Contributions of Floor Systems to Earthquake Resistance of Building Structural Frames

SEISMIC RESISTANCE CHARACTERISTICS OF REINFORCED CONCRETE BEAM-SUPPORTED FLOOR SLABS IN BUILDING STRUCTURES

by

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Ti Huang
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Report No. 422.9
October 30, 1981

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This work has been carried out as part of an investigation sponsored by the National Science Foundation.

Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the Sponsor or Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.

Department of Civil Engineering

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

April 1981

Fritz Engineering Laboratory Report No. 422.9
and Mr. Z. S. Wu who helped in the testing of the specimens; Messrs. Hugh T. Sutherland and Russell Longenbach in instrumentation, and the Fritz Engineering Laboratory technicians for handling and installation of the testing setup. Dr. Roger G. Slutter, director of the operations division, provided continuous help in the planning and conducting of the experimental studies.

Manuscript was typed by Mrs. D. Fielding and Mrs. P. Vidanage. Illustrations were prepared by Mr. J. Gera and Mrs. S. Balogh. Photographs were prepared by Mr. R. Sopko.
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ABSTRACT

This study investigates the in-plane seismic characteristics of reinforced concrete floor slabs which function as diaphragms placed between lateral load resisting systems. The paper focuses on the floor slab system with edge beams, referred to as the beam-supported floor system. The investigation consists of four phases: (1) experimental study, (2) analytical study, (3) parametric study, and (4) dynamic response analysis.

In the experimental study, scaled models (a scale ratio of 1 : 4.5) representing a portion of the floor system in a building structure were tested under various loading and support conditions. The experimental findings indicate that the development of a crack extending along the boundary between the column and middle strips controls the ultimate in-plane strength of the test panels, while the opening and closing of the crack primarily controls the behavior of the panels in post-ultimate load regions. It was found that cyclic loading or the application of the vertical load can reduce the ultimate in-plane strength by as much as 20 to 25 percent.

A non-linear finite element model was developed for the purpose of the analytical study. The model successfully predicts the ultimate strength of the test slab panels subjected to monotonic in-plane loading and duplicates the experimental load-deflection curves. The model also reproduces the unloading
stiffness of the test slab panels.

The effects of geometry, reinforcement, loading, and support conditions on the in-plane characteristics of floor slabs were investigated in the parametric study. General and practical procedures were developed to evaluate the in-plane strength and stiffness of floor slabs. The in-plane flexural strength of floor slabs can be computed by treating them as deep beams considering both flexural and shear deformations. A reduction factor is incorporated into the deep beam calculation in order to represent the stiffness degradation in the post-elastic range.

In the dynamic response analysis, a seven story and six bay symmetrical building model was selected in order to examine the influence of in-plane characteristics of floor slabs on the building response. Compared with the analysis based on the usual rigid slab assumption, the incorporation of elastic in-plane deformation of the floor slabs resulted in a 300 percent increase in the base shear applied to flexible vertical members. The base shear resisted by the flexible vertical members was increased further by 100 percent when the non-linear behavior of floor slabs and vertical members were considered.
I. INTRODUCTION

1.1 General

The dynamic response of structures to strong motion earthquakes and earthquake resistant building designs have been the subjects for a great deal of recent research. According to D'Alembert's principle, the dynamic effect of a structure can be represented in terms of time-dependent inertial forces. Once these equivalent inertial forces have been determined, the analysis and design can be performed by the conventional procedure for static loads. However, the fact that the effective external forces are controlled totally by the earthquake itself complicates the earthquake resistant building design. Since the earthquake disturbance has a displacement-oriented nature, the effective forces are dependent upon the stiffness of the structural elements. As a result, a stiffer structure will be subjected to higher earthquake loads. Structural safety is not automatically improved by arbitrarily increasing the member sizes.

Recent development of high speed digital computers has facilitated the understanding of dynamic behavior of structures during an earthquake. Much progress has already been made in both the dynamic analysis and the earthquake resisting design. Nevertheless, prediction of the inelastic response of structures to strong earthquake motions having a wide range of frequencies...
and amplitudes is still difficult because the post-elastic behavior of the structures is greatly influenced by the interaction of individual elements such as beams, columns, and shear-walls. In order to perform a true dynamic analysis, analysts first must know the characteristics of each structural element under all possible loadings. Once all the information with respect to the behavior of structural elements is known, it is possible to carry out a dynamic analysis by assembling these elements. These two steps in analytic process are interrelated obviously. For example, it is possible to model a structure more realistically when more information regarding the behavior of some structural elements is known. A refined dynamic analysis, in turn, can give more accurate evaluation of the forces applied to the individual structural elements.

While the above discussion is valid for any kind of construction material, particular care should be paid to the reinforced concrete structure. Concrete has certain intrinsic advantages over other construction material; it is highly versatile, durable and fire-resistant. Its high density and low ductility, on the other hand, bring about undesirable effects when used in regions susceptible to severe earthquakes. This disadvantage, however, does not necessarily mean that the reinforced concrete should not be used in seismic regions. Properly designed structures will perform well regardless of the material used.
Nevertheless, it should be emphasized that reinforced concrete is less tolerant of the improper design.

Much work has been performed to determine the behavior of reinforced concrete structural elements subjected to earthquake loading, particularly for the major lateral load resisting elements such as columns, beams, and shearwalls. Great improvement in the earthquake resisting design has been achieved in recent years from these studies. On the other hand, earthquake behavior of floor slab systems has not yet been given very much attention. Consequently, it is difficult to adequately include the characteristics of floor slabs in the analysis of buildings subjected to earthquake loading. Although the floor slab may not be as critical as columns or shearwalls in providing earthquake resistance, its contribution cannot be ignored.

The main purpose of this study is to investigate the effect of the behavior of floor slabs on the earthquake resistance of reinforced concrete building structures. Also it is intended to provide suggestions and recommendations about the design of floor slabs or other structural elements.

1.2 Statement of Problem

The primary function of floor slabs in a building structure is to carry vertical loads by their out-of-plane bending action and transmit these loads to the supporting elements such
as columns and walls. Most of the current design provisions for
group slabs deal with this particular function. (1.1, 1.3, 1.4,
1.7, 1.16, 1.17, 1.22, 1.26, 1.30, 1.32, 1.35, 1.36, 1.41, 1.42,
1.43) Recently, however, structural engineers have recognized
that the floor slab also performs as an important function when
buildings are subjected to lateral force. (1.19, 1.20)

In many buildings, columns and floor slabs form space
frames, and lateral loads are resisted by the flexural action of
the frames. In this type of structure, floor slabs serve as
horizontal moment resisting members in the frames. This function
of the floor slabs is frequently called Frame Action. (1.5, 1.10,
1.11, 1.18, 1.21, 1.25, 1.28) In the current ACI Building Code,
the Equivalent Frame Method described in Chapter 13 uses this con-
cept and treats the floor slabs as horizontal flexural members in
the frames. The Equivalent Frame Method, however, was developed
originally for the design under vertical loads. Consequently,
the detailed method described in ACI Building Code, including the
empirical distribution coefficients, most probably does not apply
to lateral load analysis. The important characteristics of the
Frame Action are as follows:

1. The stiffness of the floor slab as a out-of-plane flex-
ural member directly influences the lateral story stiff-
ness of a frame; therefore, appropriate evaluation for
the slab stiffness is critical. (1.5, 1.11, 1.28)
(2) It is a common practice to design a space frame as an assembly of two-dimensional frames. In this situation, floor slabs and beams running perpendicular to the plane being analyzed provide torsional restraint. Proper evaluation of the torsional stiffnesses of these elements is also important.\(^{(1.11, 1.28)}\)

(3) Transfer of moment and shear at slab-column junctions becomes a critical factor for safety. A junction must possess the required deformability to guarantee sufficient energy absorption capacity of the total frame. A premature punching failure or a slip of reinforcing bars should be avoided.\(^{(1.18, 1.21, 1.25)}\)

Yet another important function of floor slabs in a building subjected to lateral force is the Diaphragm Action.\(^{(1.19, 1.20)}\) This action is dependent on in-plane characteristics of the floor slabs, while the out-of-plane flexural characteristics control the Frame Action. When a building is subjected to an earthquake, the inertial forces are transmitted transversely through floor slabs to vertical lateral load resisting elements. In this function, the floor slabs act as diaphragms between lateral load resisting systems. The distribution of lateral load to the lateral load resisting systems depends upon the stiffness character-
istics of both the diaphragms and the vertical elements. (1.29)

In the current design practice, floor slabs are often assumed to be perfectly rigid in their own planes. Under this assumption, lateral loads are distributed into lateral load resisting elements in proportion to their lateral story stiffnesses. Although the assumption is employed widely, experience and research have shown that the maximum lateral forces on some components may be underestimated significantly. (1.29) These underestimates frequently occur in frame-wall structures. (1.13, 1.34, 1.38) These structures are favored because high story stiffness can be expected for stiff walls even though the increase of stiffness and mass causes higher lateral force to be applied to the walls. Arbitrary allocating frames and walls in these structures, however, causes significant floor slab deformation, which, in turn, results in a change in the distribution of lateral forces.

In lower stories, generally speaking, the assumption that floor slabs are perfectly rigid in their own planes leads to underestimates of the force carried by frames and overestimates of the force distributed to walls. Evaluation of the forces assigned to the frames and walls is reversed when upper stories are analyzed. It has been shown that the story stiffness of walls decreases in higher stories, while the story stiffness of frames remains relatively constant. (1.13)

An entirely reversed assumption that the in-plane floor
slab rigidity equals zero is usually adopted for low rise buildings with no more than two or three stories.\textsuperscript{(1.34)} The degree of discrepancy created by these assumptions is affected by the ratio of the story stiffness of vertical lateral load resisting elements to the true in-plane stiffness of floor slabs. Whenever in-plane stiffness of the slab is comparable to the stiffness of vertical elements, to assign extreme values such as infinite or zero for the in-plane stiffness of the floor slabs is undesirable.\textsuperscript{(1.37)} Buildings having slender cross sections possess the same potential problem.\textsuperscript{(1.15, 1.23, 1.24)} In these buildings, the floor slabs function as slender beams, and, as a result, the deformation by flexure becomes appreciable.

The above discussion is based on the assumption that the floor slab behaves elastically. There is no guarantee, however, that the floor slab maintains its elastic properties under all earthquake conditions. Shear force applied to a slab increases, as the difference between the story stiffness of two adjacent vertical lateral load resisting elements becomes greater. A high shear force may cause cracks in the slab and change its in-plane stiffness, which, in turn, will cause a change in the proportion of the lateral force distributed to each vertical element. It is quite possible that stiffness degradation and/or decrease of strength would occur during severe repeated or reversed loading cycles. Lateral force applied to each vertical member, then, will
be continuously changing due to the continuous degradation of the in-plane slab stiffness. Such a change in force is dangerous for some elements, but safe for others.

The in-plane strength of the floor slab may be as important as its stiffness. In some types of structures such as staggered wall-beam systems, the shear strength of the floor slabs is critical. Because these floor slabs carry lateral forces transmitted by shearwalls, they must possess enough in-plane shear strength to resist the total lateral load. In this regard, special attention should be paid to floor slab-wall junctions because gravity loads may have created high negative moments and high shears, before any lateral force is applied.

1.3 Objectives and Scope

Among various functions of the floor slab, the diaphragm action has received relatively little attention; consequently, floor slab designs frequently are oversimplified. Many of these simplifications result from a lack of vital information. Research, then, should aim at defining the effectiveness of the floor systems as load-transmitting diaphragms between vertical and lateral load resisting elements.
It is proposed to investigate the in-plane characteristics of reinforced concrete floor slabs under various loading and supporting conditions and to provide meaningful information for the design of the floor slabs as well as of other structural elements. This study is concentrated on the floor slab system with edge beams, referred to as the beam-supported slab (slab-on-beam) system.

The planned study presented here consists of five stages, each in a separate chapter. The five stages are: 1) the literature survey, 2) the experimental work, 3) the analytical study, 4) the parametric study, and 5) the dynamic analysis of structures including the floor slab in-plane characteristics. In Chapter II, a comprehensive survey is given to previous research work regarding the diaphragm action of floor slabs. Findings about the diaphragm action, present status of diaphragm designs, research procedure, specific research topics, and areas of needed research are discussed in detail. Chapter III describes the experimental work. A portion of the floor system in a medium to high rise building structure is modelled and tested under various loading and boundary conditions. The results of these experiments provide basic information about the in-plane characteristics of the floor slabs. Critical parameters controlling the in-plane behavior of the floor slabs are selected carefully, and the effects of these parameters on the in-plane characteristics are examined. The test results also provide a basis of comparison for the analytical model developed in Chapter IV. Chapter IV describes an analytical
model developed for investigating the in-plane characteristics of the floor slabs. The finite element method is used for the modelling. The accuracy of this model is checked by comparing the results derived from this model to the test results described in Chapter III and other existing theories. In Chapter V, the test results are again examined, with specific emphasis on three most crucial in-plane characteristics of the floor slabs: strength, stiffness, and ductility. The analytical model described in Chapter IV is utilized to provide additional data about the effects of the various parameters on these characteristics. The variables included in this parametric study are: the loading condition, the boundary condition, the relative beam size, the amount of reinforcing steel, and the aspect ratio of the floor slab. Based on the findings of the parametric study, suggestions and recommendations for the designs of the floor slabs as diaphragms are provided at the end of this chapter. In Chapter VI, a relatively simple building model representing a commonly used reinforced concrete building is analyzed for its dynamic response under a typical earthquake loading. The in-plane characteristics of the floor slabs defined in Chapter V are incorporated into this dynamic analysis. The intensity of moment and shear applied to the floor slabs during the earthquake motion is examined. The effect of the floor slab in-plane stiffness on the total lateral force and on the lateral load distribution to vertical elements are studied carefully. This study also provides
information for examining the appropriateness of the rigid floor assumption.
II. SURVEY OF PREVIOUS WORK

2.1 The Effect of Floor Slab Stiffness on Lateral Load Distribution

Floor slabs are important elements for three-dimensional structural analysis for horizontal loads since they interconnect lateral load resisting components and control the distribution of the total lateral load to these components. In order to determine the loads supported by each vertical component, the in-plane stiffness of the floor slabs must be evaluated properly.

The assumption that the floor slab is infinitely rigid in its own plane has frequently been adopted in structural analyses. This assumption reduces the number of unknown displacements significantly and, consequently, reduces the effort for computation. Various kinds of three-dimensional structural analyses were carried out based on this assumption. Clough et al.\(^{(2,12)}\) developed a computer program to analyze three-dimensional buildings subjected to static lateral loads. Floor slabs were allowed two degrees of freedom: two translation components in their own planes. Therefore, all vertical components underwent identical displacements at a given story level. On the other hand, torsional deformation about a vertical axis is significant for structures with irregular plans and those subjected to asymmetrical lateral loadings. Stamoto and
Stafford-Smith, Neville, and Macleod et al. conducted static analyses of wall-frame structures having asymmetrical plans. In the formulation of the governing equations, the displacements of walls and columns at each floor level were expressed by two translations and one rotation of the rigid floor slab. Gluck analyzed coupled wall structures according to the same procedure. The application of this procedure was extended further to solve many other problems. Nair investigated the stability of three dimensional wall-frame structures. He represented the whole structure as a beam-column and treated the overall stability as lateral torsional buckling of the beam-column. Heidebrecht and Swift, Taranath and Stafford-Smith, and Taranath examined the effect of vertical forces applied to shear walls due to torsion. They treated the whole structure as a cantilever beam with a cross section composed of walls. The axial forces carried by the walls then were evaluated by considering the bending and warping stresses of this cantilever beam. Shepherd and Donald and Gibson et al. examined the frequencies and mode shapes of asymmetrical wall-frame buildings. Douglas and Trabert studied the response of an approximately symmetrical twenty two story building under the ground motion caused by blast. A non-linear analysis of asymmetrical buildings was made by Wynhoven and
Adams\(^{(2.47)}\) They investigated the reduction of torsional stiffness of the structure caused by the yielding of columns and the accompanying change in the total torsional force. In all these studies, the floor slabs were considered rigid, with three degrees of freedom at each floor level.

The in-plane bending of floor slabs is appreciable when structures having slender cross sections are subjected to lateral forces. The bending effect is significant particularly if the lateral stiffnesses of the vertical components differ greatly. Goldberg and Herness\(^{(1.15)}\) investigated the frequencies and mode shapes of a twenty-story symmetrical building. The building with twelve bays was composed of frames at the inner bents and walls at the ends. The characteristics of the floor slabs in their in-plane directions were represented by beams in which both flexural and shear deformations were taken into account. The mode shapes showed significant bowing of the floor slabs. Goldberg and Herness also reported an appreciable difference between the forces applied to frames and the forces applied to walls. Majid and Onen\(^{(2.27)}\) performed elasto-plastic analysis of wall-frame buildings. In their analysis, the floor slabs were incorporated into the overall structures as elastic deep beams. Plastic hinges were inserted wherever the bending moments in the vertical frames
reached the yield values at the location, and the ultimate resistance of buildings was determined by a step-by-step method. Joh and Ohno\(^{1.24}\) studied the dynamic response of wall-frame structures with slender cross sections and treated the floor slabs as deep beams, using the same procedure as Majid and Onen. The parameters they studied were: the shape of slab panels, and the relative stiffness between walls and frames. Joh and Ohno showed that those parameters affected the distribution of lateral forces and also reported that large differences in lateral stiffness of neighboring vertical components caused large shear forces in the floor slabs. Kostem and Heckman\(^{2.49, 2.50}\) studied the elastic stiffness and vibrational characteristics of building structures with U-shaped concrete shear cores, reporting the significant influence of in-plane floor slab deformations on the lateral deflections and natural frequencies of the structures.

While the above studies including the flexibility of floor slabs showed the significant effects of the floor slab in-plane stiffness on the response of structures, several other studies had resulted in opposite conclusions. Shepherd and Donald\(^{2.36}\) investigated the frequencies and mode shapes of three bay frame structures, in which the stiffness of columns was four times larger in the outer bents. The floor slabs were treated as deep beams. These researchers concluded that the in-plane stiffness of the floor slabs did not affect significantly the forces distributed to vertical components although the mode shapes evidently...
showed bowing of the floor slabs. Richter et al.\(^{(2.34)}\) and Rutenberg et al.\(^{(2.35)}\) also performed static structural analysis by replacing the floor slabs with deep beams. They supported the conclusions of Shepherd and Donald that altering the floor slabs' in-plane stiffness did not cause significant change in the distribution of forces transmitted to the vertical components.

All of the studies discussed above treated the floor slabs as either rigid or linearly elastic elements. Very few studies considered potential inelastic behavior of floor slabs in structural analysis. Ramakrishnan\(^{(1.34)}\) tested scale models of wall-slab structures and examined the forces distributed to the walls. He pointed out the significant influence of the walls arranged perpendicular to the load on the stiffness of the lateral load resisting elements. These walls functioned as flanges for some vertical elements and greatly increased stiffness. Adham and Ewing\(^{(2.2)}\) analyzed the dynamic response of one-story two-bay masonry structures under earthquake ground motion. The wood roof diaphragms interconnecting the masonry walls were assumed to behave non-linearly, and a time-history analysis was executed. Because of the non-linear behavior of the diaphragms, low amplitude test results cannot be extrapolated for the
prediction of the response of these structures under severe loadings. The also found that softer wood diaphragms would attenuate the input earthquake accelerations and result in lower shear forces.

Fintel (2.16, 2.17) and Mee et al. (2.29) studied the staggered wall system, where floor slabs are part of the lateral load resisting systems. Their studies, however, focused on the evaluation of the lateral stiffness of the walls rather than the load resisted by the floor slabs. They discussed the contribution of the floor slabs as flanges on the wall stiffness. Their studies did not refer to either in-plane stiffness or strength of the floor slabs.

Many computer programs for the analysis of three dimensional structures under static or dynamic loadings have been developed during the last decade. Wilson and Dovey (2.46) have developed an elastic finite element program TABS in which floor slabs are assumed to be perfectly rigid. This program has the capacity of solving both static and dynamic problems. Guendelman-Israel and Powell (2.22) have developed a non-linear finite element program by combining two previously developed programs: TABS and DRAIN-2D, which was developed by Kanaan and Powell (2.25) to solve
static and dynamic problems of non-linear two-dimensional structures. This combined program, DRAIN-TABS, is capable of including non-linearities of structural components except floor slabs. The floor slab is assumed to be perfectly rigid as in the program TABS. Cheng and Kitipitayangkul \(^{(2.11)}\) have made another computer program for inelastic analysis of reinforced concrete steel buildings subjected to three-dimensional ground motions. All structural components except the floor slabs are allowed non-linear characteristics. The floor slabs are assumed to be linearly elastic. Since this program takes geometrical non-linearity into account, stability effects as well as effects of material non-linearities can be solved.

