
<thead>
<tr>
<th>No. Beams</th>
<th>Spacing</th>
<th>$L_{MAX}$</th>
<th>$L_{MIN}$</th>
<th>No. Bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10'-6&quot;</td>
<td>84'</td>
<td>42'</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>8'-5&quot;</td>
<td>101'</td>
<td>42'-1&quot;</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7'-0&quot;</td>
<td>105'</td>
<td>42'</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>6'-0&quot;</td>
<td>120'</td>
<td>42'</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>5'-3&quot;</td>
<td>105'</td>
<td>42'</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>4'-8&quot;</td>
<td>116'-2&quot;</td>
<td>46'-8&quot;</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Tot. = 36</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Roadway Width: 42 feet

<table>
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<th>$L_{MAX}$</th>
<th>$L_{MIN}$</th>
<th>No. Bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10'-10&quot;</td>
<td>108'-4&quot;</td>
<td>32'-6&quot;</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>9'-0&quot;</td>
<td>108'</td>
<td>36'</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>7'-9&quot;</td>
<td>116'-3&quot;</td>
<td>38'-9&quot;</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>6'-9&quot;</td>
<td>135'</td>
<td>40'-6&quot;</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>5'-5&quot;</td>
<td>135'-5&quot;</td>
<td>37'-10&quot;</td>
<td>7</td>
</tr>
<tr>
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<td>4'-6&quot;</td>
<td>135'</td>
<td>36'</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Tot. = 39</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Roadway Width: 54 feet

---

-51-
<table>
<thead>
<tr>
<th>No. Beams</th>
<th>Spacing</th>
<th>$I_{\text{MAX}}$</th>
<th>$I_{\text{MIN}}$</th>
<th>No. Bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9'-5&quot;</td>
<td>113'</td>
<td>37'-8&quot;</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>8'-3&quot;</td>
<td>123'-9&quot;</td>
<td>33'</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>7'-4&quot;</td>
<td>128'-4&quot;</td>
<td>29'-4&quot;</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>6'-0&quot;</td>
<td>120'</td>
<td>36'</td>
<td>7</td>
</tr>
<tr>
<td>14</td>
<td>5'-1&quot;</td>
<td>127'-1&quot;</td>
<td>50'-10&quot;</td>
<td>6</td>
</tr>
<tr>
<td>16</td>
<td>4'-5&quot;</td>
<td>132'-6&quot;</td>
<td>53'</td>
<td>6</td>
</tr>
<tr>
<td><strong>Tot.</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>40</strong></td>
</tr>
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</table>

Roadway Width: 78 feet

<table>
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<tr>
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<th>Spacing</th>
<th>$I_{\text{MAX}}$</th>
<th>$I_{\text{MIN}}$</th>
<th>No. Bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9'-9&quot;</td>
<td>117'-8&quot;</td>
<td>39'</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>8'-8&quot;</td>
<td>104'</td>
<td>34'-8&quot;</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>7'-10&quot;</td>
<td>117'-6&quot;</td>
<td>39'-2&quot;</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>7'-1&quot;</td>
<td>124'</td>
<td>35'-5&quot;</td>
<td>7</td>
</tr>
<tr>
<td>13</td>
<td>6'-6&quot;</td>
<td>130'</td>
<td>32'-6&quot;</td>
<td>8</td>
</tr>
<tr>
<td>15</td>
<td>5'-7&quot;</td>
<td>111'-8&quot;</td>
<td>39'-1&quot;</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>4'-11&quot;</td>
<td>123'</td>
<td>39'-4&quot;</td>
<td>7</td>
</tr>
<tr>
<td><strong>Tot.</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>50</strong></td>
</tr>
</tbody>
</table>

TABLE 3 RANGE OF BRIDGE DESIGN PARAMETERS

-52-
Roadway Width  20 feet  
(2 design lanes)

<table>
<thead>
<tr>
<th>No. Beams</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/30</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/10</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/8</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/7</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = Beam Spacing 
L = Span Length 

TABLE 4 BRIDGES ANALYZED, ROADWAY WIDTH 20 FT.
Roadway Width  30 feet
(2–3 design lanes)

<table>
<thead>
<tr>
<th>No. Beams (S/L)</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/30</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/10</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/8</td>
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<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/7</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = Beam Spacing  
L = Span Length

TABLE 5  BRIDGES ANALYZED, ROADWAY WIDTH 30 FT.
Roadway Width 42 feet
(3-4 design lanes)

<table>
<thead>
<tr>
<th>No. Beams</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/30</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/10</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/8</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

S = Beam Spacing
L = Span Length

TABLE 6 BRIDGES ANALYZED, ROADWAY WIDTH 42 FT.
Roadway Width  54 feet
(4-5 design lanes)

<table>
<thead>
<tr>
<th>No. Beams</th>
<th>13</th>
<th>11</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/L</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/30</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/8</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/7</td>
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<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

S = Beam Spacing
L = Span Length

TABLE 7 BRIDGES ANALYZED, ROADWAY WIDTH 54 FT.
### Roadway Width 66 feet

(5-6 design lanes)

<table>
<thead>
<tr>
<th>No. Beams S/L</th>
<th>16</th>
<th>14</th>
<th>12</th>
<th>10</th>
<th>9</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/30</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/10</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/8</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1/7</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
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<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = Beam Spacing  
L = Span Length

**TABLE 8** BRIDGES ANALYZED, ROADWAY WIDTH 66 FT.
Roadway Width  78 feet
(6-7 design lanes)

<table>
<thead>
<tr>
<th>No. Beams</th>
<th>17</th>
<th>15</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/30</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/25</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/20</td>
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<td>X</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/17.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/15</td>
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S = Beam Spacing
L = Span Length