2.2 In-plane Characteristics of Floor Slabs

Few studies have been published concerning the in-plane characteristics of reinforced concrete floor slabs. As a result, there exists little information to aid the design of these floor slabs in their function as diaphragm. Cervenka and Gerstle\(^{(2.10)}\) tested two-span continuous reinforced concrete slabs under in-plane loads and examined their strength and stiffness. The tests however, were utilized only to verify the accuracy of their finite element model. Consequently, no design suggestion was made.
Floor slabs as diaphragms are viewed frequently as horizontally oriented shearwalls. This assumption is reasonable because both floor slabs and shearwalls have similar dimensions, and both are subjected to in-plane loads. Shearwalls often have boundary frames. These boundary frames are analogous to the edge beams integral with the floor slabs since both function to confine the flat plate element. Nevertheless, the shearwall and the floor slab are quite different in many respects. Reinforcing bars in shearwalls usually are placed symmetrically about the middle plane of the shearwalls. On the other hand, the arrangement of reinforcing bars in floor slabs is distinctively unsymmetrical; many reinforcing bars in floor slabs are cut off according to the requirement on design flexural moments, while the bars in shearwalls extend from one edge of the walls to the other. Vertical loads applied to structures are resisted differently by shearwalls and floor slabs. These vertical loads cause in-plane compression in shearwalls, while floor slabs must resist these loads by bending. The floor slabs subjected to combined vertical and lateral loads, therefore, are truly three-dimensional, while the shearwalls can still be analyzed as plane stress problems. Boundary frames attached to shearwalls are usually arranged symmetrically about the middle planes of the walls, while integrated beams in floor systems are attached to the lower side of the floor slabs. These restraining
members also differ in size. They are usually much larger in shearwalls than in floor slabs.

Despite these differences, the two structural elements still show very similar behavior, particularly in their function as diaphragm. Consequently, analysis and design of floor slabs as diaphragms frequently make use of experimental findings, theoretical analyses, and design procedures originally developed for shearwalls. Benjamin and Williams (2.7) conducted a comprehensive research of one-story shearwalls subjected to static loadings. They made experimental investigation of the effects of various parameters on the ultimate strength of the shearwalls. The variables chosen in their study were: the aspect ratio, the relative size of boundary members, the amount of reinforcing steel in the shearwalls or the boundary members, and any additional reinforcement arranged diagonally in the shearwalls. Tomii and Osaki (2.41) and Tsuboi et al. (2.44) also studied the behavior of low-rise shearwalls (with height-to-length ratio less than 1.0) surrounded by reinforced concrete or steel frames. They pointed out that the boundary frames improved significantly the ductility as well as the strength of the shearwalls. Barda (2.6) also investigated low-rise shearwalls. He reported on the significant effect of the vertical reinforcement on the shear strength of the shearwalls.
Cardenas and Magura\textsuperscript{(2.8)} presented the results of a laboratory investigation on the strength of shearwalls for high-rise buildings. They chose as variables the amount and distribution of vertical reinforcement and the moment-to-shear ratio. Their test results indicated that the strength of high-rise shearwalls was generally controlled by flexure. They also showed that the flexural strength of these walls could be calculated by using the same assumption used for reinforced concrete beams. Fiorato et al.\textsuperscript{(2.19)} investigated the behavior of high-rise shearwalls with the emphasis on examining the performance of confinement reinforcement in vertical boundary columns and anchorage of horizontal wall reinforcement when the walls were subjected to severe cyclic loadings. The behavior of shearwalls under cyclic loadings also was investigated by Oesterle et al.\textsuperscript{(2.32)}, Alexander et al.\textsuperscript{(2.3)}, and Shiga et al.\textsuperscript{(2.38)} The items studied by these researchers included: stiffness in post-elastic regions, ductility, energy absorption capacity, and stiffness degradation due to cyclic loadings.

While the achievement of the above cited studies has been based mainly on experimental studies, there have also been several theoretical investigations. Tomii and Tokuhiro\textsuperscript{(2.42)} proposed an analytical method to solve the problems of isolated shearwalls surrounded by boundary frames. They used the Airy stress function
to solve the governing elastic equations and offered elastic solutions for several types of loadings. The finite element method has been used in many theoretical studies. Yuzugullu and Schnobrich (2.48) developed a finite element model to simulate the behavior of shearwalls with boundary frames under static loadings. Darwin (2.14) presented a new finite element model and developed load-deformation curves of shearwalls under cyclic loadings. A more detailed discussion of the use of the finite element method in two dimensional plane stress problems will be given in Chapter IV.

2.3 Current Design Procedures

As pointed out in preceding sections, the distribution of lateral forces to vertical lateral load resisting components must be carefully evaluated to enable a proper design. The Uniform Building Code (2.24) stipulates that the total lateral force should be distributed in proportion to the stiffness of vertical components. This procedure is based upon the rigid slab assumption, which, as already discussed, may not be adequate for some types of structures. In order to compensate this deficiency, the Code also stipulates that frames in wall-frame structures should be capable of resisting at least twenty five percent of the total lateral force. On the other hand, the
Applied Technology Council (ATC)\cite{2.4} allows for the flexibility of the floor slabs by recommending that:

"The design lateral forces should be based on an analysis which explicitly considers diaphragm deformations and satisfied equilibrium and compatibility requirements."

Alternately, the ATC suggested that design could also be based upon the envelope of the two solutions: first by assuming the diaphragms to be infinitely rigid, then very flexible. It also suggests that where the horizontal diaphragm is not continuous, the story shear can be distributed to the vertical components based on their tributary areas and torsional moments can be ignored. The ATC also discusses the design forces for diaphragms:

"The seismic forces to be resisted by diaphragms area a minimum force equal to 0.5 $A_v$ times the weight of the diaphragm and other elements of the building attached thereto plus the portion of $V_x$ required to be transferred to the components of the vertical seismic resisting system because of offsets or changes in stiffness of the vertical components above and below the diaphragms."

$A_v$: The seismic coefficient representing the Effective Peak Velocity-Related Acceleration

$V_x$: The seismic shear force at any level

Codes and regulations stipulate that the distribution of the lateral force be determined based on the lateral stiffness
of vertical elements. Evaluation of the lateral stiffness of vertical elements like frames and walls, however, is a cumbersome task. Muto\textsuperscript{(1.29)} has developed a simple method to compute the stiffness of frames and walls. The story stiffness of frames is calculated by assigning inflection points in beams and columns. The wall stiffness, on the other hand, is computed by treating the wall as a deep beam which includes flexural and shear deformations and base rotation.

Once the forces on floor slabs are determined, designers can design these floor slabs according to available codes or specifications. The ACI Code\textsuperscript{(1.1)} does not have specific provisions for the design of floor slabs as diaphragms. Consequently, designers frequently employ the design procedures developed for shearwalls. Cardenas and Magura\textsuperscript{(2.8)} and Fintel\textsuperscript{(2.18)} offered design methods to determine the flexural capacity of slender shearwalls. The shear capacity of shearwalls has been the subject of research for many years. Cardenas et al.\textsuperscript{(2.9)} and ACT-ASCE Committee 426\textsuperscript{(2.1)} have given comprehensive reviews of previous studies and summaries of various design procedures for shearwalls subjected to shear forces. Tomii\textsuperscript{(2.43)} gave a review of Japanese practices of shear wall designs.
When inelastic behavior of structures under severe lateral loadings is considered, the ductility of the structural components should be evaluated appropriately. The ductility of reinforced concrete components, however, is very difficult to define on account of many complicated phenomena, such as the cracking of concrete, the yielding or slipping of reinforcing bars, or the crushing of concrete. Mattock (2.28), Corley (2.13), and Backmann (2.5) offered empirical formulas to predict the ductility for flexural beams. Uzumeri and Paulay (2.45) later extended Mattock's and Corley's works and proposed design procedures for the ductility of shearwalls. The proposed procedures deal with only slender shearwalls whose performance is controlled basically by flexure and consequently do not accurately evaluate the ductility of stocky shearwalls. Stocky shearwalls, in which shear behavior dominates, are known to have rather limited ductility (2.6, 2.7).

2.3 Summary

As described in Section 2.1, studies including the diaphragm action of floor slabs have reached different conclusions as to the effect of the floor slabs on the lateral load distribution to vertical elements. One of the primary reasons of the inconsistent results can be the lack of decisive information about the in-plane characteristics of the floor slabs. According to Sections 2.2 and 2.3, little research has contributed to the
understanding of the in-plane behavior of floor slabs, and, practically speaking, there is no design provision for the diaphragm design of floor slabs. The appropriateness of using the design provisions developed for shearwalls to design diaphragms has not yet been verified. Furthermore, procedures to determine the design forces controlling the in-plane behavior of floor slabs have not been established, either.

The following chapters will provide data needed to discuss these unclarified problems and propose practical procedures of the diaphragm design.
III. EXPERIMENTAL STUDY

3.1 General

This chapter describes the experimental study. Scaled models representing floor slab panels in medium to high rise building structures were tested under various loading and support conditions. The results of these experiments provide general information about the in-plane characteristics of the floor slabs. Critical parameters controlling the in-plane behavior of the floor slabs are varied carefully, and the effects of these parameters on the in-plane characteristics are examined.

3.2 Design of Test Structures

3.2.1 Prototype Floor System and Scaled Model

The prototype floor slab for test specimens was taken from a rectangular multi-story, multi-bay reinforced concrete building, in which earthquake resistance was provided by shearwalls located in selected bents. Seismic forces at various floor levels were transmitted to the walls by the diaphragm action of the floor slabs. Structural dimensions were chosen to represent a building of medium to high rise. The center-to-center span length of slab panels were 7320 mm (24 ft) in both directions, the columns were 610 mm x 610 mm (24 in. x 24 in.) with no capital, the slab was 180 mm (7 in.) thick, and the beams were 610 mm x 310 mm
(24 in. x 12 in.) in their cross sections. A portion of the plan view of the prototype floor system is shown in Fig. 3.1.

The basic test specimen chosen for this experimental study represented an interior panel of the prototype building and was supported on one edge by a shearwall and on the opposite edge by columns. The fabricated test specimen consisted of three consecutive panels supported by two shearwalls and four columns. Overhanging slabs, equal to one quarter of the panel dimension, were added on all non-continuous sides to represent parts of the floor slabs of the adjacent bays. (Fig. 3.2) Full scale modelling was abandoned because of economy, space, labor, loading, and measurement. Small scale modelling with its many disadvantages, on the other hand, was also unacceptable on account of the difficulties in modelling the aggregate particles, the reinforcing bars, and the bonding effect between concrete and steel. An intermediate scale ratio of 1 : 4.5 was selected. Fig. 3.2 illustrates the test specimen and the several support conditions used in the experiment. The basic panel is 1630 mm x 1630 mm (64 in. x 64 in.) and 40.0 mm (1.56 in.) thick.

3.2.2 Design of Test Specimen

The prototype floor slab was designed according to the current ACI Code (1.1) for a service live gravity load of 3.8 kPa
(80 psf). The direct design method described in Section 13.6 of the Code was employed. The columns were designed for combined axial force and bending moment caused by the vertical load at an intermediate floor level. The test specimen then was designed by scaling down these forces and moments. The shearwalls, on the other hand, were designed so that they would possess sufficient strength to carry the expected maximum load which may emerge during the testing. This design was performed to insure failure in the slabs. Based on a preliminary analysis, the design shear force for the shearwalls was selected to be 230 kN (50 kips). Adopted concrete strength was: 27.6 MPa (4000 psi) for the floor slabs and walls and 34.5 MPa (5000 psi) for the columns. The yield strength of reinforcing bars was taken to be 410 MPa (60 ksi). Table 3.1 lists the critical dimensions of the specimen.

Special care was given to the size of reinforcing bars. The smallest bars available were larger than what was needed by the adopted scale ratio (1 : 4.5). Fig. 3.3 shows the arrangement of the reinforcing bars.

Table 3.3 lists the dimension, the design moment, and the area. The next to the last column of the table shows the ratio of the amount of steel provided in each individual strip.
to the amount of steel required by the Code. The last column lists the ratio of the amount of steel provided in the strip to the amount of steel required to carry the design moment. The table shows that the temperature requirement (0.0018 times the gross area) controls the amount of steel required at many critical sections. The average over-supply ratio of reinforcement is 28 percent for one direction and 21 percent for the other direction. In comparison with the flexural requirements, the amount of reinforcement used is 3.42 and 2.81 times larger respectively than that required.

Table 3.4 lists the amount of steel provided in the beams. Additional reinforcing bars were placed in the bottom layer of negative moment regions and in the top layer of positive moment regions in order to meet the requirement stipulated in Appendix A of the Code.

3.2.3 Fabrication of Specimen

After cutting and bending reinforcing bars, strain gages were placed in specified locations. Reinforcing bars then were placed and tied together in a form work. In addition to reinforcing bars, a total of fifty five inserts also were installed. These inserts were placed in the specimen to serve as hooks through which vertical loads can be applied. Additional explanation of the function of the inserts and the application of
vertical loads will be given in Section 3.4.2. The formwork was elevated about 1.2 m (4 ft) from the floor level of the testing laboratory in order to facilitate construction of the specimen.

Two specimens were constructed and casted at the same time by using concrete mixed at the testing laboratory. The two specimens are labelled B-1 specimen and B-2 specimen. Two kinds of concrete were prepared: 27.6 MPa (4000 psi) for the floor slabs and the walls and 34.5 MPa (5000 psi) for the columns. Table 3.5 shows the detail of concrete mix. A slump of 130 mm (5 in.) was specified in view of the maze of reinforcement in the specimens. Plastisizer was added to further facilitate the placement of concrete around closely spaced reinforcing bars. The added plastisizer was WRDA-19, which is formulated to comply with ASTM Specification Designation C 494, a Type A water reducing admixture. The aggregate size was limited to 6.4 mm (0.25 in.) in order to preserve approximately the prototype relationship between the aggregate size and the slab thickness. Separate batches of concrete were used for each specimen.

Sixteen 76 mm x 150 mm (3 in. x 6 in.) and ten 150 mm x 300 mm (6 in. x 12 in.) concrete test cylinders were made from each batch of concrete. They were placed next to the freshly cast test specimens so that both the specimens and test cylinders
would experience the same curing conditions. The specimens and cylinders were cured for fourteen days under moist burlap at room temperature. On the fifteenth day after the placing, the burlap was removed, and the cylinders were stripped. The specimens and concrete test cylinders were then air-cured until tested. Table 3.6 lists the slump, the 7-day strength, and the 28-day strength of the concrete.

Mechanical properties of concrete and reinforcing bars were obtained by the concrete cylinder test at appropriate intervals and by the steel tension test. Section 3.3 describes these properties in further detail.

3.3 Mechanical Properties of Materials

3.3.1 Reinforcing Bars

Deformed reinforcing bars of three sizes were used in the test slab specimens: D2.0, D2.5, and D3.0. The mechanical characteristics of these bars were determined by basic tension tests. The test was repeated four times for each size of reinforcing bar. An electric extensometer with a 57 mm (2.25 in.) gage length was used to measure the strain. Table 3.7 lists the yield stress, the yield strain, the ultimate stress, the ultimate strain, and the modulus of elasticity of these bars. The values listed in the table represent the averages of the results of the four tests.
3.3.2 Concrete

Two kinds of concrete were used in the test specimens: 27.6 Mpa (4000 psi) concrete and 34.5 MPa (5000 psi) concrete for each of the two specimens, B-1 and B-2. Standard cylinder tests were performed at ages of seven and twenty eight days. On the first day of slab specimen testing, four more concrete test cylinders were tested to obtain modulus of elasticity, Poisson's ratio, and compressive strength. The ages of concrete at these tests were fifty two days for specimen B-1 and 109 days for specimen B-2.

In order to calculate the modulus of elasticity and Poisson's ratio, the longitudinal strain of the concrete cylinder was measured by a pair of clip gages mechanically attached to the cylinder, while the transverse strain was measured by two electric strain gages mounted on the cylinder in its circumpherential direction. After the cylinder was preloaded three times with 90 kN (20 kips), the load was monotonically increased until the cylinder failed. The split cylinder test was carried out during the same time. Table 3.8 lists the compressive strength, the tensile strength, the modulus of elasticity, and the Poisson's ratio of the concrete at the beginning of slab specimen testing. The ratios indicated in the table are those in the initial stage.
3.4 Testing Procedure

3.4.1 Test Setup

A group of special fixtures was developed to perform the experiment effectively. In order to facilitate the application of vertical load and provide access to the underside, the specimen was supported on four heavily reinforced concrete pedestals anchored to the floor of the testing laboratory. The tops of these pedestals were equipped to receive either a wall or a pair of columns. In either case, a variety of support fixidity was provided. A wall can be attached to the pedestal by means of twelve bolts along each side of the wall. Assisted by two pairs of heavy steel braces, these bolts completely prevented the wall from moving in the floor plane. Under this support condition, no force can be transmitted between slab panels on opposite sides of the wall. The slabs were effectively isolated from each other, thus enabling the testing of single panels. (Fig. 3.4) On the other hand, by removing the braces and loosening all anchoring bolts, the wall can be supported on several sets of ball bearings and, therefore, can become free to move about on top of the pedestal in any horizontal direction. The third alternative was to secure only four of the twenty four bolts near the middle of the wall to eliminate all translatory movements, while allowing the walls essentially free to rotate about a vertical axis. The column base fixtures were also adjustable to provide either a free sliding or a fixed
condition. The column in the free sliding condition did not offer any resistance to the applied lateral load and provides only a vertical reaction to the gravity load. With a fixed base, the only possible freedom was rotation about a horizontal hinge in the fixture, located 405 mm (16 in.) below the slab center plane. The columns in this condition participated in resisting lateral load. An overall view of the test setup is shown in Fig. 3.5.

3.4.2 Loading System

The in-plane load was generated by a double-acting mechanical jack placed at the slab center-plane and acting against a heavy steel frame. To simulate the desired shear action, a steel frame was used to distribute the jack load to five embedded studs along the loading line at uniformly spaced distances of 540 mm (21.3 in.). (Fig. 3.6 and 3.7) The frame and studs were carefully designed so that each stud would transmit approximately one fifth of the total applied load and the action would lie in the slab center-plane. Fig. 3.7 shows the dimensions of both the frame and studs. The total jack load was measured by a concentric loadcell between the loading jack and distribution frame. (Fig. 3.8) The load on each stud was not individually measured.
The out-of-plane (vertical) load was applied as a series of concentrated forces, spaced at 540 mm (21.3 in.) (one-third panel dimension) center-to-center in each direction. Inserts were placed at the center of each ninth portion of each panel for the application of these loads. (Fig. 3.9) All point loads within one panel width, including those in the quarter panel extension portions, were controlled by a single vertical (gravity) load simulator. A series of statically determinate levers was devised so that all point loads would be equal. (Fig. 3.10) A preliminary elastic analysis showed that a series of concentrated forces could reasonably simulate the uniformly distributed vertical load on the slabs. The vertical (gravity) load simulator was designed so that substantial displacement of the specimen would be permitted in the direction of the in-plane loading without affecting either the direction or the magnitude of the applied vertical load. The out-of-plane (vertical) load was monitored by two loadcells installed between the loading jack and the distribution levers.

3.4.3 Instrumentation and Recording of Data

Deflections and strains were monitored throughout the test. In-plane deflections were measured by linear variable differential transformers (LVDT's) connected to selected reference points. Figs. 3.11 and 3.12 show the arrangement of LVDT's at different phases of the experiment. All LVDT's were
connected to a single data acquisition system, and signals from the LVDT's were recorded on paper tapes. Several of the LVDT's also were connected to X-Y plotters together with the loadcell measuring in-plane loading, and the load-displacement relationship was continuously monitored. Fig. 3.13 schematically shows the whole system of instrumentation established at the single panel strength test.

Out-of-plane (vertical) deflections were measured at several points by means of a transit. A scale was fixed to the top of the tested slab panel at each measuring points. Fig. 3.14 illustrates the target locations as well as the measuring technique. W32 paper electrical wire strain gages were used to measure strains in reinforcing bars. After smoothing the surfaces of reinforcing bars, the strain gages were carefully mounted, protected, and waterproofed to avoid damage before and during the testing. A total of 196 strain gages were installed in each specimen: sixty strain gages in each slab panel and four gages in each column. Fig. 3.15 shows the location of the strain gages. The strain gages were connected to the data acquisition system, and their signals were recorded on paper tapes. Six sets of rosette gages were mounted on the surface of the tested slab immediately before the load was applied. Fig. 3.16 indicates the location of the rosette gages.
3.5 Testing Program and Sequence

A series of testing was programmed for each of the two specimens. The program's sequence was carefully planned to maximize information that can be obtained from the testing but minimize the time and effort required to prepare for each testing program.

The testing program consisted of four kinds of tests: the stiffness test, the strength test, the repaired strength test, and the free vibration test. In the stiffness test, each specimen was tested as a whole unit, and elastic in-plane stiffness characteristics of the floor system were examined. In the strength test, each slab panel of specimens was tested separately according to predetermined loading and boundary conditions. Ultimate strength, stiffness in post-elastic regions, and ductility of the panel were examined. The panel damaged after the strength test was repaired by using the epoxy-injection technique and tested again under the same testing procedures used in the strength test, labelled the repaired strength test. In this test, the performance of the repaired panel on ultimate strength, stiffness and ductility was investigated. In the free vibration test, each slab panel of specimen was examined about its vibration characteristics such as frequency and damping. The vibration test was repeated several times at various stages in the entire testing
process of each panel: the initial stage, after strength test when the panel was severely damaged, after repair, and after repaired strength test. The changes in vibrational characteristics at these stages were examined.

This report deals with the first two tests only, namely, the stiffness test and the strength test. The description of the entire testing program as well as the test results are presented elsewhere. (3.2)

3.5.1 Stiffness Test

For each specimen, a stiffness test was carried out prior to the strength tests. Each specimen was tested as a whole unit, with both walls supported in the free-to-rotate condition and all columns in the free-to-slide condition. Small in-plane loads were applied simultaneously along both column lines (Fig. 3.11). Under these loading and boundary conditions, the specimen behaved like a simply supported beam with overhanging ends. Two loading conditions were used, both with loads of equal magnitudes. First, the loads were applied in the same direction, causing a symmetrical loading condition in the specimen, as shown in Fig. 3.11(a). In the second test, the loads were applied in opposite directions, causing an anti-symmetrical loading condition as in Fig. 3.11(b). The loads applied were limited to 15 kN (3.5 kip), 12 percent of the ultimate load, to ensure that the specimen would remain in its linear
elastic range. Deflection of the panels and rotation of the walls were measured with LVDT's, as shown in Fig. 3.11. In this figure, $\Theta$ is the rotation of the wall measured in the plane of the slab specimens. The LVDT pair, #9 and #10, or #11 and #12, measured this rotation. The displacement $\delta$, monitored by the LVDT #1 or #5, was measured parallel to the loading at the end of the specimen.

3.5.2 Strength Test

In the strength test, the wall supporting the slab panel to be tested was securely fastened to a pedestal by means of bolts and braces. The other wall and all columns were supported on pedestals in the free-to-slide condition. The in-plane load was applied along the column line parallel to the fastened wall and through the horizontal load distribution frame described in Section 3.4.2. The out-of-plane (vertical) load was also applied when required by the testing program. LVDT's were installed at critical locations and monitored throughout the test as shown in Fig. 3.12. Signals from all strain gages both inside and on the surface of the tested panel were also recorded.

Two types of in-plane loads were used in the strength tests: monotonic loading and cyclic loading. In monotonic loading tests, the in-plane load was gradually increased until the resistance of the test panel decreased significantly after
reaching the ultimate strength. The load then was released and
applied in the opposite direction until failure again took place
in a similar manner. The load then was released again, and the
test was completed. In the subsequent discussion, the first
direction of loading is referred to as positive, and the second
as negative. In cyclic loading tests, the in-plane load
followed the spectrum shown in Fig. 3.17. The load was applied
in complete cycles with gradually increasing displacement ampli­
tudes. Three complete cycles were used at each amplitude. The
spectrum was controlled by the displacement along the loading
line. The amplitudes specified in the spectrum were: 0.25 mm,
0.76 mm, 1.3 mm, 2.8 mm, 4.3 mm, 5.8 mm, 7.6 mm, and 8.9 mm.
This type of loading spectrum most effectively provides data
regarding the hysteretic behavior of members or structures when
the number of test specimens is limited.

For the tests including vertical (out-of-plane) loading,
the total load was chosen to simulate full service dead and live
loads. It is noted that the relative weight of the scaled model was
less than that of the prototype. Therefore, the vertical load applied
to the test panels represented, in addition to full service live
load of 3.8 kPa (80 psf), also a supplement of service deal load
of 3.9 Pa (83 psf). The total vertical load applied to the entire
panel was 45.8 kN (10.3 kip).
The test was conducted according to the following procedure:

1. Initial readings were taken from all measuring devices: loadcells, LVDT's, strain gages, and vertical scales.

2. The out-of-plane (vertical) load up to the specified load level was applied if required. This load was held constant throughout the remaining steps of the test.

3. Strain and displacement readings were taken again after the application of out-of-plane loading. The data provided the characteristics of the test panel subjected to the design service vertical load.

4. The test panel was then preloaded by several cycles of a small in-plane load. The load was limited to 9 kN (2 kip) for tests without vertical load and to 5 kN (1.5 kip) for tests with vertical load. The purpose of this operation was to ensure that all instruments were working properly and make the whole testing system to stabilize by adjusting itself.

5. After preloading, in-plane load was applied according to the specified load spectrum. The in-plane load was applied quasi-statically and stopped at frequent
load or displacement increments for strain and displacement measurements. During the in-plane loading, the vertical load was continuously monitored and adjusted to maintain its specified level.

6. The test was terminated when a significant drop in in-plane load resistance was observed, or when it became impossible to maintain the desired vertical load.

7. At the end of each test, a set of final readings were taken from all measuring devices after complete unloading.

3.5.3 Designation of Testing Programs

For the sake of convenience, each test in the program is identified by a five character alphanumeric code. The first character of the code is always B, signifying Beam-supported slab specimens. The second is either H or V. H indicates that no vertical load is applied (in-plane, horizontal load only), while V indicates that vertical load is applied together with in-plane load. The third character identifies the slab being tested. Numerals 1, 2, and 3 respectively refer to panels 1, 2, and 3. (Refer to Fig. 3.2) The stiffness test, in which specimens were tested as whole units, is identified by numeral 6 as the third character. A combination of the fourth and fifth characters defines the loading condition: MN for Monotonic loading, CY for
Cyclic loading, SS for Symmetrical loading in the stiffness test, and AS for Anti-Symmetrical loading in the stiffness test. A chart of the alphanumeric codes used in the testing program is tabulated in Table 3.9(a) with an example code. This table also summarizes the entire testing program and gives brief explanations of the characteristics of each testing program.

3.6 Test Results

3.6.1 Results of Stiffness Tests

A stiffness test was executed for each of the two specimens B-1 and B-2. The displacements (δ) along the edges parallel to the loading and the rotation of the wall (θ) were measured respectively by the LVDT pair #1 and #5 and the LVDT pairs #9 and #10, and #11 and #12. Table 3.10 lists the average values of the displacement and rotation per unit load as shown in Fig. 3.11. Specimens B-2 has slightly larger displacements and rotations than specimen B-1, hence lower stiffness. Shrinkage cracks were observed in specimen B-2 before the testing and they were believed to have contributed to the lower stiffness. The modulus of elasticity of concrete in the two specimens on the other hand, were nearly the same. (Table 3.8)
3.6.2 Behavior of Test Slab under Service Vertical Load

The service vertical load was applied to panels 1 and 2 of specimen B-2 prior to in-plane loading. Table 3.11 lists the vertical displacements of the panels under this condition measured at three points as shown in Fig. 3.14. Fig. 3.18 illustrates the strain distribution under the service vertical load. At this stage, two lines of cracks were observed on the top surface of the slab, one along the slab-wall junction, and the other along the column line parallel to the wall. (Fig. 3.19) Apparently, negative bending moment due to the service vertical load exceeded the cracking moment of the slab.

3.6.3 Results of Strength Test

The extreme loads and displacements from the strength tests are listed in Table 3.12. The displacements are measured at a point directly opposite the in-plane loading as shown in Fig. 3.12 (LVDT #3). The displacements shown in the figures represent the true deflections of the tested slab panels. All contributions of the fixed edge's movements were removed from the displacement measured by LVDT #3. Consequently, the true displacement spectrum applied to these test panels was slightly less severe than intended. Figures 3.20 through 3.25 show the load-displacement curve of the test slab panels.
Table 3.13 lists the stiffness of the panels under small in-plane loads applied at the beginning of the test step No. 4, Section 3.5.2. The elastic stiffness is defined as the load per unit displacement along the loading line (LVDT #3). The stiffness in post-elastic regions, on the other hand, is not clearly defined since it varies according to load levels as well as previous loading histories. In order to evaluate the stiffness and stiffness degradation of the panels subjected to cyclic loading, five different values are selected and observed as schematically shown in Fig. 3.26. They are:

1) the slope labelled the secant stiffness 1, defined by the line linking the points corresponding to the extreme displacements of a hysteretic loop, 2) the slope labelled the secant stiffness 2, defined by the line linking the points corresponding to the maximum positive displacement and preceding zero displacement of a hysteretic loop, 3) the slope labelled the secant stiffness 3, defined by the line linking the points corresponding to the maximum negative displacement and preceding zero displacement of a hysteretic loop, 4) the slope labelled the tangent stiffness 1, defined by the line linking the points corresponding to zero and 1.3 mm (0.05 in.) displacements intersected on the way to the maximum positive displacement of a hysteretic loop, and 5) the slope labelled the tangent stiffness 2, defined by the line linking the points corres-
ponding to zero and -1.3 mm (-0.05 in.) displacements intersected on the way to the maximum negative displacement of a hysteretic loop. The secant stiffness 1 measures the total load change as the panel is subjected to a complete reversal of cyclic displacements. The tangent stiffness 1 or 2 represents the stiffness when a small reversed load is applied after a severe load cycle. Test results have verified that the stiffness is almost constant in the region between zero and ±1.3 mm displacements. The secant stiffness 2 and 3 measure two additional stiffness values referring to positive or negative loading only. The difference between the secant stiffness 2 or 3 and the tangent stiffness 1 or 2 reflects the pinching effect of the hysteretic loop. Figures 3.27 through 3.29 show the stiffness of the tested slab panels subjected to cyclic loading.

Figures 3.30 to 3.35 show the crack pattern of the tested panels when the tests were completed. Numerals attached to cracks represent the loads when the cracks were observed. Solid lines indicate cracks developed during positive loading, and broken lines indicate cracks developed during negative loading. In addition, bold solid lines indicate "major cracks", which are defined later. Only the top surfaces are shown for slab panels tested with in-plane load alone since the crack patterns of both top and bottom surfaces are nearly identical. For panels tested
with both vertical and in-plane loads, crack patterns of the top and bottom surfaces were significantly different. Both top and bottom crack patterns, therefore, are shown for these panels.

Figs. 3.36 and 3.37, respectively, show the vertical deflections of BV1MN and BV1CY at measured locations. (Fig. 3.14)

For each strength test, comments and discussions are given below.

**BH2MN** The ultimate load, 120 kN (27.0 kip), was reached when several cracks, started at lower load levels, suddenly started growing. This development was followed by a significant loss in resistance of about 31 kN (7.0 kip). Although the load again increased, approaching the ultimate load level 116 kN (26.0 kip), this time its stiffness was less. Another significant loss of resistance took place when several reinforcing bars broke at a total displacement of 7.62 mm. After the bars broke, the system regained its equilibrium at a load of 70 kN (16 kip) and at a displacement of 8.6 mm. During negative loading, defined as loading in the negative direction, the maximum load attained was 89 kN (19.9 kip). At this point, several reinforcing bars broke (~7.24 mm of displacement), and the resistance was reduced greatly.
At 36 kN (8 kip), a crack developed along the slab-wall junction, extending from the edge of the slab to the beam. At 116 kN (26.0 kip), a point near the ultimate load, three cracks developed at the slab edge and rapidly extended inward. One of the cracks, which increased its width significantly, primarily controlled the behavior of the panel in the post-ultimate load region. This crack was labelled the major crack. Additional cracks developed diagonally between the first and second significant losses in resistance. The breaking of several reinforcing bars caused the second significant loss in resistance. One bar embedded in the quarter overhang and two in the beam broke at their intersections with the major crack. The major crack extended parallel to the wall at a distance of approximately 360 mm (14 in.). This line nearly coincided with the boundary between the column and middle strips of the panel, where a number of negative and positive reinforcing steel were cut off. During negative loading, the resistance decreased several times by about 5 to 8 kN (1.0 to 1.5 kip). These losses in resistance corresponded to the development of new cracks. When the load reached 89 kN (19.9 kip), one reinforcing bar in the overhang and two bars in the beam broke, which caused the resistance to decrease about 27 kN (6.0 kip).
The ultimate load was obtained at 56.9 kN (12.8 kip), followed by a decrease in resistance of 13 kN (3 kip). Afterward, the load was unchanged, while the displacement increased from 2 to 7 mm. Cracks were accumulated in the post-ultimate load region, and another significant loss in resistance of about 13 kN (3 kip) caused critical damage to the panel. During negative loading, the load gradually increased to 39 kN (8.9 kip), followed by a sudden decrease by 11 kN (2.5 kip).

At the load of 19 kN (4.3 kip), a crack which had developed along the slab-wall junction extended from the tension edge to the beam. The development of a second crack parallel to the wall caused a significant loss in resistance immediately after the ultimate load was reached. This crack, which later increased in width, controlled the deformation in the post-ultimate load region. At 5.1 mm (0.20 in.) of displacement, one reinforcing bar in the overhang broke, causing a slight decrease in resistance of 2 kN (0.5 kip). At 7.3 mm (0.29 in.) of displacement, three bars in the beam and two additional bars in the slab broke, reducing the resistance significantly. The width of the major crack at the slab edge was 9.5 mm (3/8 in.) at this displacement. During negative loading, the first crack was observed at the load of 22 kN (5.0 kip). A second crack which developed at 24 kN (5.5 kip) later merged with the major
crack developed during positive loading. After resistance decreased slightly several times due to the development of new cracks, three bars in the beam and two bars in the slab broke at 6.2 mm (0.24 in.) of displacement. At this point, the load decreased from 30 kN (6.8 kip) to 20 kN (4.5 kip), which was 52 percent of the negative ultimate load.

BH1CY The resistance never decreased in each of the cycles with the three lowest amplitudes: 0.25 mm, 0.76 mm, and 1.3 mm. The hysteretic loops, then, were stable during these cycles. The resistance decreased twice during the loading in each of the 2.8 mm, 4.3 mm, and 5.8 mm amplitudes. The resistance decreased once during the negative loading of the first cycle and again during the positive loading of the second cycle in the 2.8 mm amplitude. During the positive and negative loadings of the first cycle in the 4.3 mm and 5.8 mm amplitudes, the resistance decreased again. The development of cracks caused these losses of resistance. In the first cycle of the 5.8 mm amplitude, the critical cycle, both positive and negative ultimate loads were attained [94.7 kN (21.3 kip) for the positive ultimate load and 96.5 kN (21.7 kip) for the negative ultimate load]. A crack extending parallel to the wall at a distance of 360 mm (14 in.) developed during this cycle (the major crack). During later loadings, new cracks did not develop, and the major crack controlled the deformation. During cycles
of the 7.6 mm (0.23 in.) amplitude, the resistance decreased gradually to 52 percent of the ultimate resistance for positive loading and 63 percent of the ultimate resistance for negative loading. The breaking of reinforcing bars at their intersections with the major crack caused losses in resistance in this cycle.

A crack developed along the slab-wall junction at 28 kN (6.3 kip) during positive loading and at 28 kN (6.3 kip) during negative loading. The major crack began at 38 kN (8.5 kip) from the edge subjected to tension under negative loading and at 69 kN from the edge subjected to tension under positive loading. The cracks, which developed at the edges of the panel, extended inward while shifting their directions toward the wall; these cracks developed substantially during the cycles of the 4.3 mm (0.17 in.) amplitude for positive loading and during the cycles of the 5.8 mm (0.23 in.) amplitude for negative loading. The complete formation of the major crack, which occurred during the first cycle of the 5.8 mm (0.23 in.) amplitude, prevented the development of other cracks. During negative loading in the first cycle of the 7.6 mm (0.23 in.) amplitude, one reinforcing bar in the slab broke, causing a loss in resistance of 6.7 kN (1.5 kip). During positive loading in the second cycle of the 7.6 mm (0.23 kip) amplitude, one bar in the slab broke, causing a loss in resistance of
19 kN (4.3 kip). During the succeeding cycles of this amplitude, several additional bars in the beams and slab broke.

Unlike the previous strength tests, no substantial loss in resistance was observed during testing. The load reached 95 percent of the ultimate load, 97 kN (22 kip), at 6.3 mm (0.25 in.) of displacement. The panel deformed farther by about 2.5 mm (0.1 in.) without any loss in resistance. The ultimate load, 102 kN (23 kip), was reached at 8.4 mm (0.33 in.) of displacement and followed by a small portion having a slight negative slope. The in-plane load was then removed at 8.8 mm (0.35 in.) of displacement since the loading device arrived at its limit. During negative loading, the resistance gradually increased to 90 kN (20.3 kip), followed by a slight loss in resistance. The test was terminated after the panel reached -8.8 mm (-0.35 in.) of displacement.

Vertical deflections measured by the scales gradually increased under positive loading as shown in Fig. 3.36. The deflection at the center of the panel was 5.4 mm (0.21 in.) immediately before unloading. Upon unloading of the in-plane load, the deflections did not decrease but increased slightly. They continued to increase under negative loading until near the end of the test. The maximum deflection, reached immediately before unloading, was 7.8 mm (0.31 in.) at the panel's center.
As mentioned in Section 6.3.2, the vertical load caused two cracks on the top surface of the panel. One of these cracks, developed along the slab-wall junction, extended downward and finally reached the bottom surface at 22 kN (4.9 kip). As evidenced from Fig. 3.34, the crack patterns between the top and bottom surfaces differed significantly. Many cracks on the bottom surface extended from the center of the panel in radial direction, whereas cracks on the top surface were confined near the slab-wall junction and more parallel to the wall. The major crack, however, was observed almost simultaneously on both top and bottom surfaces. At 27 kN (6.0 kip), a crack (the major crack) developed and extended from the edge of the slab at a distance of 330 mm (13 in.). During negative loading, the major crack was observed for the first time at 35 kN (7.8 kip). This crack rapidly extended inward and merged with the major crack developed during positive loading. No reinforcing bars broke during this test.

BH3CY The resistance never decreased in each of the cycles with the three lowest amplitudes: 0.25 mm, 0.76 mm, and 1.3 mm. The hysteretics loops, then, were stable in these cycles. During the first cycle of the 2.8 mm (0.11 in.) amplitude, a crack, which eventually became the major crack, developed parallel to the wall at a distance of 280 mm (11 in.), and a slight decrease in resistance occurred for the first time. The hysteretic
loops remained stable during the succeeding cycles of the 2.8 mm (0.11 in.) amplitude as well as the three cycles of the 4.3 mm (0.17 in.) amplitude. The pinching effect was appreciable during the cycles of the 4.3 mm (0.17 in.) amplitudes. The maximum resistance reached during the second and third cycles of the 4.3 mm (0.17 in.) amplitude was slightly lower than that reached during the first cycle. The first cycle of the 5.8 mm (0.23 in.) amplitude yielded the ultimate load for both positive and negative loadings. [41.8 kN and 40.5 kN (9.4 kip and 9.1 kip)]. The resistance continuously decreased during the succeeding two cycles of this amplitude and was only about 60 percent of the ultimate resistance in the third cycle. Additional cracks did not develop during the second and third cycles of the 5.8 mm (0.23 in.) amplitude. At the end of the first cycle of the 7.6 mm (0.30 in.) amplitude, the resistance was reduced to 14.9 kN (3.36 kip), 36 percent of the ultimate resistance, and the test was terminated.

A crack along the slab-wall junction was observed at 18 kN (4.2 kip) during positive loading and at 14.2 kN (3.2 kip) during negative loading. Most of the other cracks developed from the edge of the slab and extended inward during the 2.8 mm (0.11 in.) amplitude. They extended almost directly across the panel without shifting their directions. During negative loading in the first cycle of the 5.8 mm (0.23 in.) amplitude,
a reinforcing bar in the overhang broke at 38.6 kN (8.68 kip). During negative loading in the second cycle of the 5.8 mm (0.23 in.) amplitude, two bars in the beam broke at 29.5 kN (6.65 kip), causing a loss in resistance of 10.2 kN (2.3 kip).

During positive loading in the first cycle of the 7.6 mm (0.30 in.) amplitude, one reinforcing bar in the slab and two bars in the beam broke at 26.2 kN (5.91 kip). All of the broken bars were cut off at their intersections with the major crack.