TABLE 9  BRIDGES ANALYZED, ROADWAY WIDTH 78 FT.
ROADWAY WIDTH = 42 FT. \hspace{1cm} AASHTO \hspace{0.5cm} \frac{S}{5.5} = 1.27

NO. OF BEAMS = 7

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TABLE 10 DISTRIBUTION FACTORS 42 FT. WIDE, 7 BEAM BRIDGES
10. FIGURES
Fig. 1 Rectangular Plate Element and Basic Displacement Components
Fig. 2  Eccentrically Attached Beam Element
Fig. 3(a) Coordinate System and Positive Sign Convention

Fig. 3(b) Coordinate System and Generalized Displacements

\[ \theta_y = -\frac{\partial w}{\partial x} \]
Fig. 4  Cross-section of Lehighton Bridge
DESIGN DIMENSIONS

Fig. 5 Cross-section of Bartonsville Bridge
Fig. 6  Discretization of Bartonsville Bridge

-66-
Fig. 7 Test Vehicle
Fig. 8 2 PL Mesh Discretization and Orthotropy Factor for Lehighton Bridge
Fig. 9 Comparison of Moment Percentages Derived From Analyses and Field Test Results
Slab Elements Over Flanges $D_Y=100.0$
Slab Elements Between Beams $D_Y=1.0$

Fig. 10  4 PL Mesh Discretization and Orthotropy Factors
Fig. 11 Comparison of Moment Percentages Derived from Analysis Using 2 PL Mesh and 4 PL Mesh
Fig. 12 2 PL and 1 PL Mesh Discretization
Fig. 13  Comparison of Moment Percentages Derived from Analyses and Field Test Results
Fig. 14 Comparison of Moment Percentages Derived from Analyses and Field Test Results
Fig. 15 Transverse Cross-section of Lehighton Bridge Slab
\[ I' = 294.3^4 \text{ in.} \]

\[ I = 198.48^4 \text{ in.} \]

\[ D_Y = \frac{I'}{I} = 1.48 \]

Fig. 16 Determination of Orthotropy Factor for Slab - Lehighton Bridge
Fig. 17  Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 18  Curb-Parapet Section – Lehighton Bridge
Fig. 19 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 20  Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 21 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 22 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 23 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Lehighton Bridge
Fig. 24 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Bartonsville Bridge
Fig. 25 Comparison of Moment Percentages Derived from Analyses and Field Test Results - Bartonsville Bridge
Fig. 26 Influence Line for Moment Percentages
42 ft. Wide Bridge, 7 Beams, Length 105 ft. - Beam 1
Fig. 27 Influence Line for Moment Percentages
42 ft. Wide Bridge, 7 beams, Length 105 ft. Beam 4
Fig. 28 Distribution Factors for Interior Beam 42 Ft. Wide, 7 Beam Bridges
Fig. 29  Distribution Factors for Interior Beam 42 Ft. Wide Bridges ($N_L = 4$)
Fig. 30 Distribution Factors for Interior Beam 20 Ft. Wide Bridges (N_1 = 2)
Fig. 31  Distribution Factors for Interior Beam 30 Ft. Wide Bridges ($N_L = 2$)
Fig. 32 Distribution Factors for Interior Beam 30 Ft. Wide Bridges ($N_L = 3$)
Fig. 33 Distribution Factors for Interior Beam 42 Ft. Wide Bridges ($N_L = 3$)
Fig. 34 Distribution Factors for Interior Beam 42 Ft. Wide Bridges ($N_L = 4$)
Fig. 35  Distribution Factors for Interior Beam 54 Ft. Wide Bridges (N_L = 4)
Fig. 36 Distribution Factors for Interior Beam 54 Ft. Wide Bridges ($N_L = 5$)
Fig. 37  Distribution Factors for Interior Beam 66 Ft. Wide Bridges ($N_L = 5$)
Fig. 38  Distribution Factors for Interior Beam 66 Ft. Wide Bridges ($N_L = 6$)
Fig. 39  Distribution Factors for Interior Beam 78 Ft. Wide Bridges ($N_L = 6$)
Fig. 40  Distribution Factors for Interior Beam 78 Ft. Wide Bridges ($N_L = 7$)
Fig. 41 Distribution Factors for Exterior Beam 20 Ft. Wide Bridges ($N_L = 2$)
Fig. 42 Distribution Factors for Exterior Beam 30 Ft. Wide Bridges ($N_L = 2$)
Fig. 43  Distribution Factors for Exterior Beam 30 Ft. Wide Bridges (N_L = 3)
Fig. 44  Distribution Factors for Exterior Beam 42 Ft. Wide Bridges ($N_L = 3$)
Fig. 45  Distribution Factors for Exterior Beam 42 Ft. Wide Bridges ($N_L = 4$)
Fig. 46  Distribution Factors for Exterior Beam 54 Ft. Wide Bridges ($N_L = 4$)
Fig. 47 Distribution Factors for Exterior Beam 54 Ft. Wide Bridges ($N_L = 5$)
Fig. 48  Distribution Factors for Exterior Beam 66 Ft. Wide Bridges (N_L = 5)
Fig. 49  Distribution Factors for Exterior Beam 66 Ft. Wide Bridges ($N_L = 6$)
Fig. 50  Distribution Factors for Exterior Beam 78 Ft. Wide Bridges ($N_L = 6$)
Fig. 51 Distribution Factors for Exterior Beam 78 Ft. Wide Bridges ($N_L = 7$)
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