**BV2CY** The hysteretic loops remained stable in cycles with the four lowest amplitudes: 0.25 mm, 0.76 mm, 1.3 mm, and 2.8 mm. A crack, which eventually became the major crack, developed during the first cycle of the 1.3 mm (0.05 in.) amplitude at a distance of 250 mm (10 in.) and extended parallel to the wall. During negative loading in the first cycle of the 4.3 mm (0.17 in.) amplitude, a loss in resistance was observed for the first time from 82.8 kN (18.6 kip) to 77.8 kN (17.5 kip). During the succeeding two cycles of this amplitude, the maximum resistance was slightly lower than that during the first cycle. The ultimate loads were reached during the first cycle of the 5.8 mm (0.23 in.) amplitude [85.0 kN (19.2 kip) and 83.1 kN (18.7 kip)]. The resistance then decreased significantly during the succeeding two cycles of this amplitude. Two cycles with the 7.6 mm (0.30 in.) amplitude were further applied. At the end of this amplitude's second cycle, the resistance measured about half of the ultimate resistance.
Vertical deflection increased during the loading. At the end of the third cycle of the 5.8 mm (0.23 in.) amplitude, the deflection at the center of the panel was 6.4 mm (0.25 in.), which was measured by the scale #2 in Fig. 3.14. The deflection rapidly increased by 2.5 mm (0.10 in.) during the loading of the 7.6 mm (0.30 in.) amplitude. Permanent deflection after both in-plane and vertical loads were completely released was 5.3 mm (0.21 in.). (Fig. 3.37)

The deflections on the beams, which were measured by the scales #1 and #3, showed the behavior similar to the deflection at the center of the panel. These deflections also rapidly increased during the cycles with the 7.6 mm (0.30 in.) amplitude as shown in Fig. 3.37(a) and (c). The permanent deflections were 4.1 mm (0.16 in.) for the scale #1 and 3.5 mm (0.14 in.) for the scale #3.

Two cracks developed under the vertical load. One of the cracks, developed along the slab-wall junction during the first cycle of the 0.76 mm (0.03 in.) amplitude at the load of 22 kN (14.9 kip). Cracks were completely formed by the end of the first cycle of the 5.8 mm (0.23 in.) amplitude; afterward, the opening and closing of the major crack controlled the deformation of the panel. During positive loading in the first cycle of the 7.6 mm (0.30 in.) amplitude, one
reinforcing bar in the overhang and one bar in the beam broke, causing a loss in resistance from 80.0 kN (18 kip) to 70 kN (15.7 kip). During negative loading in the first cycle of the 7.6 mm (0.30 in.) amplitude, one reinforcing bar in the overhang and two bars in the beam broke, and the resistance decreased from 71.6 kN (16.0 kip) to 60.9 kN (13.7 kip).

3.7 Discussion

3.7.1 Stiffness Test

In-plane stiffnesses of the slab panels were calculated by an elastic finite element analysis in order to be compared with the experimental results. The standard SAP IV(3.1) finite element program was used with the following assumptions and specifications:

1. Inelastic action was not considered.
2. The concrete material was taken to be isotropic and homogeneous. Reinforcing bars were not included.
3. The material properties for concrete: the modulus of elasticity and poisson's ratio, were assigned values as obtained from concrete cylinder tests. (Table 3.8)
4. The analysis was treated as a two dimensional plane stress problem. Forces, stresses, and deflections in the third direction were ignored.

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5. Slabs were represented by a number of square plane stress elements. Beams were represented by flexural beam elements. The eccentricity between the neutral axes of slabs and beams was neglected. Then, no consideration was made on the effect of out-of-plane deformation caused by the in-plane loading on the in-plane stiffness.

6. A rather coarse discretization was used as shown in Fig. 3.38. This is referred to as Type I.

The third column of Table 3.10 lists the deflection ($\delta$) and rotation ($\theta$) obtained from the finite element analysis. The ratio of the values from this analysis to the experimental results range from 0.87 to 1.03 for the B-1 specimen and from 0.83 to 0.95 for the B-2 specimen. The analysis produces greater stiffnesses than the test by an average of 8 percent for the B-1 specimen and 12 percent for the B-2 specimen. The fact that the finite element method gives an upper bound of the solution is responsible for a fraction of the discrepancy. The remainder of the discrepancy can be attributed to minute cracks caused by shrinkage or accidental forces which might have been applied.
during the transportation of the specimens. In addition, the material properties used in the analysis, based upon the concrete cylinder tests, may not have represented the material properties in the specimens. In general, however, the correlation between the stiffness test and elastic finite element analysis is considered reasonable.

3.7.2 Behavior Under Design Service Vertical Load

An elastic finite element analysis using SAP IV was carried out on the behavior of a single slab panel subjected to design service vertical load. In this analysis, assumptions with respect to the concrete material properties followed those described in Section 3.7.1. This time, however, the panel was solved as a plate problem, and square plate elements were used. The panel was discretized as shown in Fig. 3.38, referred to as Type II.

The strain distribution obtained by the analysis as well as the strains measured in the tests are shown in Fig. 3.18. The analysis shows that the strain in the middle portion of the slab-wall junction exceeds the cracking strain, which is calculated based upon the modulus of rupture, $7.5\sqrt{f_c'}$, 

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and the elastic modulus. The strain, however, is less than the cracking strain in the remaining portion of the panel. Once a part of the panel falls into the inelastic region, the stress is completely redistributed; therefore, elastic analysis is no longer valid. The strains measured in the tests, on the other hand, apparently exceed the cracking strain in the middle portions of the slab-wall junction (Section 1-1 in Fig. 3.18) and the column line parallel to the loading (section 3-3 in Fig. 3.18). Both the elastic finite element analysis and test results show evidence of the crack development under the design service vertical load.

As shown in Table 3.11, measured vertical deflections are greater than the vertical deflections obtained in the analysis. The difference between the analytical and measured deflections is appreciable particularly at points #1 and #3 which were directly above the beams (Fig. 3.14); the measured deflections are nearly twice the calculated values. The two cracks that developed in the panels probably caused additional deflection.

The difference between the analysis and test results could be attributed primarily to any non-linear and/or inelastic action of the panel, which was not considered in the analysis.
In order to further evaluate the stress and deflection of the panel subjected to the design service vertical load, various non-elastic behaviors such as cracking of concrete and yielding of concrete or steel should be incorporated into the analysis.

3.7.3 Strength

Test results show that the type of loading (monotonic vs. cyclic loading), the moment-to-shear ratio, and the intensity of vertical load all affect the ultimate strength of the slab panels. In all cases, the ultimate strength was reached immediately preceding the development of the major crack which extended parallel to the wall at a distance of about 350 mm (14 in.). This major crack extended along the boundary between the column and middle strips of the slab panel. Many reinforcing bars which function as either positive or negative reinforcement were terminated at this location. (Fig. 3.3)

The panels' resistance decreased after the formation of the major crack. Very few new cracks developed afterwards, while the overall deformation of the panel was controlled primarily by the opening and closing of the major crack. (The section at the major crack acted like a plastic hinge).
3.7.3.1 Effect of Type of Loading or Strength

The three pairs of test panels: BH2MN and BH1CY, BV1MN and BV2CY, and BH3MN and BH3CY, were compared to determine the effect of the type of loading on the ultimate strength.

Cyclic loading resulted in a significant decrease of the ultimate resistance approximately by 20 to 25 percent. The ratio of the ultimate load under cyclic loading to that under monotonic loading was 0.79 (BH1CY/BH2MN), 0.83 (BV2CY/BV1MN), and 0.73 (BH3CY/BH3MN) respectively, for the three pairs. Cumulative damages such as cracks were considered the most probable reason for such a reduction in the ultimate load. Under cyclic loading, the ultimate strength would depend upon the compressive strength of concrete which has been cracked previously. Even though the cracks are closed, the bearing may not be complete, and the effective strength may be reduced.

In monotonic loading tests, the ultimate load under negative loading was always lower than under positive loading. The ratios of the ultimate load under negative loading to that under positive loading were 0.74 (BH2MN), 0.88 (BV1MN), and 0.68 (BH3MN), respectively. This strength reduction was attributed to damages caused during the last stage of positive loading. By the time positive loading was completed, many
cracks had developed, and major cracks had extended almost to the opposite edges of the panels. Several reinforcing bars also had broken. These damages would logically have weakened the panel under negative loading.

In cyclic loading tests, on the other hand, the ultimate loads under positive and negative loadings were almost identical. The ratios of the ultimate load under positive loading to that under negative loading were 1.01 (BH1CY) 0.98 (BV2CY), and 0.97 (BH3CY). Here, damages were accumulated gradually by cyclic loading, and directional preference was minimal.

3.7.3.2 Effect of Vertical Load on Strength

The effect of vertical (out-of-plane) loading on the in-plane strength was studied by comparing the results of test panels BH2MN vs. BV1MN and BH1CY vs. BV2CY. As might be expected, the vertical load lowered the ultimate in-plane resistance. The ratio of the ultimate load with the vertical load to that without such load was 0.85 for positive loading and 0.98 for negative loading from the pair BV1MN/BH2MN and 0.89 for positive loading and 0.86 for negative loading from the pair BV2CY/BH1CY. The design service vertical load reduced the resistance by no more than 15 percent.
3.7.3.3 Effect of Moment-to-Shear Ratio on Strength

Two moment-to-shear ratios were used in the strength tests of various panels. For panels 1 and 2, the ratio was 1.63 m (64 in.), while for panel 3, the ratio was doubled. Two sets of tests can be compared to determine the effect of the moment-to-shear ratio on their strength: BH2MN and BH3MN, and BH1CY and BH3CY. Direct comparison showed strength ratios of 0.47 for positive loading and 0.44 for negative loading for panels tested monotonically (BH3MN/BH2MN) and 0.44 for positive loading and 0.42 for negative loading for panels under cyclic loading (BH3CY/BH1CY). The doubling of the moment-to-shear ratio is seen to reduce the resistance to about 40 to 45 percent. Interestingly, this ratio of 0.40 to 0.45, agrees closely to the ratio of distances of the major cracks from the applied in-plane load. As described in Section 3.7.1, the major crack developed along the boundary between the column and middle strips. The distance from the loading line to the major crack is 1220 mm (48 in.) for panels 1 and 2, and 2840 mm (112 in.) for panel 3. The ratio of the distances is 1220/2840 or 0.43, which is nearly the same as the experimental ultimate strength ratio, 0.40 to 0.45. This correspondence implies that the ultimate strength of the slab panel was controlled primarily by the flexural capacity at the major crack section. The transverse shear force had only a secondary effect.
3.7.4 Stiffness

3.7.4.1 Changes in Stiffness under Monotonic Loading

The tangent stiffness of BH2MN and BH3MN continuously decreased as load increased until the maximum resistance was reached. Several times in the post-ultimate load region, resistance decreased suddenly as each new crack opened. However, each time the load was recovered quickly, and, consequently, the level of resistance remained relatively stable until the breaking of reinforcing bars drastically reduced the resistance. The initial tangent stiffness, the tangent stiffness immediately before the ultimate loads, and the secant stiffness at the ultimate loads were 218 MN/m, 33.8 MN/m, and 42.9 MN/m for BH2MN and 166 MN/m, 22.2 MN/m, and 28.5 MN/m for BH3MN. The tangent stiffness at the ultimate load was 16 percent of the initial tangent stiffness for BH2MN and 13 percent for BH3MN. As evidenced in Fig. 3.24, the behavior of BV1MN is significantly different from that of BH2MN or BH3MN. The tangent stiffness was 222 MN/m initially and monotonically decreased to practically zero. The ultimate load was reached at a rather large displacement, and there was no sudden changes of resistance as displacement continued to increase with the formation of additional cracks. The vertical load apparently prompted the development of cracks. This process, in turn, resulted in a more rapid stiffness degradation for BV1MN than for BH2MN or BH3MN. When
cracking occurs, the force carried by concrete at the interface of the crack before cracking must be transferred to reinforcing bars. The early development of cracks in BV1MN apparently enabled a more gradual transfer of force from concrete to reinforcing bars. The gradual transfer of force not only prevented the sudden loss in resistance caused by cracking, but also retarded the attainment of the ultimate load.

Stiffness was substantially recovered during unloading. For BH2MN, the tangent stiffness at the beginning of unloading was 67.2 MN/m, approximately 31 percent of the initial tangent stiffness and nearly double the tangent stiffness before unloading. For BH3MN and BV1MN, the tangent stiffnesses at the instant of unloading were 33.7 MN/m and 83.3 MN/m, which were 21 and 37 percent of their respective initial tangent stiffnesses. As the load was applied in the negative direction, stiffness degraded until the load reached approximately 50 percent of the ultimate strength. Unlike the development during positive loading, the tangent stiffness remained nearly constant for loads above this level. The tangent stiffness in this region was 4.2 MN/m for BH2MN, 2.8 MN/m for BH3MN, and 3.2 MN/m for BV1MN, less than 2 percent of the initial tangent stiffness.
The load level in which the initial tangent stiffness was applicable did not exceed 36 kN (8 kip) for BH2MN, 10 kN (4.3 kip) for BH3MN, and 13 kN (3 kip) for BV1MN, which represent 13 to 20 percent of the ultimate resistance. Table 3.14 tabulates the stiffness values at various load stages.

3.7.4.2 Changes in Stiffness under Cyclic Loading

For all cyclic loading tests, BH1CY, BV2CY, and BH3CY, stiffness rapidly degraded during the cycles with two lowest amplitudes, 0.25 mm and 0.76 mm. At the end of the sixth cycle, the stiffness had reduced to 90 MN/m for BH1CY and BV2CY, and 50 MN/m for BH3CY. During the succeeding cycles, stiffness continued to decrease as amplitude increased. At the end of tests, the secant stiffness 1, as defined in Section 3.6.3, was 6.5 MN/m for BH1CY, 8.6 MN/m for BV2CY, and 3.9 MN/m for BH3CY. These values represent only 2 to 4 percent of the respective initial stiffnesses.

With few exceptions, stiffness remained the same for the three cycles at each amplitude. Occasionally, a significant crack developed, and stiffness decreased during the succeeding cycles. Damage like the breaking of reinforcing bars and the widening of cracks, on the other hand, caused continual degradation of stiffness in greater amplitudes (5.6 mm and 7.6 mm amplitudes).
As shown in Figs. 3.28 and 29, the two types of stiffness defined in Section 3.6.3, the secant stiffness 2 and 3, were almost identical for BH3CY and BV2CY, indicating that the stiffness degraded symmetrically in both positive and negative directions during cyclic loading. In addition, secant stiffness 1 also did not differ much from the other secant stiffnesses, indicating that the widths of hysteretic loops were narrow and that energy absorption in a complete cycle was small. Specimen BH1CY behaved quite differently; the three secant stiffnesses were all different. Secant stiffness 3 degraded more quickly than secant stiffness 2 and was about 40 percent of the secant stiffness 2 in the cycles of the five lowest amplitudes (0.25 mm, 0.76 mm, 1.3 mm, 2.8 mm, and 4.3 mm). The test of BH1CY also showed that cracks developed more often under negative loading than positive loading during these cycles. Secant stiffness 1 of BH1CY corresponded to the average of secant stiffnesses 2 and 3.

The difference between secant stiffness 1, 2, or 3 and tangent stiffness 1 or 2 was small for BV2CY, while for BH1CY and BH3CY, tangent stiffness 1 and 2 were relatively smaller than secant stiffness 1, 2, and 3 particularly during cycles of larger amplitudes. The difference resulted primarily from the pinching effect of hysteretic loops. The
difference was appreciable during the cycles of the 5.8 mm and 7.6 mm amplitudes for BHICY and the cycles of the 2.8 mm, 4.3 mm, and 5.8 mm amplitudes for BH3CY. During these cycles, tangent stiffness 1 or 2 was about 50 percent of secant stiffness 1, 2, or 3.

3.7.4.3 Effect of Vertical Load on Stiffness

In monotonic loading tests, the trends of stiffness degradation on BH2MN and BV1MN are substantially different as described in Section 3.7.4.1. In BV1MN, stiffness gradually degraded smoothly without sudden decreases of resistance, measuring near zero at the ultimate load. In BH2MN, on the other hand, stiffness was approximately 15 percent of the initial stiffness when the ultimate load was reached. Afterward, resistance fluctuated without significant decrease, while displacement increased many fold until the resistance was drastically reduced by the breaking of reinforcing bars.

In cyclic loading tests, the trends of stiffness degradation in BHICY and BV2CY are similar. In fact, values of secant stiffness 1 in these tests are close particularly for amplitudes 1.3 mm or greater. In these ranges, secant stiffness 1 gradually degraded from 50 MN/m to 10 MN/m as amplitudes increased and continued to degrade during each loading cycle of the 7.6 mm amplitude.
As listed in Table 3.13, the initial stiffness was 218 MN/m for BH2MN, 272 MN/m for BH1CY, 222 MN/m for BV1MN, and 201 MN/m for BV2CY. During initial loading stages, all conditions were identical for BH2MN and BH1CY and for BV1MN and BV2CY. The initial stiffness of BH2MN, however, was 80 percent of that of BH1CY, while the initial stiffnesses of BV1MN and BV2CY were nearly the same. The reason for BH2MN to have a 20 percent lower initial stiffness than BH1CY was not clear although imperfection or damage was strongly suspected for BH2MN at the beginning of the testing. Referring to the initial stiffness of BH1CY, 272 MN/m, as a datum, the initial stiffness was reduced by about 20 percent due to the design service vertical load.

3.7.4.4 Effect of Moment-to-Shear Ratio on Stiffness

As described in Section 3.7.4.3, the initial stiffness of BH2MN was suspiciously smaller than the initial stiffnesses of other tested slabs and, therefore, not appropriate for the use in meaningful comparisons. In monotonic loading, however, the tangent stiffnesses of BH3MN and BH2MN at all other stages: under positive loading, at the beginning of unloading, and under negative loading (except the initial stiffness), showed very consistent relationship. The stiffness of BH3MN ranged from 50 to 60 percent of that of BH2MN.
In cyclic loading, the relative stiffness ratio of BH1CY to BH3CY was 0.61 initially and ranged from 0.47 to 0.60 during the succeeding loading cycles, based on secant stiffness 1, 2, or 3. The ratio had a tendency to gradually decrease as amplitudes increased. The relative stiffness ratio, however, decreased about 0.25 to 0.30 in amplitudes equal to or greater than 4.3 mm (0.17 in.) if tangent stiffness 1 or 2 was substituted. In these amplitudes, the pinching effect was more appreciable for BH3CY than for BH1CY as shown in Fig. 3.23.

3.7.5 Behavior in Post-Ultimate Region

3.7.5.1 Definition of Deformability and Ductility

Table 3.15 facilitates the discussion of deformability and ductility of tested slab panels. The first two columns list the displacements, labelled critical displacements, at which the resistance of panels decreased substantially. The first and second columns respectively list the critical displacements under positive and negative loadings. As described in Section 3.6.3, critical displacements are much greater than the displacements corresponding to the maximum resistance and immediately followed by the breaking of reinforcing bars.

Ductility, listed in the third and fourth columns, is defined as the ratio of the critical displacement to the displacement at which 90 percent of the ultimate resistance was first reached. The load-deflection curves of all tested panels except
BV1MN had some plastic plateaus. The curve of BV1MN did not have a plastic plateau in either direction as shown in Fig. 3.24, indicating that stiffness gradually degraded as load increased. Ductility was not calculated for BV1MN since the test was terminated before any reinforcing bars broke. The table's final column lists the deflections, labelled total deflections, which equal the sums of the positive and negative critical displacements.

3.7.5.2 Monotonic Loading vs. Cyclic Loading

The comparison between BH2MN and BH1CY shows that total deflection and ductility respectively are 10 percent and 45 percent larger for monotonic loading. The ductility of the two panels differed primarily because the deflection at which 90 percent of the ultimate resistance reached for the first time was larger for BH1CY than for BH2MN. The comparison of BH3MN and BH3CY shows the same tendency. BV1MN, on the other hand, deformed more than BV2CY by about 60 percent.

3.7.5.3 Effect of Vertical Load

Ductility was nearly the same for BH1CY (without vertical load) and BV2CY (with vertical load), while BH1CY deformed about 15 percent more than BV2CY. A comparison between BH2MN (without vertical load) and BV1MN (with vertical load), on the other hand, shows that BV1MN deformed 25 percent more than BH2MN.
Note, however, that BV1MN deformed more than any other tested panels. In general, the design service vertical load affected ductility slightly.

3.7.5.4 Effect of Moment-to-Shear Ratio

The slab panels with a moment-to-shear ratio of 3.25 m (128 in.) (BH3MN and BH3CY) were 1.2 times more ductile than the slab panels with a moment-to-shear ratio of 1.63 m (64 in.). Apparently, ductility was improved for panels in which bending moment (flexural deformation) was dominant. Total deflection, on the other hand, is less in BH3MN and BH3CY than in BH2MN and BH1CY by about 15 percent.

3.8 Summary and Remarks

The experimental work has provided information about the in-plane characteristics of the slab panels under various supporting and loading conditions. Elastic analysis by means of the finite element method has provided some verifications and comparisons. The results of these comparisons are summarized below.

1. The elastic finite element analysis predicted a slightly higher stiffness of the 3-panel test specimens under symmetrical and anti-symmetrical in-plane loads. (Section 3.7.1) The analysis
produced stiffnesses 10 to 20 percent higher than the experiments. Minute residual stresses due to shrinkage or accidental loads prior to the testing are believed to be responsible for these discrepancies.

2. Two cracks, one along the slab-wall junction and the other along the column line parallel to the wall, developed on the top surface of the panel when subjected to the design service vertical load. (Section 3.6.2) Plate analysis confirmed the occurrence of these cracks. (Section 3.7.2)

3. The development of a crack, labelled a major crack, which extended parallel to the wall at a distance of about 350 mm, controlled the ultimate in-plane resistance of the test slab panel. (Section 3.6.3) This location of the major crack coincided with the boundary between the column and middle strips of the slab panel. The ultimate load was reached when the major crack extended completely across the panel. Afterward, the opening and closing of the major crack controlled the deformation. The flexural capacity of the section through which the major crack extended primarily governed the ultimate resistance. The magnitude of shear affected the ultimate resistance very slightly. (Section 3.7.3.3)
4. Cyclic loading, specified by the spectrum shown in Fig. 3.17, reduced the ultimate resistance of the panels by 20 to 25 percent. (Section 3.7.3.1) Cumulative damages like cracks most likely caused this reduction.

5. The application of the design service vertical load reduced the ultimate in-plane resistance by not more than 15 percent. (Section 3.7.3.2) The vertical load, however, did not alter the general behavior of the slabs; the major crack still developed along the boundary between the column and middle strips, and the complete formation of the major crack governed the ultimate resistance.

6. In panels under monotonic in-plane loading, tangent stiffness gradually degraded until the ultimate resistance was reached. (Section 3.7.4.1) The tangent stiffness immediately before this occurred was 10 to 15 percent of the initial tangent stiffness. After reaching the ultimate resistance, resistance decreased several times due to the development of cracks. The load, however, was recovered and the level of resistance remained relatively constant until the breaking of reinforcing bars drastically reduced the resistance. (Figs. 3.20 and 3.22)
7. In panels under monotonic in-plane loading, the design service vertical load significantly affected the pattern of stiffness degradation. (Section 3.7.4.3) Tangent stiffness degraded gradually, measuring near zero at the ultimate load. The vertical load apparently enhanced the development of cracks, and cracking in low load levels made the transfer of forces from concrete to reinforcing bars more gradual. (Fig. 3.24)

8. For cyclic loading, stiffness continually degraded as amplitude increased. During each cycle having a small to medium amplitude, stiffness remained constant. During cycles with a larger amplitude, in which some reinforcing bars broke, stiffness continuously degraded as the load was reversed. (Section 3.7.4.2)

9. Three kinds of stiffness, secant stiffness 1, 2, and 3, defined in Section 3.6.3 differed little, indicating that stiffness changed only slightly in a complete cycle. (Section 3.7.4.2) In the panels with no vertical load, the pinching effect was appreciable during the cycles with large amplitudes but confined in relatively small regions (in the vicinity of neutral displacement). In the panels with vertical load, on the other hand, the pinching effect was not evident. (Section 3.7.3.4)
10. Regardless of the loading condition, the definition of stiffness, or the level of deflection, the increase of the moment-to-shear ratio from 1.63 m (64 in.) to 3.25 m (128 in.) reduced the in-plane stiffness by approximately 40 to 50 percent.

11. Compared with monotonic loading, cyclic loading reduced the range of deformation (total deflection defined in Section 3.7.5) by about 10 percent and the range of ductility by about 45 percent. (Section 3.7.5.2) The design service vertical load reduced neither total deflection nor ductility. (Section 3.7.5.4)

12. The panels with a moment-to-shear ratio of 3.25 m (128 in.) were 1.2 times more ductile than the slabs with a moment-to-shear ratio of 1.63 m (64 in.). Total deflection, on the other hand, was approximately 15 percent less in the panels with the larger moment-to-shear. (Section 3.7.5.3)
IV ANALYTICAL STUDY

4.1 General

This chapter deals with the development of an analytical model, which can predict the behavior of concrete floor slabs beyond the linear elastic range. The experimental study described in Chapter III has provided valuable information on the behavior of floor slabs under various loadings. Because of the limited number of tests, however, experimental data were insufficient for the development of appropriate design procedures. The analytical study was needed to supplement the experimental findings.

It is difficult to obtain closed-form solutions for the behavior of reinforced concrete members because of the non-linear inelastic material behavior as well as the composite behavior of the two components. It is necessary to simulate the changing of concrete modulus, the developing of cracks, and the yielding of reinforcing bars. The finite element method was used to develop the model.

The characteristics and formulation of the proposed model are described in Sections 4.4 and 4.5. In Section 4.6, the model is applied to various example problems, and the accuracy of this model is evaluated by comparing the analytical solutions with the experimental result and other available data.
4.2 Review of Application of the Finite Element Method to Problems of Reinforced Concrete Structures

The finite element method has been applied to the study of reinforced concrete structures ever since this method was devised. Many studies have dealt with beams, frames, walls, plates, and shells. Scordelis, (4.24) Darwin, (2.14) and Ngo (4.21) have reviewed extensively the literatures describing the application of this method.

4.2.1 Application to Two-Dimensional Problems, (I) Crack-Line Approach

Ngo and Scordelis (4.20) were the first to demonstrate the application of the finite element method to reinforced concrete beams. They modelled concrete and steel as linear elastic materials connected by linear elastic 'bond links'. Cracks were predefined as the separation of nodal points. Nilson (4.19) extended this method to include non-linear material properties and a non-linear bond-slip relationship. He used an incremental load procedure to analyze reinforced concrete members subjected to concentric or eccentric loads. Whenever cracking was indicated, the tracing of the crack development was performed manually. The finite element mesh layout was modified accordingly, and the analysis was started again from zero load and incrementally loaded to the next stage of failure. Later, Ngo (4.21) extended this method to automatically produce crack-lines to simulate progressing crack growth. He developed an finite element program which was capable of automatically generating nodal points in one continuous execution.
Kawai et al.\(^{(4.12, 4.13, 4.15)}\) proposed another method to analyze reinforced concrete members. Their model represented a concrete plate as an assembly of triangular elements interconnected by normal and shear springs. Each triangular finite element was assumed to be rigid, and therefore displacements at any point in this element were expressed by the displacements and rotations at its centroid. The coefficients of springs at the interfaces were determined to reflect the material properties of the concrete plates. The development of cracks was then simulated by reducing the spring constants at cracked interfaces. The concept of this model is somewhat equivalent to the yield line theory. These researchers executed nonlinear incremental analyses of plane concrete members subjected to direct tension.\(^{(4.14)}\)

4.2.2 Application to Two-Dimensional Problems, (II) Crack-Zone Approach

Cervenka and Gerstle\(^{(1.6, 2.10, 4.7)}\) developed the idea of employing a finite element with composite concrete-steel material properties at uncracked, cracked, and plastified stages. They carried out the non-linear incremental analysis of reinforced concrete panels under both monotonic and cyclic loadings. The program included the closing and re-opening of cracks within the finite element. They treated the concrete as an elasto-plastic material in compression and as an elastic brittle material in tension. Once a crack was opened, stiffness perpendicular to the
crack and shear stiffness parallel to the crack were set at zero. The program had a capacity of automatically altering these material properties in one continuous execution. The newly defined orthotropic material could take stress parallel to the crack only. The same general approach was used by later studies. Valliappan and Doolan, \(^{(4.26)}\) and Suidan and Schnobrich \(^{(4.25)}\) applied this method to cantilever beams. Yuzugullu and Schnobrich \(^{(2.48)}\) used Cervenka's and Gerstle's procedure for shearwall problems. Yuzugullu and Schnobrich pointed out that better results were obtained if a finite value was employed for shear stiffness parallel to the open cracks instead of zero. Salem \(^{(4.23)}\) also applied this method to solve shear panel and beam problems. He incorporated the hardening rule into the formulation of the concrete constitutive relationship. Adham et al. \(^{(4.1)}\) included bond slips and the dowel action in this model.

The studies already mentioned produced workable results for monotonic loading. The investigations which attempted to determine the behavior of reinforced concrete members under cyclic loading, \(^{(2.10, 4.7)}\) however, met with indifferent results. While the experimental results showed a continuous loss of stiffness and strength with each cycle of load, the analytical model did not show such degradation. Darwin and Pecknold \(^{(2.14, 4.9)}\) developed a new material model to represent the hysteretic behavior of concrete under biaxial loading. The constitutive law of concrete derived form this material model was expressed in terms of the stress-increment and the strain-increment depending
upon the current stress state as well as the previous history of concrete. Darwin and Pecknold combined this constitutive law with the procedures developed by Cervenka (2.10) and solved several shear panel problems under cyclic loading. Aktan (4.2) later used this constitutive law to investigate shearwall behavior under cyclic loading.

4.2.3 Application to Plate Problems

While aforementioned studies deal with reinforced concrete beams and panels subjected to in-plane loadings, the finite element method has also been used to analyze the reinforced concrete plates under out-of-plane loadings. Jofriet and McNeice (4.11) combined the finite element analysis with the concept of the effective moment of inertia, originally offered by Branson (4.6) and Beeby (4.5) in order to analyze reinforced concrete slabs subjected to out-of-plane loadings. In their analysis, the effective moment of inertia calculated the stiffness of cracked plate elements because setting the flexural stiffness for the cracked elements at zero usually leads to underestimates of the flexural rigidity. Hand et al., (4.10) on the other hand, solved reinforced concrete plate and shell problems by using layered finite elements. In their analysis, each plate or shell element was divided into several layers through the thickness, and each subdivided element was treated as a two dimensional plane stress element. Kirchhoff-Love's hypothesis was employed to satisfy
the compatibility of these subdivided elements. Like Suidan's
and Schnobrich's(4.25) and Yuzugullu's and Schnobrich's(2.48)
studies, their analysis also adopted non-zero shear stiffness for
cracked elements. The results of their analysis showed that the
magnitude of the shear stiffness in cracked elements was not
crucial for the overall behavior as long as a non-zero value was
used. Bashur and Darwin(4.4) also used the layered finite
elements for their study, combining these elements with the
material model proposed by Darwin and Pecknold(4.9).

4.3 Modelling of Material Properties

4.3.1 Inelastic Behavior of Material

Concrete Many of the previous studies(2.10, 2.14, 4.1,
4.24,4.26) modelled concrete as a linear-elastic perfectly-
plastic material and applied the Von Mises yield criterion and
the associated flow rule to develop the elasto-plastic consti-
tutive equation formulated by Zienkiewicz et al.(4.29)
Salem(4.23) modified that model, introducing the hardening
rule first developed by Ziegler.(4.28) Kupfer and Gerstle(4.17)
and Liu,(4.18) proposed constitutive models for monotonic biaxial
loading of plane concrete based upon their experimental investi-
gations. Darwin and Pecknold(4.9) formulated a constitutive
equation of concrete subjected to biaxial cyclic stresses. Aktan (4.3) and Bashur and Darwin (4.4) later used that model for their studies.

**Steel** Many studies treated reinforcing bars as uniaxially loaded members, idealizing the stress-strain relationship of those bars to be linear-elastic perfectly-plastic (2.10, 2.14, 4.4, 4.23, 4.25, 4.26) or bilinear (2.48, 4.1, 4.3). Peterson and Kostem (4.22) on the other hand, used the Ramberg-Osgood material model for representing the stress-strain relationship of reinforcing bars. Aktan et al. (4.3) studied the cyclic behavior of reinforcing bars, concluding that the Ramberg-Osgood or linear-elastic perfectly-plastic models satisfactorily duplicates the experimental hysteretic stress-strain curves.

**Bond Slip** Some studies investigating the behavior of bond slips (4.19, 4.20, 4.21) introduced special link elements representing the bond slip between concrete and reinforcing bars. The bond slip, however, was neglected in most research studies of the general behavior of structural elements.

**Cracks** Two types of models for the crack development were used in the previous studies. One was the crack-zone model, in which cracking was assumed to take place within a finite region of the structure. (1.6, 2.10, 4.1, 4.2, 4.7, 4.9, 4.10, 4.25, 4.26, 4.44) Cracks in a finite element modified the element
stiffness matrix to reflect the loss of tensile concrete strength in the direction perpendicular to the cracks. The second model represented cracks as lines between two nodes. *(4.12, 4.13, 4.14, 4.15, 4.19, 4.20, 4.21)* Whenever cracks developed, the interface tractions along the cracked lines were released.

4.3.2 Related Experimental Findings

The experimental study described in Chapter III has offered specific information about the post-elastic behavior of the floor slabs, which was used to select appropriate models. The post-elastic behavior of the slab panels was controlled primarily by the development of cracks and the yielding of reinforcing bars. The region in which concrete was compressed beyond its elastic limit was small. The strength of the slab panel was controlled by the flexural capacity of the critical section into which the major crack penetrated, while the shear force had little effect.

The dowel action of reinforcing bars also had very little influence on the ultimate capacity of the slab panels since the diameters of reinforcing bars were small (not more than 4.6 mm), and the slab panels were lightly reinforced. The experimental investigation did not assess the effect of bond slips on the strength or ductility. The development length and surface condition of reinforcing bars primarily control the
performance of bond between the bars and concrete. Even though all reinforcing bars in the slab panel were deformed, many of them were cut off. In addition, the points of termination were selected according to the out-of-plane bending moment rather than the in-plane bending moment applied to the slab panel. In this regard, the bond performance in the slab panel under in-plane loadings was not clearly understood.

The opening and closing of the major cracks controlled the behavior in post-elastic regions; consequently, few cracks were developed in those regions. The ductility of the floor slabs depended upon the deformability of the reinforcing bars at the critical sections. The breaking of bars at those sections brought about the ultimate failure of the slab panels.

4.4 Proposed Model

Based on the findings cited above, it was decided to use the model developed by Kawai et al. (4.16) because this model easily simulates the development of cracks, which has been proven to be of primary importance to the post-elastic behavior. This decision, however, is irrelevant to the argument that the crack line model could be more accurate than the crack zone model or vice versa. In both models, the accuracy of solution depends a great deal on the size of finite elements and can only be checked by numerical experimentation and existing data such as experimental results. The method developed by Ngo (4.21) also is appealing be-
cause of its capacity to directly trace the development of cracks. This method, however, was not used because the stiffness matrix tends to be sparsely populated as additional nodes are generated in the process of computation. The algorithms of automatically generating nodal points and solving a sparse matrix are extremely complicated. The proposed model does not take either the dowel action or bond-slip into account.

The analyzed floor slabs consist of three different components: the concrete slab, the reinforcing bar, and the concrete beam. The concrete plate is represented as an assemblage of rigid triangles interconnected by normal and shear springs, as proposed originally by Kawai et al. Each triangle with springs is labelled the triangular element in this study. The proposed program has two options for representing the reinforcing bars in slabs. In the first option, each bar is treated as a bar element, labelled the truss element, which resists axial forces but has no flexural rigidity. In the second option, all reinforcing bars in the slab are smeared together and treated as an orthotropic plate. This orthotropic plate is then divided into rigid triangles with springs in the same manner as for the concrete panels. These triangle elements are referred to as "smeared triangular elements". The spring constants are determined so that they can reflect the orthotropic properties of the reinforcing bars. Concrete beams running underneath the concrete plate are modelled as truss elements resisting axial forces only. The total neglection of the flexural resistance
of the beams was justified by the following reasons. First, the experimental investigations have shown that reinforcing bars at the four corners of each beam are strained almost equally at the several gaged cross sections, indicating that the axial deformation of the beam is dominating. Second, elastic finite element analyses were performed to investigate the effect of the bending of the beams on the overall behavior. One of the available finite element programs \( (3.1) \) was used for this purpose. Truss members with no flexural rigidity represented the beams in one execution, whereas beam elements with appropriate flexural rigidity were used in the other execution. The two executions yielded practically the same results. Hence, the flexural rigidity of these beams is significantly smaller than the in-plane rigidity of the concrete plate.

4.4.1 Formulation of Triangular Elements

In Fig. 4.1 are shown two rigid triangular elements with nodes 1-2-4 (element 1) and 3-2-4 (element 2). The elements are connected along their interface (24) by uniformly distributed normal and shear springs, with spring constants \( k_d \) and \( k_s \) respectively. (see Fig. 4.4) \( G_1 \) and \( G_2 \) are the centroids of the elements. Since the elements are assumed to be rigid, the displacements at any point in an element are characterized by the
translations \((u, v)\) of the centroid, and the rotation \((\Theta)\) of the element (Fig. 4.2). Point \(W\), located at the interface in the two elements before displacement, will be moved to separate locations \(W_1'\) and \(W_2'\) after displacement (Fig. 4.3). These displacements, \(<U_1, V_1, U_2, V_2>\) can be related with the displacements at both centroids \(<u_1, v_1, \Theta_1, u_2, v_2, \Theta_2>\) as follows:

\[
\begin{bmatrix}
U_1 \\
V_1 \\
U_2 \\
V_2
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & (y - y_{g1}) \\
0 & 1 & (x_{g1} - x) \\
1 & 0 & (y - y_{g2}) \\
0 & 1 & (x_{g2} - x)
\end{bmatrix}
\begin{bmatrix}
u_1 \\
v_1 \\
u_2 \\
v_2
\end{bmatrix}
\]

or

\[
\{U\} = [Q] \{U_1\}
\]

Subscripts 1 and 2 indicate that the displacements belong to the triangular element 1 with nodes 1, 2, and 4 and the triangular element 2 with nodes 2, 3, and 4. As shown in Fig. 4.3, the
separation of $W_1'$ and $W_2'$ after displacement, measured in the normal and tangential directions of the interface, are:

$$\begin{bmatrix}
\delta_d \\
\delta_s
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\bar{U}_1 \\
\bar{V}_1 \\
\bar{U}_2 \\
\bar{V}_2
\end{bmatrix}
$$

or

$$\{\delta\} = [M] \{\bar{U}\}$$

Here, $\delta_d$ and $\delta_s$ are measured with respect to the local coordinates $(\bar{X}, \bar{Y})$ as shown in Fig. 4.3. The displacement components $<\bar{U}_1, \bar{V}_1, \bar{U}_2, \bar{V}_2>$ also are measured with respect to the local coordinates. A transformation matrix $[R]$ relates the displacement vector in the local coordinates $\{\bar{U}\}$ to the displacement vector in the global coordinates $\{U\}$:

$$\begin{bmatrix}
\bar{U}_1 \\
\bar{V}_1 \\
\bar{U}_2 \\
\bar{V}_2
\end{bmatrix} =
\begin{bmatrix}
n_1 & m_1 & 0 & 0 \\
0 & n_1 & m_1 & 0 \\
n_2 & m_2 & 0 & 0 \\
0 & n_2 & m_2 & 0
\end{bmatrix}
\begin{bmatrix}
U_1 \\
V_1 \\
U_2 \\
V_2
\end{bmatrix}
$$

(4-3)
or

\[ \{ \bar{U} \} = [R] \{ U \} \]

where:

\[ n_1 = \cos (\bar{x}, x) \]
\[ n_2 = \cos (\bar{x}, y) \]
\[ m_1 = \cos (\bar{y}, x) \]
\[ m_2 = \cos (\bar{y}, y) \]

According to equations, 4-1 to 4-3, the relative displacement vector \( \{ \delta \} \) is:

\[ \{ \delta \} = [M] [R] [Q] \{ U_1 \} = [B] \{ U_1 \} \quad (4-4) \]
\[ [B] = [M] [R] [Q] \]

The normal strain \( (\varepsilon_d) \) and the shear strain \( (\varepsilon_s) \) corresponding to the separation are defined as:

\[ \{ \varepsilon \} = \begin{bmatrix} \varepsilon_d \\ \varepsilon_s \end{bmatrix} = \frac{1}{h_1 + h_2} \begin{bmatrix} \delta_d \\ \delta_s \end{bmatrix} = \frac{1}{h} \times \{ \delta \} \quad (4-5) \]

\( h_1 \) and \( h_2 \) are the legs from the centroids of elements 1 and 2 to the interface, and \( h \) is the total distance between the centroids.

The constitutive law between the stress and the strain is defined as:

\[ \begin{bmatrix} \sigma_d \\ \tau_s \end{bmatrix} = \begin{bmatrix} k_d & 0 \\ 0 & k_s \end{bmatrix} \begin{bmatrix} \varepsilon_d \\ \varepsilon_s \end{bmatrix} \quad (4-6) \]
or

\[ \{\sigma\} = [D] \{\varepsilon\} \]

The total strain energy stored in the springs distributed along line (24), therefore, is:

\[ V_c = \frac{1}{2} \int_{A_c} \{\delta\}^T \begin{bmatrix} D \end{bmatrix} \{\varepsilon\} ds \]

\[ = \frac{1}{2} \{U_1\}^T \int_{A_c} \frac{1}{h} \begin{bmatrix} B \end{bmatrix}^T \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} B \end{bmatrix} ds \{U_1\} \quad (4-7) \]

Here, \( A_c \) is the area of the interface (24), and \( ds \) indicates an infinitesimal length. By applying Castigliano's theorem, the displacements at the centroids \( \{U_1\} \) can be related to two forces \( (P_x, P_y) \) and moment \( (M) \) at the centroids as follows:

\[
\begin{bmatrix}
P_{x1} \\
\vdots \\
P_{y1} \\
M_1 \\
P_{x2} \\
P_{y2} \\
M_2
\end{bmatrix} =
\begin{bmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33} \\
K_{41} & K_{42} & K_{43} \\
K_{51} & K_{52} & K_{53} \\
K_{61} & K_{62} & K_{63}
\end{bmatrix}
\begin{bmatrix}
\text{SYM.} \\
K_{44} \\
K_{54} & K_{55} \\
K_{64} & K_{65} & K_{66}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
v_1 \\
\theta_1 \\
u_2 \\
v_2 \\
\theta_2
\end{bmatrix} \quad (4-8)
\]
or.

\[ \{P\} = [K] \{U_i\} \]

Where: \([K]\) is a 6 x 6 element stiffness matrix. Although each component of \([K]\) matrix can be derived in a closed form, the three-point Gaussian integration is employed for the computation of the matrix. Once \(\{U_i\}\) is known, the stress vector \(\langle \sigma_d, \sigma_s \rangle\) is computed in the following form:

\[ \{\sigma\} = [D] \frac{t}{h} [B] \{U_i\} \]

(4-9)

The same procedure can be used to formulate the element stiffness matrix of smeared reinforcing bars. Fig. 4.5 shows two triangular elements representing reinforcing bars which are arranged orthogonally and inclined by \(\theta_x\) from the global coordinates. The axes of the local coordinates parallel to the reinforcing bars. Equations 4-1 to 4-4 are used to formulate this stiffness matrix. Equation 4-5, however, are modified to:

\[ \varepsilon_1 = \frac{\delta_1}{\ell_1} \]
\[ \varepsilon_2 = \frac{\delta_2}{\ell_2} \]

(4-5')

\(\ell_1\) and \(\ell_2\) respectively are the distances between the two centroids projected on the local \(\bar{x}\) and \(\bar{y}\) axes. The separation, \(\delta_1\) and \(\delta_2\), also are measured with respect to the local coordinates. The new stress-strain relationship is:
\[ \sigma_1 = E_{s1} \times \varepsilon_1 \]  
\[ \sigma_2 = E_{s2} \times \varepsilon_2 \]  

(4-6')

In the above expressions, \( E_{s1} \) and \( E_{s2} \) are the moduli of elasticity of steel in directions, 1 and 2. The total strain energy stored in the springs which are distributed along the interface is:

\[ V_s = \frac{1}{2} \sum_{i=1}^{2} (\delta_i \times E_{s1} \times \varepsilon_i) \, ds \]  

(4-7')

Where \( A_i \) is the area of reinforcing bars placed in the \( i \) direction and intersecting the interface. The element stiffness matrix is then obtained by applying Castilgano's theorem to equations 4-4, 4-5', 4-6' and 4-7'.

4.4.2 Formulation of Truss Elements

Fig. 4.6 shows a typical truss element connecting two rigid triangular elements. This truss element interconnects these triangular elements at their centroids rather than at their nodal points. The flexural rigidity of the truss element is not considered; that is, this element behaves as a uniaxial tension and compression member. Buckling under compressive force is not considered, either. The strain (\( \varepsilon_b \)) is:
\[
\varepsilon_b = \frac{1}{\ell} x < - \ell_3 - m_3 0 \left| \begin{array}{c}
\ell_3 \\
m_3
\end{array} \right|
\left\{ \begin{array}{c}
u_1 \\
v_2 \\
\theta_1 \\
\theta_2
\end{array} \right\} \quad (4-10)
\]

or
\[
\varepsilon_b = \frac{1}{\ell} [B_b] \{U_i\}
\]

where:
\[
\ell = \text{Length of the truss element}
\]
\[
n_3 = \cos \alpha = \frac{x_{g2} - x_{g1}}{\ell}
\]
\[
m_3 = \sin \alpha = \frac{y_{g2} - y_{g1}}{\ell}
\]

The strain \((\sigma_b)\) is:
\[
\sigma_b = E_b \times \varepsilon_b = E_b \times \frac{1}{\ell} [B_b] \{U_i\} \quad (4-11)
\]

\(E_b = \text{Axial stiffness of the truss member}\)

The strain energy stored in the truss element, therefore:
\[
V_b = \frac{1}{2} \int_0^\ell \sigma_b^T \times E_b \times \varepsilon_b \, d\ell
\]
\[
= \frac{A_b E_b}{2\ell} \times \{U\}^T [B_b]^T \{U_i\}
\]

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Here, $A_b$ is the cross sectional area of the truss member.

According to equation 4-12, the displacements $\{U_i\}$ and forces $\{P\}$ at the centroids can be related as follows:

$$\{P\} = \frac{A_b E_b}{\ell} \left[ B_b \right] \left[ B_b \right]^T \left[ U_i \right]$$

or

$$\{P\} = \left[ K_b \right] \left[ U_i \right]$$

$$\left[ K_b \right] = \frac{A_b E_b}{\ell} \left[ B_b \right] \left[ B_b \right]^T$$

4.4.3 Material Model of Concrete Slab in Compression

Concrete under compression is assumed to be isotropic and linear-elastic and perfectly-plastic. The constitutive matrix of the triangular elements representing concrete slab in the elastic range is defined as:

$$\left[ D_e \right] = \begin{bmatrix} D_{11}^e & 0 \\ 0 & D_{22}^e \end{bmatrix}$$

$$D_{11}^e = \frac{E}{1-\nu^2}$$

$$D_{22}^e = \frac{E}{2(1+\nu)}$$
E and ν respectively are the modulus of elasticity and Poisson's ratio of concrete. As long as the concrete is in the elastic range the spring constants $k_d$ and $k_s$ are respectively:

\[ k_d = \frac{D^{e}}{11} \]
\[ k_s = \frac{D^{e}}{22} \]  \hspace{1cm} (4-15)

The compressive yielding of concrete is assumed to follow a yield criterion with an associated flow rule. The yield function defined here is:

\[ f = \left( \frac{\sigma_d}{\sigma_p} \right)^2 + \left( \frac{\tau_s}{\tau_p} \right)^2 = 1 \]  \hspace{1cm} (4-16)

\[ \sigma_p = \sigma_y \]
\[ \tau_p = \sigma_y / \sqrt{3} \]

$\sigma_y$ is the yield stress of concrete under uniaxial compression. If $d\varepsilon_p$ denotes the increment of plastic strain, then:

\[ \{d\varepsilon_p\} = \lambda \left( \frac{\partial f}{\partial \sigma} \right) \]  \hspace{1cm} (4-17a)

or for each component 1 and 2:
\[d\varepsilon_p^d = \lambda \frac{\partial f}{\partial \sigma_d} \quad d\varepsilon_p^s = \lambda \frac{\partial f}{\partial \tau_s}\] (4-17b)

where:
\[d\varepsilon_p^p = \begin{bmatrix} d\varepsilon_d^p \\ d\varepsilon_s^p \end{bmatrix}\]

\(\lambda\) is a proportionally constant, as yet undetermined in these equations. During an infinitesimal increment of stress, changes of strain are assumed to be divisible into elastic and plastic parts:

\[\{d\varepsilon\} = \{d\varepsilon_e\} + \{d\varepsilon_p\}\] (4-18)

The elastic strain increments are related to stress increments by the elastic matrix \([D_e]\):

\[\{d\sigma\} = [D_e]\{d\varepsilon_e\}\] (4-19)

According to the yield criterion:

\[df = \frac{\partial f}{\partial \sigma_d} d\sigma_d + \frac{\partial f}{\partial \tau_s} d\tau_s = 0\] (4-20)

From equations 4-17 and 4-20, we can derive \(\lambda\) as:

\[\lambda = \frac{D_{11}^e \left(\frac{\partial f}{\partial \sigma_d}\right) d\sigma_d + D_{22}^e \left(\frac{\partial f}{\partial \tau_s}\right) d\tau_s}{D_{11}^e \left(\frac{\partial f}{\partial \sigma_d}\right)^2 + D_{22}^e \left(\frac{\partial f}{\partial \tau_s}\right)^2}\] (4-21)

The derivation of equation 4-21 is illustrated in Appendix.
According to equations 4-17 through 4-21, the elasto-plastic constitutive equation can be expressed in an incremental form as:

\[ \{d\sigma\} = [D_{ep}]\{d\varepsilon\} \]  

(4-22)

Where:

\[
[D_{ep}] = \begin{bmatrix}
D^{e}_{11} & 0 \\
0 & D^{e}_{22}
\end{bmatrix} - \frac{1}{S} \begin{bmatrix}
S^2 & S_1 S_2 \\
S_2 S_1 & S^2
\end{bmatrix}
\]

\[ S_1 = D^{e}_{11} \left( \frac{\partial f}{\partial \sigma_d} \right) \]
\[ S_2 = D^{e}_{22} \left( \frac{\partial f}{\partial \tau_s} \right) \]

As long as the concrete stress is in the yield condition (equation 4-16), equation 4-22 is used to relate the stress to the strain. Unloading is assumed to occur if \( \lambda \) is negative.

Strain hardenings such as kinematic and isoparametric hardenings are ignored, as evidenced from the yield function equation 4-15. Zienkiewitz et al. presented the formulation of the elasto-plastic matrix in further detail.

4.4.4 Material Model of Concrete Slab in Tension

Concrete is assumed to be linear elastic until cracks occur, and tension cracking follows the maximum normal stress theory. Once cracks take place, the normal spring constants \( k_d \)
is set at zero. The shear spring constants \( k_s \), on the other hand, hold a finite value in order to maintain numerical stability throughout the matrix algebra. Recontact of cracked surfaces under unloading or reloading is considered by introducing a criterion, \( \delta_d < 0 \). Once two cracked surfaces contact, both the normal and shear constants of the springs located in the interface are assumed to regain their elastic values. The spring constants maintain these values as long as the interface remains closed.

4.4.5 Material Model of Reinforcing Bars

The stress-strain relationship for the reinforcing bars is assumed to be linear-elastic perfectly-plastic in both tension and compression, taking neither strain hardening nor the Baushinger effect into account. These bars are assumed to break under tension when they reach their ultimate strains. The buckling of bars under compressive force is ignored.

4.4.6 Accuracy of the Proposed Model

As described in Sections 4.4.1 and 4.4.2, the proposed model is considerably different in its formulation of potential energy from most other finite element analyses. In this section, the accuracy of the model in the elastic range is demonstrated by comparing the model with one of the
Table 4.1 lists the ratios of the displacements at several selected points shown in Fig. 4.8. The deflection ratio of the proposed model to SAP IV, ranges from 1.01 to 1.07; the model always gives larger displacements than SAP IV. The discrepancy, however, appears to be minimal despite the difference in models' mesh size and shape and procedure for formulating potential energy. The proposed model is suitable for solving elastic plane stress problems.

4.5 Procedure of Analysis

4.5.1 Scheme of Computation

The basic input for the analysis procedure consists of a description of the topology and material properties of the structure. The loads are expressed as forces acting on the centroids of triangular elements. The material properties for concrete as well as reinforcing bars are specified for each element. The first step in the analysis is to form the structure stiffness matrix from the individual element stiffness matrices. Initially, the element stiffness matrices are determined based on the virgin material properties of the concrete and steel.

The structure is then analyzed under monotonically increasing loads. For each load level, the solution is derived through several iterations after specific convergence criteria are met. The structure is assumed to behave linearly within
existing elastic finite element programs, SAP IV. (3.1) For this purpose, a plane stress problem is solved by means of the two programs. The analyzed structure is a 64 (in.) x 64 (in.) square panel with the thickness of 1 (in.) as shown in Fig. 4.8. One of the edges is clamped, while the others are held free. The panel is homogeneous and isotropic with a modulus of elasticity of 3,500 (ksi) and a poisson's ratio of 0.15. The dimensions and material properties, similar to those of the tested concrete panels, however, do not matter since the analysis is made for comparison between two different finite element models. Two cases of loading are considered. In loading case 1, equally distributed forces are applied along the free edge opposite the clamped edge. In loading case 2, these forces are applied along all three free edges, resulting in a pure shear condition in the panel. The intensity of the forces is chosen arbitrarily. The panel is discretized to finite elements as shown in Fig. 4.9. It is known that the size of finite elements significantly affects the accuracy of solutions. The finite element sizes shown in the figure are selected after several executions with different element sizes. Although a square finite element in SAP IV is four times greater in area than a triangular element in the proposed model, this difference does not necessarily leads to a conclusion that the proposed model is more accurate than SAP IV; the formulation of the stiffness matrix, after all, is different.
an iteration. Following each iteration, the structure stiffness matrix is reconstructed using the tangent stiffness properties of the material, and forces within the structure are corrected to reflect the nonlinear behavior of the material model. The force correction procedure, which is explained below, uses the Initial Stress Method of Zienkiewicz et al. (4.29). Following the solution of the element equilibrium equations, the element displacements are used to obtain the strains within each element. The material strains then determine the apparent changes in stresses for the concrete and steel. These changes are corrected to reflect the nonlinear properties of the materials. The differences between the apparent stresses and corrected stresses are the residual stresses used to calculate residual element loads. With each iteration, the state of each material is updated, stresses are corrected, and a new tangent stress-strain matrix is calculated. The element and structure stiffness matrices are reconstructed, and the residual loads are applied until the solution for that load step converges. This force correction procedure is shown schematically in Fig. 4.7.

The analysis, on the other hand, is terminated when the structure reaches its maximum load. When the imposed load approaches the structure's maximum resistance level, the load
increment becomes too large to converge. If it takes place, the load increment at the present load step is reduced by one third, and the iteration is restarted. Once the load increment is lowered to within a specified fraction of the load reached in the previous load step, the execution then stops, assuming that this load be the ultimate load.

4.5.2 Solution Technique of the Nodal Equilibrium Equations

Wilson, et al. (4.27) have developed an efficient computer program for the direct solution of large numbers of simultaneous linear equations. Basically, the program used the Gauss elimination in order to solve positive-definite symmetrical systems. The specific features are that systems of very large size and bandwidth can be solved and that operations on zero elements are effectively eliminated to reduce the time for computation. This program is used as a subroutine in the analysis.

4.6 Numerical Examples

4.6.1 Selection of Examples

In order to evaluate the usefulness and applicability of the proposed finite element model, several numerical examples are presented in this section. For this purpose, the model is used to analyze the tested panels for both monotonic and cyclic loadings, and the results are compared with the test results: BH2MN, BH3MN, and BH1CY. To achieve close match between the analysis
and experiment, material properties used in the analysis are determined based upon the results of the concrete cylinder test and steel tension test. In the proposed model, concrete subjected to compression is assumed to be linear-elastic perfectly-plastic, while concrete subjected to tension is assumed to be linear up to a specified cracking stress. The yield strength and tensile strength obtained by the concrete cylinder test are used to represent the yield stress and cracking stress respectively. The yield stress and ultimate strain of reinforcing bars are assumed to be those derived from the steel tension test. (Table 3.7)

4.6.2 Monotonic Loading on Beam-Supported Slab Panels (BH2MN and BH3MN)

The ability of the proposed model to simulate the behavior of floor slabs under monotonic in-plane loading is demonstrated by using two tested slab panels (BH2MN and BH3MN). In order to save the time required for computation, the individual panel under test is analyzed. (Fig. 4.10) For simulating the behavior of BH2MN, a 410 mm (16 in.) overhang of the slab panel, outside of the load line, is ignored. The significance of the overhang is minimal because this neglect only disregards the effect of the self-equilibrium stresses acting on the panel section along the load line. A preliminary elastic analysis of BH2MN has also shown that this ignored part has
little effect on the overall behavior of the panel. The edge connected to the vertical wall (refer to Fig. 3.2) is clamped, and, therefore, the wall is not included in the analysis. The other three edges are assumed to be stress-free. The concrete panel is discretized to 216 triangular elements. (Fig. 4.11) Three concrete beams, two running perpendicular and one running parallel to the loading direction, are represented by truss elements. Since the truss elements are jointed at the centroids of triangular elements, the eccentricity between the neutral axes of slab and beams are neglected. Reinforcing bars are represented by truss elements in one execution (case 1) and by smeared triangular elements in another execution (case 2). In the case 1 analysis, all reinforcing bars located along the interface of two triangular elements are treated as one truss element for each direction (X and Y directions in Fig. 4.11). The applied load is assumed to be distributed uniformly along the loading line parallel to the clamped edge. The distributed load, therefore, is represented by nine equal element forces along the loading line. (Fig. 4.11) The proportion of the forces remains unchanged during the loading.

In order to simulate the behavior of BH3MN, the entire middle panel (1630 mm x 2440 mm) is analyzed (Fig. 4.10). The edge connected to the fixed vertical wall (refer to Fig. 3.2) is clamped, while the remaining edges are stress-free; therefore,
the walls also do not appear in the analysis. The concrete panel is discretized to 216 triangular elements like BH2MN. Three concrete beams are treated as truss elements, and reinforcing bars are treated as smeared triangular elements. In order to simulate the loading employed in BH3MN, both moment and shear force are applied to the free edge parallel to the clamped edge. The applied moment equals the shear force multiplied by a 1630 mm moment arm. The shear force is represented by nine equal element forces in the y direction, and the moment by nine linearly distributed element forces in the x direction. These directions follow those specified in Fig. 4.11. The element forces are increased proportionally in the analysis.

The results of the analyses are compared to the experimental load-displacement and load-rotation curves in Figs. 4.12 and 4.13. For BH2MN, analyses of cases 1 and 2 constantly show a higher load than the experiment. Once the load exceeds 100 kN (80 percent of the experimental ultimate load), however, the analytical curves approach the experimental curve rapidly and meet it at 120 kN, which is 95 percent of the experimental ultimate load. The analytical curves reach the ultimate loads at 116 kN (case 1) and 115 kN (case 2), which are 97 and 96 percent of the experimental ultimate load, respectively. The
experimental curve shows a sudden decrease of resistance at the displacement of 2.8 mm, and after the decrease takes place, the resistance does not reach its maximum again. Since the analytical model is loaded with a series of imposed load increments, it is not possible to deal with structural unloading, and the curves rise continuously until the ultimate load, defined in Section 4.5.1, is reached. According to the case 1 analysis, in which the reinforcing bars are treated as truss elements, the displacement corresponding to the ultimate load is 3.0 mm, while 2.8 mm is obtained from the experimental curve. In the second analysis (case 2), in which the smeared triangular elements are used for the reinforcing bars, the transfer of stresses caused by the cracking of concrete is more gradual than in the case of the real slab panel, which contained discrete reinforcing bars. The smooth transfer of forces appears to retard the onset of instability. The case 2 analytical curve, however, reaches 99 percent of its ultimate load at 2.6 mm and travels on additional 0.9 mm with a load increase of 1 percent. The region between 2.6 mm and 3.5 mm can be said as a plastic plateau. Except for the displacement of the ultimate load, the load-displacement curves in cases 1 and 2 practically are the same. In later analyses, the smeared triangular elements are used to represent reinforcing bars, for the use of these elements reduces the band-widths of global stiffness matrices and improves the numerical stability particularly near the ultimate load.
For BH3MN, the experimental and analytical load-displacement curves show agreement up to about 30 kN, which is 60 percent of the ultimate load. At this load level, the slope of the experimental curve decreases suddenly, while the analytical curve shows a smooth reduction of the slope up to the ultimate load. The analytical ultimate load is 51.1 kN, which is 93 percent of the experimental ultimate load. The analytical curve reaches 99 percent of the maximum load at 2.0 mm displacement, followed by a 1.6 mm plastic plateau. On the other hand, the experimental curve reaches the ultimate load of 56.9 kN at 1.9 mm displacement, and a sudden decrease of resistance, which is never fully recovered as the panel undergoes an additional 4.9 mm of displacement. In both BH2MN and BH3MN, the experimental and analytical load-end rotation curves show agreement up to about 90 percent of the ultimate loads (Fig. 4.13). At this load level, the analytical curves depart from the experimental curves, showing a rapid decrease of the slopes. The characteristics of the experimental curves, the loss and recovery of resistance in the post-ultimate load regions are replaced by the smooth transition of the slopes in the analytical curves.

The ability of the proposed model to duplicate experimental crack patterns is demonstrated in Fig. 4.14. The experimental and analytical crack patterns are compared at three different load levels for both BH2MN and BH3MN. Since the pro-
posed model assumes that cracks occur at interfaces between two triangular elements, cracks can develop in only one of the three directions: the direction parallel to the loading, the direction perpendicular to the loading, and the direction inclining by 45 degrees from the loading direction. Considering this limitation on the crack direction, the model simulation of the crack development is good.

In general, the proposed model successfully simulates the overall behavior of the slab panels. The model offers a close duplication of the ultimate resistance as well as the deformation corresponding to the ultimate resistance. The model also duplicates the deformation of most tested panels; it did not, however, duplicate the displacement of BH2MN. For BH2MN, the analytical load-displacement curve always shows higher load than the experimental curve. Several reasons for this discrepancy can be noted. Material properties used in the model, based upon the concrete cylinder tests and steel tension tests (Tables 3.7 and 3.8), may not be the same as those of the tested slab panels. The tested slab panels might have initial minute cracks because of the shrinkage of concrete or the accidental forces applied during the process of setting the specimen. Significant residual stress due to creep or shrinkage might exist since the slab panels were tested more than two months after they were cast. The analyses, on the other hand,
do not consider any of such possibilities. Additional experiments are required to discuss more specifically the reasons for this discrepancy.

The analytical curves do not reach the experimental failure displacements: 7.6 mm for BH2MN and 6.9 mm for BH3MN. The reason is clear; the experimental curves show that the resistance is never completely recovered after the first significant loss of resistance. That is, the region after the point corresponding to the ultimate resistance is a post-ultimate load region. On the other hand, the analytical model can simulate the behavior only up to the ultimate load. The experimental results verify that the breaking of the reinforcing bars embedded in the concrete beam subjected to tension causes the ultimate failure. To evaluate the breaking of reinforcing bars in cracked regions, however, is difficult since the strains of these bars depend heavily upon various local effects such as bond slips and dowel action. The model, on the other hand, has been developed to simulate the overall behavior rather than to investigate the local behavior of individual components.

Nevertheless, the failure displacements are evaluated by using the stiffness of the structure as well as the stress and strain of concrete and reinforcing bars at the ultimate load. Assuming that the stiffness at the ultimate load remains unchanged in the post-ultimate load region, the displacement
at which the strain of the reinforcing bars in the concrete beam located 400 mm from the tension edge reaches the ultimate strain can be computed. In other words, the model is assumed to be linear, having the same stiffness as that of the ultimate load. This assumption is rather reasonable because the experimental results show that few cracks take place once the ultimate load is reached and that the behavior in the post-ultimate load region is controlled primarily by the opening of the major crack. The computed failure displacements are 6.65 mm for BH2MN and 6.60 mm for BH3MN as shown in Fig. 4.12. They are 88 and 93 percent the experimental failure displacements respectively. The correlation seems reasonable for the considerably crude assumption.

4.6.3 Cyclic Loading on Beam-Supported Slab Panels (BH2MN and BH1CY)

The ability of the proposed model to simulate the behavior of floor slabs under reversed or cyclic loading tested with two tested slab panels (BH2MN and BH1CY). For BH2MN, the tested panel was unloaded after it reached failure and reloaded in the opposite direction (defined as the negative direction in Chapter III) until another failure occurred. The analytical model follows this loading pattern; first it is unloaded after the ultimate load is reached and then reloaded in the opposite direction until another ultimate load is reached. For BH1CY, the panel was subjected to a series of load
reversals in both directions with gradually increasing displace-
ment specified by the loading spectrum in Fig. 3.17. Since
enormous computation time is required to completely duplicate
the loading spectrum, a simplified loading spectrum is selected
for the analytical model. It contains one complete cycle with
the 1.3 mm amplitude, followed by one cycle with the 2.8 mm
amplitude. In the third cycle, the load continues to increase
until failure occurs. The discretization of the slab panel is
identical to that used for BH2MN subjected to monotonic loading.
Smeared triangular elements are used for reinforcing bars in this
analysis.

The analytical load-displacement curve of BH2MN under
the reversed loading is compared with the experimental curve
in Fig. 4.15. Since the analytical curve does not reach the
experimental failure displacement in the initial loading as
mentioned in Section 4.6.3, these two curves are far apart on
the unloading. In the reloading, the analytical curve con-
sistently shows a higher slope than the experimental curve
and reaches the ultimate load at -4.0 mm displacement as com-
pared with the -6.0 mm experimental displacement. The curve also
shows a clear pinching effect in the neighborhood of zero dis-
placement. Since the ultimate displacement which the analytical
model can reach is substantially less than the experimental failure
displacement, the model fails to simulate the experimental curve
in the reloading region. In addition, the analytical curve
reaches the same ultimate resistance in both directions (115 kN for positive loading and 110 kN for negative loading), although the experimental curve shows a substantial decrease in ultimate resistance due to the reversed loading by about 26 percent. The analytical ultimate load in the negative direction is 110 kN, which is 96 percent of the analytical positive ultimate load and 125 percent of the experimental negative ultimate load. The damage that the tested slab panel (BH2MN) received during the first half cycle of loading is not simulated properly by the analytical model. This unsatisfactory simulation explains the discrepancy on the ultimate loads.

In Fig. 4.16, the analytical crack pattern at the negative ultimate load is compared to the experimental crack pattern at the point of failure in the negative direction. Despite the discrepancy between the analytical and experimental curves, the two crack patterns are considerably alike.

The analysis for cyclic loading of test panel BH1CY resulted in a load-displacement curve unlike the experimental curve. The analytical ultimate load after two complete cycles is 107 kN, which is 112 percent of the experimental ultimate load of BH1CY. As shown in Fig. 4.17, the analytical curve consistently shows a higher load than the experimental curve. The difference is probably caused because the analytical model does not
accurately evaluate the damage accumulated in the slab during the cyclic loading. The tested slab panel experienced six complete cycles immediately preceding the first cycle of the 1.3 mm amplitude. In the analysis, on the other hand, load up to this displacement level is applied from the virgin state. Furthermore, the tested panel reached the ultimate load at 4.3 mm displacement after twelve cycles were completed, while in the analysis, only two complete cycles are employed before the ultimate load. It is suspected that the model does not properly represent the causes for cumulative damage. In cyclic loading with increasing amplitudes, cracks develop gradually from both sides. The cracks open and close alternately as the load is reversed. The stiffness of the panel changes at each opening and closure of the cracks. It is also known that load reversal commonly enhances bond slips. The proposed model does not simulate these potential sources of cumulative damage.

4.6.4 Concluding Remarks

In summary, the proposed model properly simulates the behavior of slab panels subjected to in-plane loading when they are loaded monotonically. The model accurately predicts both the ultimate resistance and the corresponding displacement. The analytical load-deformation curves reasonably coincide with the experimental curves. The analytical curves do not reach the
the experimental failure displacements since these displacements are in post-ultimate load regions. Nevertheless, an evaluation of the failure displacement based upon the stiffness at the ultimate load is still workable.

On the other hand, the ability of the model to simulate the cyclic behavior of floor slabs is not satisfactory. For BH2MN, the model does not duplicate the damage that the tested slab panel possessed upon unloading and, consequently, fails to simulate either the weakened stiffness or the reduced strength which the tested panel disclosed during the reversed loading. The model fails to simulate the behavior of the floor slab under cyclic loading with gradually increasing amplitudes (BH1CY). The deficiency of the model to simulate cumulative damages due to cyclic loading most likely creates the discrepancy.
V PARAMETRIC STUDY

5.1 General

In this chapter, the experimental findings described in Chapter III are further examined in order to define the parameters which control the in-plane behavior of floor slabs and evaluate the effects of these parameters on the in-plane characteristics of floor slabs. This study focuses on three major characteristics: strength, stiffness, and ductility. Parameters which may affect these characteristics are chosen carefully, and the significance of each parameter is examined. The selected parameters are: 1) the aspect ratio of floor slabs, 2) the relative beam size, 3) the amount of reinforcing steel, 4) the arrangement of reinforcing bars, 5) the boundary condition, 6) the loading condition (monotonic, reversed, or cyclic loading), and 7) the intensity of gravity load.

This Chapter also describes a simplified representation of the floor slab subjected to in-plane loading as a deep beam considering both flexural and shear deformations. The appropriateness and accuracy of this representation are discussed in detail.
5.2 **Elastic Characteristics**

The stiffness test and accompanying elastic finite element analysis, described in Chapter III, have shown that the in-plane stiffness characteristics of floor slabs in the elastic region can be reasonably estimated by the finite element analysis. When the analysis of the entire structure is needed, however, dividing each slab panel to smaller elements is impractical. A more practical representation of the floor slabs, in which they are treated as deep beams, is demonstrated in this section. The effect of several parameters such as the relative beam size or the aspect ratio on the stiffness characteristics is discussed.

5.2.1 **Comparison with Stiffness Test**

In order to simulate the stiffness test by a simplified beam model, the tested specimen, composed of three panels separated by two walls (refer to Fig. 3.2), is treated as a simply supported beam with a projection at each end, labelled an equivalent beam. As shown in Fig. 5.1, two forces are applied to the equivalent beam at the distance ($\ell$) from the supports. The forces are applied in the same direction to simulate the symmetrical loading test and in opposite directions to simulate the anti-symmetrical loading test. Fig. 5.1 also illustrates the resulting moment and shear diagrams created by these forces. Both bending and shear de-
formations are included to calculate the deflection of the equivalent beam. The following equations, 5-1 to 5-4, determine the rotations \((\Theta_s, \Theta_a)\) at the supports and the deflection \((\delta_s, \delta_a)\) at the edges of the beam.

For symmetrical loading:

\[
\Theta_s = \frac{\ell^2}{2EI} \times P \tag{5-1}
\]

\[
\delta_s = \left( \frac{5\ell^3}{6EI} + \frac{\ell}{AG} \right) \times P + \frac{\ell^2}{EI} \times (\ell_1 - \ell) \times P \tag{5-2}
\]

For anti-symmetrical loading:

\[
\Theta_a = \left( \frac{\ell^2}{6EI} + \frac{2}{AG} \right) \times P \tag{5-3}
\]

\[
\delta_a = \left( \frac{\ell^3}{2EI} + \frac{3\ell}{2AG} \right) \times P + \left( \frac{2\ell^2}{3EI} + \frac{2}{AG} \right) \times (\ell_1 - \ell) \times P \tag{5-4}
\]

where: \(P\) = applied force

\(E\) = modulus of elasticity of the equivalent beam

\(G\) = shear modulus of the equivalent beam

\(I\) = moment of inertia of the equivalent beam

\(A\) = shear area of the equivalent beam

\(\ell\) = span length (1630 mm)

\(\ell_1\) = length of the projection (2030 mm)
In the application of equations 5-1 to 5-4, the material properties, E and G are taken as those of concrete obtained from the cylinder test. (Table 3.8) The effect of reinforcement as well as the stiffening beams are ignored. The moment of inertia is taken to be that of the gross section of the panel, including the two beams running parallel to the span (or perpendicular to the loading) as shown in Fig. 5.5. The nominal shear area is taken to be 0.833 times the area of the same cross section, 0.833 being the theoretical coefficient for the shear area of a rectangular cross section.

The results based upon these properties are listed in Table 3.14. For symmetrical loading, the equivalent beam calculation gives an approximately 20 percent higher stiffness than the experiment but closely approximates the finite element analysis. For anti-symmetrical loading, on the other hand, the equivalent beam calculation gives an 40 percent higher stiffness than the experiment or the finite element analysis. Note that the finite element analysis correlates well with the stiffness test for anti-symmetrical loading. Further investigation revealed that the combined effect of the high shear force applied in the middle panel and the boundary condition employed for the stiffness test and finite element analysis caused additional rotations in the walls. Since the tested panels were supported only at walls' mid-points, at a distance of 1220 mm (48 in.) from the
panel edges perpendicular to the loading, the shear force applied to the walls forced them to either contract or elongate in the direction parallel to the shear force. The equivalent beam calculation, however, did not account for such deformation due to the shear force. Further finite element analysis verified that this additional rotation of the walls due to the shear force disappeared when the walls were prevented from deforming in the direction parallel to the shear force. In this finite element analysis, artificially stiff beams were substituted for the walls to restrict the deformation along their lines. As indicated by Table 3.14, the stiffnesses of the slab specimen were increased significantly, and the results were nearly the same as the equivalent beam calculation. In most building structures in which two adjoined floor slabs are separated by shearwalls or frames, either elongation or contraction of the boundary between the two slabs is unlikely to occur. The equivalent beam calculation, therefore, seems to give reasonable evaluation in most cases.

In general, the technique which treats floor slabs as deep beams is acceptable to simulate the in-plane stiffness characteristics of floor slabs. First, the equivalent beam calculation reasonably duplicates the finite element results. Second, the finite element results indicate good
correlation with the stiffness test results. Chapter III describes the comparison between the stiffness test and finite element analysis in detail.

5.2.2 Effective Moment of Inertia and Effective Shear Area

In the previous section, the suitability of the equivalent beam calculation to evaluate the in-plane stiffness characteristics of floor slabs is demonstrated. In order to properly evaluate the in-plane stiffness characteristics, however, the moment of inertia (I) and the shear area (A) of equivalent deep beams should be selected carefully. Although an equivalent beam calculation using the nominal moment of inertia and shear area reasonably simulates the results of the stiffness test and finite element analysis, a floor slab with a different combination of aspect ratio and relative beam size may require a different set of moment of inertia and shear area.

A parametric study was made to determine the moment of inertia and shear area of equivalent beams so that the equivalent beam calculation would properly reflect the true in-plane characteristics of floor slabs. The moment of inertia and shear area determined by this parametric study are labelled the effective moment of inertia (I_e) and the effective shear area (A_e) respectively. Elastic finite element analysis was made
to provide the data necessary for computation. In the previous section, this analysis has been verified to be capable of properly simulating the true behavior of slab panels under in-plane loadings. As in the finite element analyses described in Chapters III and IV, eccentricities between the mid-planes of floor slabs and beams were neglected; therefore, the floor slabs with beams were analyzed as two-dimensional plane stress problems. Cantilever floor slabs were analyzed instead of three panel floor system analyzed in the previous section. This treatment is reasonable because the finite element analysis also simulated adequately the behavior of the specimens' outer panels in the stiffness test. The aspect ratio and the relative beam size were selected as parameters. The relative beam size was defined as the ratio of the nominal moment of inertia of a floor slab with beams to the nominal moment of inertia of the floor slab without such beams. The relative beam size of 1.0, then, indicated a flat slab. The width of floor slabs, or likewise, the depth of the equivalent beams, designated as d in Fig. 5.2, was defined as the distance between the center lines of the two beams. The beams were placed at the edges of the slab panels. This parametric study did not include floor slabs with projecting parts. The dimensions and material properties of the analyzed floor slabs are listed in Table 5.1.
The following procedure was adopted to compute the effective moment of inertia \( I_e \) and shear area \( A_e \):

1. A cantilever floor slab with a particular set of aspect ratio and relative beam size was selected. (Fig. 5.2)

2. A unit rotation \( \theta \) without allowing any translation at the free edge was applied and the shear force \( P \) and the end moment \( M \) at the edge were computed.

3. \( I_e \) and \( A_e \) were calculated from the following equations:

\[
I_e = \frac{l}{2E} (2M - P\ell) \quad \text{(5-5)}
\]

\[
A_e = \frac{3P(2M - P\ell)}{G(3M - 2P\ell)}
\]

These equations can be derived by means of the basic beam theory which takes both flexural and shear deformations into account.

The values used in this parametric study ranged from 0.25 to 3.0 for the aspect ratio and from 1.0 to 2.7 for the relative beam size. The calculated \( I_e \) and \( A_e \) are listed in Table 5.2.

According to Table 5.2, the values of \( I_e \) are very close to the nominal moment of inertia regardless of the aspect ratio or the relative beam size. On the other hand, both the aspect
ratio and relative beam size affect the effective shear area; a smaller aspect ratio or a larger relative beam size gives a larger effective shear area. This change in the effective shear area is consistent with the generally accepted concept of the shear area. For the flat slab (the relative beam size is 1.0) with 3.0 of the aspect ratio, the ratio of the effective shear area to the cross sectional area 0.872, is very close to the standard 0.833 value for a beam with a rectangular cross section. For the floor slab with 0.25 of the aspect ratio and 2.7 of the relative beam size, the effective shear area nearly is the same as the cross sectional area of the floor slab (not including the area of the beams). The shear area of cross sections with stiff edges like wide-flange cross sections, is the area of the part between the edges.

For practical design purposes, nevertheless, the cross sectional area of floor slabs reasonably represents the shear area. The area of beams, if any, need not to be added to the area of the floor slab. This simplification creates less than a 12 percent error in the aspect ratio and relative beam size covered in this study. Since the stiffness of equivalent beams are computed on the basis of the sum of flexural and shear deformations and thereto the computation based upon the nominal moment of inertia properly provides flexural deformation, the total

-128-
error brought about by considering the cross sectional area of floor slabs as the shear area can be further reduced. Although the analysis was made only for the slabs with edge beams, supplemental analysis disclosed that the use of nominal moment of inertia and shear area is also suitable to evaluate the stiffness of slabs in which beams are attached inside the slab edges (like the tested slab panels).

5.3 Strength of Floor Slabs

5.3.1 Description of Problems

The proposed finite element model described in Chapter IV was used to predict the strength of eight slab panels with a variety of loading conditions, boundary conditions, relative beam sizes, and reinforcing bar arrangements. These analyzed panels are designated as cases 1 to 8 in the following discussion.

All eight cases dealt with a slab panel 1630 mm x 2440 mm in plane dimensions with one of the long edges clamped and the other three edges free. Three beams of 96 mm x 68 mm cross section were attached to the slab panel. Two beams were placed symmetrically in the short direction, 1630 mm apart (405 mm from the edges). A beam was placed along the loading line. These dimensions followed those of the tested slab panels. Cases 1 and 2 respectively were the original tested floor slab, BH2MN and BH3MN. In BH2MN, the basic slab panel was
analyzed with distributed shear forces along the free edge. In BH3MN, the slab panel was analyzed with a combined shear force and bending moment along the free edge. In case 3, the slab panel was subjected to bending moment at the free edge. Unlike cases 1 and 2, the slab panel was under pure bending, and no shear force existed. Case 4 was similar to case 1 except that the loaded edge was prevented from rotating. This boundary condition simulates floor slabs supported by vertical members whose torsional rigidity is very large. The cases 1 and 4, therefore, marked the upper and lower bounds of the strength of floor slabs supported by vertical members with finite torsional rigidity. In case 5, uniformly distributed forces were applied along the three free edges so that the floor slabs would be subjected to pure shear. Case 6 had the same boundary and loading conditions as case 1 (BH2MN), but the area of longitudinal reinforcing bars, which extended perpendicular to the loading, was doubled while the area of transverse reinforcing bars remained unchanged. The slab's flexural strength was expected to increase. Case 7 also had the boundary and loading conditions of case 1. Additional reinforcing bars, however, were placed near the floor slab's edges so that its flexural strength could be increased. In case 8, the dimensions of the concrete beams were doubled, while their locations remained unchanged. A detailed explanation about the
analysis procedures of these slab panels has already been made in Chapter IV. In addition, Fig. 5.3 schematically shows the conditions employed to analyze each of the eight floor slabs.

5.3.2 **Ultimate Strength and Failure Mode**

In cases 1, 2, 3, and 8, cracks developed on the finite element boundary parallel to the loading and at a distance of 270 mm (10.5 in.) from the clamped edge, and the ultimate loads were reached when the cracks nearly penetrated the floor slabs (Fig. 5.4). Most reinforcing bars intersecting the cracks yielded at the ultimate loads. These cracks and cross sections corresponded to the major cracks and the critical sections defined in Chapter III. These failures were then labelled flexural failures.

In cases 4 to 7, on the other hand, the slab panel developed diagonal crack which completely separated a triangular portion from the remainder of the slab at the failure point. All reinforcing bars crossing the diagonal crack also yielded. This failure was designated as shear failure. It was felt that this failure would be inhibited in a slab supported by a shearwall or a strong beam attached along the loading edge because the shearwall or the beam would function as a link between the two parts (Fig. 5.4). Even after a diagonal crack completely penetrated the floor slab, the two parts can remain connected at the loading edge. To test this speculation, three additional analyses were
made, designated cases 4', 6', and 7'. These were identical
to cases 4, 6, and 7, respectively, except that a stiffer and
stronger beam was added along the loading edge. (100 times
stiffer than the normal beam) (Fig. 5.4) Table 5.3 also lists
the strength of these floor slabs. The shear failure was
completely prevented because the beam was stronger and stiffer
and the ultimate resistance of these floor slabs was controlled
by flexural failure. The major crack locations at ultimate
load levels coincided with those of cases 1, 2, 3, and 8
(Fig. 5.4).

5.3.3 Evaluation of Flexural Capacity

Since the flexural capacity of cross sections in which
major cracks developed (critical sections) seemed to control the
flexural failure of floor slabs, an attempt was made to evaluate
the ultimate resistance of floor slabs on the basis of the
theoretically computed flexural capacity of the critical sections.
Assumptions and procedures used to calculate the flexural capa-
city of critical sections are described below.

1. The Navier-Bernoulli hypothesis, that a plane section
remains plane after deformation, was adopted to specify
the strain distribution in the critical section.
2. The stress-strain relationship of reinforcing steel was assumed to be linear-elastic perfectly-plastic. The yield stress and modulus of elasticity of the steel were taken as those derived from the tension tests (Table 3.8).

3. The stress-strain relationship of concrete in compression was expressed by a cubic equation. Four constants were selected from the report published by Rao,\(^{(5.1)}\) in which the values were determined by the curve-fitting of stress-strain curves originally proposed by Kriz and Lee.\(^{(5.2)}\) The original curves were not used because the stress therein was not expressed as an explicit function of strain and, therefore, not suitable for trial-and-error computation. The cubic equation to represent the stress-strain relationship of 27.6 MPa (4000 psi) concrete is:

\[
\begin{align*}
f_c &= 31.5 \varepsilon - 11.0 \varepsilon^2 + 0.980 \varepsilon^3 \\
f_c &= \text{stress of concrete (MPa)} \\
\varepsilon &= \text{strain of concrete (m/m)}
\end{align*}
\]

Many equations defining the stress-strain relationship of concrete under uniaxial compression and bending are available in both explicit and implicit forms.\(^{(5.3)}\) No
An attempt was made to calibrate the propriety of these equations on this flexural capacity calculation. Nevertheless, the discrepancy which could be caused by using different equations appears to be minimal. For example, the computation by using a linear stress-strain relationship instead of equation 5-5 gave only a 15 percent discrepancy.

4. The tensile strength of concrete was completely neglected. Consequently, concrete did not carry any stress as long as the strain was in tension.

5. Strain was specified at the cross section's outermost fiber in tension, and a neutral axis which satisfied the equilibrium of axial force on the cross section (resulting axial force of zero) was located by trial-and-error. The corresponding moment then was computed from the stresses in concrete and reinforcing steel. The procedure was repeated by alternating the level of strain at the outermost fiber in tension. Finally, the maximum moment obtained was taken as the flexural capacity of the cross section. Figure 5.5 schematically shows the procedure used to compute the moment with a specified strain.
As listed in Table 5.3, the ultimate strength computed from the flexural capacity of critical sections reasonably approximates the ultimate strength computed from finite element analyses. In case 4', the finite element analysis gave a 20 percent higher strength. This difference may have occurred because the slab panel in case 4' was divided into elements too large to guarantee the accuracy of the analysis. Moment gradient in case 4', in which the loading edge was prevented from rotating, was twice as large as in case 1. The length from zero to maximum moment locations was divided into three sections in case 4', whereas the length in case 1 was divided into six sections. (Fig. 4.11) Case 1 then may have reflected the moment gradient more reasonably than case 4'. Except for the slight discrepancy in case 4', however, the strength derived from the finite element analysis agrees with the strength calculated according to the flexural capacity of critical sections. The comparison between cases 1 and 8 indicates that the larger concrete beams increased the elastic in-plane stiffness and the load corresponding to the onset of the post-elastic behavior of the slab panel; however, they did not affect the ultimate resistance. Once the major crack penetrated the concrete beam subjected to tension, the beam no longer contributed to the flexural resistance of the critical section.
5.3.4 Evaluation of Shear Capacity

Cases 4, 5, 6, and 7, exhibited a shear mode of failure. In each of the four cases, the amount as well as the arrangement of transverse reinforcing bars, equivalent to stirrups in beams, were the same. The shear capacities, however, differed. Table 5.3 shows that the shear capacity increases as the moment-to-shear ratio becomes greater. This increase may be caused by the flexural compressive stresses in concrete which tend to retard the onset of development of the diagonal crack. The ratio of compressive force to shear force increased in slab panels with large moment-to-shear ratios; such compressive force did not exist in case 5'.

Since all reinforcing bars intersecting the diagonal crack yielded when failure occurred, the shear force \( V_s \) resisted by the reinforcing bars could be evaluated by the standard ACI Code formula (11.5.6.2):

\[
V_s = \frac{A_v \times f_y \times d}{s}
\]

- \( A_v \) = area of shear reinforcement within a distance \( s \), or area of shear reinforcement perpendicular to flexural tension reinforcement within a distance \( s \) for deep flexural members
- \( s \) = spacing of shear or torsional reinforcement in direction parallel to longitudinal reinforcement
distance from extreme compression fiber to centroid of longitudinal tension reinforcement but need not be less than 0.80h for pre-stressed members

\[ f_y = \text{specified yield strength of non-prestressed reinforcement} \]

According to this formula, the shear force \( (V_s) \) in cases 4, 5, 6, and 7 equals 122 kN, which closely corresponds to the maximum shear resistance in case 4. The ACI code also presents a formula (11.3.1.1) for the shear force carried by cracked concrete \( (V_c) \):

\[
V_c = 0.17 \times \sqrt{f'_c} \times b_w \times d
\]

- \( b_w \): web width
- \( f'_c \): specified compressive strength of concrete (MPa)

This formula calculates 67 kN of \( V_c \). On the other hand, the results derived from cases 4, 5, 6, and 7 show that \( V_c \) does not exceed 33 kN, less than 50 percent of the value suggested by the ACI code. The formula (11.3.1.1), therefore, is not suitable to evaluate the effect of cracked concrete on shear resistance in the slab panels covered by this study.
A comparison between cases 4, 6, and 7 and cases 4', 6' and 7' stipulates another criterion for the shear capacity of floor slabs. If floor slabs are integrated with shearwalls or strong beams along their boundaries (Fig. 5.4), the shear failure, in which a triangular portion is separated from the remainder of the floor slab did not occur. As a result, the flexural capacity controls the ultimate resistance of the floor slabs. In floor slabs whose aspect ratio is more than 1.0, however, the shear failure could still take place even when the slabs are supported by shearwalls. As shown in Fig. 5.4, full penetration of diagonal cracks can occur in the floor slabs without being affected by the supporting shearwalls. In these slabs, therefore, the shear capacity controls the ultimate resistance.

5.4 Evaluation of Stiffness

5.4.1 Elastic Limit

As described in Chapter III, the in-plane stiffness of slab panels significantly degraded as loads increased. The elastic in-plane stiffness of floor slabs defined in the Section 5.2, therefore, is not applicable if loads exceed a certain limit.

First, the range within which the elastic stiffness can be applied was evaluated. The experimental results reported in Section 3.6.3 show that the development of cracks along
slab-wall junctions significantly reduces the stiffness of the tested panels for the first time. The load levels at which the cracks developed were 36 kN (8.0 kip) for BH2MN and 19 kN (4.3 kip) for BH3MN, 30 and 34 percent of the ultimate load, respectively. The load which would cause cracking along the slab-wall junction was calculated based upon the following assumptions:

1. Strain was distributed linearly along the junction. (Navier-Bernoulli's hypothesis)
2. Stress-strain relationship was linear.
3. Cracking occurred when the strain exceeded a cracking strain.
4. Reinforcing bars were not included in the calculation of moment at the junction.

The modulus of elasticity (E) and the split cylinder tensile strength ($\sigma_t$) were used to compute cracking strain ($\varepsilon_t$). Thus calculated, the cracking strain was 0.00010. The calculated cracking load was 64 kN (15 kip) for BH2MN and 32 kN (7.3 kip) for BH3MN, which are substantially higher than the experimental cracking loads.

An elastic finite element analysis was carried out to further investigate the strain distribution along the wall-slab junction. Assumptions and procedures for the analysis
followed those described in Section 3.7.1. The analyzed slab referred to as Type II was discretized as shown in Fig. 3.38. Fig. 5.6 illustrates the analytical strain distribution along the slab-wall junction together with the strain distribution derived from Navier-Bernulli's hypothesis. This diagram indicates that the analytical strain is 1.5 times larger at the slab edges than the strain derived from this hypothesis. If the analytical strain is adopted, the cracking load is 43 kN (9.7 kip) for BH2MN and 21 kN (4.8 kip) for BH3MN, 120 and 110 percent of the experimental cracking load, respectively. Minute cracks and residual stress which may have existed in the test panels before testing are likely to have reduced the cracking load by about 15 percent.

In summary, the cracking along the slab-wall junction most probably gives the upper limit of the elastic behavior. The strain distribution, however, is highly non-linear; consequently, linear strain distribution significantly underestimates the strain at the slab edges (by about 40 percent for the tested slab panels).
5.4.2 **Stiffness in Post-Elastic Regions**

The evaluation of the in-plane stiffness of floor slabs with various loading conditions as well as previous histories is complicated particularly in post-elastic regions. Here, the technique of treating floor slabs as deep beams for the stiffness estimation is extended to post-elastic regions. Seven slab panels, each of which had a particular set of geometrical, loading and boundary conditions, were selected to investigate their in-plane stiffnesses. The seven slab panels were: 1) the tested slab panel, BH2MN, 2) case 1 of the finite element analysis (Section 5.3.1), 3) the tested slab panel, BH3MN, 4) case 2 of the finite element analysis (Section 5.3.1), 5) case 3 of the finite element analysis (Section 5.3.1), 6) case 6 of the finite element analysis (Section 5.3.1), and 7) case 4 of the finite element analysis (Section 5.3.1).

In order to evaluate the elastic in-plane stiffness of floor slabs, as described in Section 5.3, the slabs were represented by deep beams that had particular combinations of effective moment of inertia and shear area, and the elastic material properties $E$ and $G$. To extend this procedure to post-elastic regions, two hypothetical properties, named equivalent flexural stiffness $(EI)_e$ and equivalent shear stiffness $(AG)_e$.
were introduced. The in-plane stiffness of floor slabs in post-elastic regions then was evaluated by combining the basic beam theory with these hypothetical stiffnesses. The calculation of these stiffnesses followed the procedure explained below:

1. $(EI)_e$ and $(AG)_e$ of the investigated floor slab are assumed to be constant throughout the length, from the clamped edge to the loading edge. Either $(EI)_e$ or $(AG)_e$ varies obviously depending upon the bending moment and shear force applied to the cross section. The calculated $(EI)_e$ and $(AG)_e$, therefore, represent the averaged stiffnesses of the floor slab rather than the actual properties.

2. According to the loading condition imposed on the floor slab, load-deformation relationships are defined as:

$$\Theta = \frac{pl^2}{2(EI)_e} \quad \left\{ \begin{array}{l}
\delta_f = \frac{pl^3}{3(EI)_e} \quad \text{for BH2MN, case 1, and case 6} \\
\delta_s = \frac{pl}{(AG)_e}
\end{array} \right. \quad (5-6)$$

(5-7)
\[ \Theta = \frac{3P\ell^2}{2(EI)_e} \]  \hspace{1cm} (5-9)

\[ \delta_f = \frac{5P\ell^3}{6(EI)_e} \]  for BH3MN and case 2  \hspace{1cm} (5-10)

\[ \delta_s = \frac{P\ell}{(AG)_e} \]  \hspace{1cm} (5-11)

\[ \Theta = \frac{M\ell}{(EI)_e} \]  \hspace{1cm} (5-12)

\[ \delta_f = \frac{M\ell^2}{2(EI)_e} \]  for case 3  \hspace{1cm} (5-13)

\[ \delta_s = 0 \]  \hspace{1cm} (5-14)

\[ \Theta = \frac{P\ell^2}{8(EI)_e} \]  \hspace{1cm} (5-15)

\[ \delta_f = \frac{P\ell^3}{12(EI)_e} \]  for case 4  \hspace{1cm} (5-16)

\[ \delta_s = \frac{P\ell}{(AG)_e} \]  \hspace{1cm} (5-17)

\[ \delta = \delta_f + \delta_s \]  \hspace{1cm} (5-18)
The equations 5-6 through 5-18 equal those derived from the basic elastic beam theory except that equivalent flexural and shear stiffnesses substitute for the elastic flexural and shear stiffnesses. Fig. 5.7 shows the loading and boundary conditions of the seven investigated slab panels.

3. \( \Theta \) and \( \delta \) are measured at various load levels, and at each load level, a pair of \( (EI)_e \) and \( (AG)_e \) are computed by substituting the measured \( \Theta \), \( \delta \), and \( P \) or \( M \) in the proper equations. Since total deflections and total load rather than deflection increments and load incre-
ment are used to compute the equivalent flexural and shear stiffnesses, these stiffnesses are associated with the secant stiffness of the floor slab at specified load levels.

In Figs. 5.8 and 5.9, the computed (EI)_e's and (AG)_e's are plotted against the largest bending moments, which were applied at the clamped edges. The (EI)_e's and (AG)_e's, and the bending moments are non-dimensionalized respectively by the initial (EI)'s and (AG)'s, and the measured ultimate moments. The curves of BH2MN and case 1 or the curves of BH3MN and case 2 match closely in the figures, demonstrating the ability of the proposed finite element model described in Chapter IV to simulate the in-plane behavior of slab panels in post-elastic regions. Figs. 5.8 and 5.9 show rapid decrease in equivalent flexural and shear stiffnesses once elastic limits are exceeded. The stiffnesses continuously decrease as the moment increase. With the exception of case 6, investigated slab panels have stiffnesses that are about 30 percent of their initial stiffnesses when the moments reach about 40 percent of the ultimate moments. At the ultimate stages, both equivalent stiffnesses range from 10 to 13 percent of the initial elastic stiffnesses. In case 6, on the other hand, the equivalent stiffnesses degrade less than in the other slabs. They are reduced to about 30 percent of the initial
stiffness when the moment reaches about 70 percent of the ultimate moment, and to approximately 15 percent of the initial stiffnesses at the ultimate moment.

Fig. 5.10 shows the composition of the total deflections in the flexural and shear components for the investigated slab panels. It is seen that the compositions remain relatively constant regardless of load level; in BH2MN and cases 1 and 6, the shear proportion ranges from 0.5 to 0.7. In BH3MN and case 2, it ranges from 0.35 to 0.45, and in case 4, from 0.8 to 0.9. On the other hand, the proportions are 0.62 in BH2MN and cases 1 and 6, 0.39 in BH3MN and case 2, and 0.87 in case 4 in the elastic range. These values are calculated from the equations, 5-6 to 5-17, combined with the elastic material properties and effective moment of inertia and shear area defined in Section 5.2. These values are shown by dotted lines in Fig. 5.10. It is interesting to note that the proportions remain unchanged from elastic to inelastic ranges. In Fig. 5.11, the same compositions are plotted against total displacement for BH2MN and BH3MN, showing the trend in the post-ultimate load ranges. As pointed out previously, behavior beyond the ultimate load point was not obtained by the finite element analysis. The proportions essentially remain unchanged even in the post-ultimate load ranges.
Two important characteristics of the equivalent flexural and shear stiffnesses can be inferred from this study. First, the observation that the proportions remain constant indicates that the equivalent flexural stiffness degrades in the same proportion as the equivalent shear stiffness. That is, at a given load, the identical coefficient to represent the stiffness reduction, referred to as the reduction factor ($\alpha$), can be used to evaluate both stiffnesses. Second, the observation that the proportion in the post-elastic region is the same as that in the elastic region makes it convenient to use the basic elastic beam theory considering both flexural and shear deformations. The total deformation of floor slabs subjected to in-plane loads in post-elastic regions, then, can be calculated by means of the beam theory together with the degradation factor $\alpha$.

Although the discussion so far has been devoted to evaluating the secant stiffness of floor slabs, the tangent stiffness of floor slabs can also be evaluated by using the same procedures; the proportions of flexural and shear deformations to total deformation in the elastic range do not change in post-elastic regions.
5.4.3 Degradation Factor $\alpha$

The discussion in Section 5.4.2 has verified that the in-plane stiffness of floor slabs in post-elastic regions can be approximated by using the fundamental beam theory which includes both flexural and shear deformations together with the stiffness degradation factor $\alpha$. The evaluation of $\alpha$ is developed by examining of the flexural stiffness in post-elastic regions. As shown in Fig. 5.8, the flexural stiffness degradation curves of all investigated slab panels except case 6 are confined in a narrow band. This situation suggests that the moment gradient in the panel has little effect on the degradation of the equivalent flexural stiffness. (Fig. 5.7) The parameter which differentiated case 6 from the others was the amount of reinforcement; the amount of longitudinal (flexural) reinforcement in this panel was twice the amount in the other panels. On the other hand, the amount of flexural reinforcement is known to control the flexural stiffness of cracked cross sections. The observation that the flexural reinforcement controls both the equivalent flexural stiffness in the post-elastic regions and the flexural stiffness at cracked cross sections infers the possibility to evaluate the equivalent stiffness of floor slabs in post-elastic regions based upon the flexural characteristics at their cracked (critical) sections. A simple method to compute the equivalent flexural stiffness of floor slabs in post-elastic regions is proposed below:
According to Fig. 5.12, the end rotation of the slab panel in the post-elastic region is the sum of the elastic rotation due to bending and the hinge rotation at the critical section. The end rotation is:

\[ \Theta = \Theta_f + \phi \times \ell_p \]  \hspace{1cm} (5-19)

\( \Theta \) = total end rotation
\( \Theta_f \) = rotation due to elastic bending
\( \phi \) = curvature at the critical section
\( \ell_p \) = length of the hinge

\[ \Theta_f = \frac{P\ell^2}{2(EI)}_e, \text{ elastic} \]

for BH2MN and cases 1 and 6

\[ \Theta_f = \frac{3P\ell^2}{2(EI)}_e, \text{ elastic} \]

for BH3MN and case 2

\[ \Theta_f = \frac{M\ell}{(EI)}_e, \text{ elastic} \]

for case 5

Although the hinge introduced at the critical section is hypothetical but represents the sum of non-elastic rotations caused by flexural cracks in the slab panel. The procedure described in Section 5.3.3 computes \( \phi \)'s at various load levels. Fig. 5.13
illustrates the moment-curvature relationship of two critical sections: the critical section of cases 1, 2, 3, and 4, and the critical section of case 6. The technique used to divide the total deformation into elastic deformation and hinge deformation is analogous to the technique which Mattock (2.28) and Corlay (2.13) devised to determine the ductility of reinforced concrete flexural members. Knowing the elastic rotation \( \Theta_f \), hinge length \( l_p \) can be calculated from the total end rotation \( \Theta \) derived from either the experiment or finite element analysis and from the curvature \( \phi \) calculated according to the procedure described in Section 5.3. As shown in Fig. 5.14, the calculated \( l_p \)'s range from 190 mm to 300 mm. The range is limited to a relatively narrow band despite the wide variations of the parameters such as load level, loading condition, and amount of steel among the investigated cases.

The average of the \( l_p \)'s, 250 mm, is used to reconstruct the equivalent flexural stiffness \( (EI)_e \) in post-elastic regions. Once the \( l_p \) is specified, data from either experiments or analyses are not needed to compute \( (EI)_e \). The procedure to compute \( (EI)_e \) is explained for BH2MN:

\[
\Theta = \frac{P\ell^2}{2(EI)_{e, \text{elastic}}} + \phi \times l_p \quad (5-20)
\]

\( \phi \) is selected from Fig. 5.15, and \( l_p = 250 \) mm.
As evidenced in Figs. 5.8 and 5.15, the calculated equivalent flexural stiffness and consequently the degradation factor reasonably duplicate those derived by means of vigorous experiments or finite element analyses. It should be pointed out, however, that 250 mm of the hinge length \( \ell_p \) is valid within the range covered in the investigated slabs. Matlock (2.28) and Corlay (2.13) found that the depth of beams and the moment gradient primarily controlled the hinge length. On the other hand, the observation in this section does not show that the moment gradient significantly influences the hinge length. Further investigation is needed to define the hinge length \( \ell_p \) more precisely for floor slabs with different geometry, loading, and reinforcement.

5.5 **Ductility of Floor Slabs**

To evaluate the ductility of reinforced concrete members is difficult because of the interaction of several complicated phenomena like cracking of concrete, the yielding or slipping of reinforcing bars, or the crushing of concrete. Matlock (3.28) and Corlay (2.13) produced two of the earliest works discussing the ductility of reinforced concrete flexural beams. Their basic
procedures used to evaluate the ductility of a simply supported beam with a concentrated force at mid-span are:

1) to assume that the total deflection of the beam consists of elastic deflection and inelastic deflection caused by a plastic hinge at the mid-span, 2) to compute the ultimate curvature at the critical section (the section at the mid-span) of the beam by specifying the ultimate compressive strain of concrete, 3) to assume that the ultimate curvature spreads over a finite length, and next to compute the inelastic rotation at the critical section, and 4) to relate the total inelastic deformation to the inelastic rotation at the hinge according to an empirical formula. Mattock reported that the plastic hinge length was controlled by the effective depth of beam, the gradient of the applied bending moment measured by the distance between the locations of the maximum and zero moments, and the degree of confinement which limits the ultimate compressive strain. Mattock offered the following formula that defines the hinge length:

\[ \ell_p = 0.4d + 0.05z \]  

(5-23)

d = the effective depth of beams

z = the distance along span from section of maximum moment to adjacent section of zero moment
Fiorato et al.\(^{(2.19)}\) computed the ductility of their tested slender shearwalls by using the procedures devised by Mattock. They found that experimental and computed ductilities were comparable.

The formula 5-23 and the moment-curvature curve shown in Fig. 5.13 calculates 14.9 mm of the ultimate deflection for both BH2MN and BH3MN. The calculated deflection is approximately twice as large as the ultimate deflection observed during the tests. The breaking of reinforcing bars controlled post-ultimate load deformation in the tested slab panels described in Chapter III, while Mattock's and Corley's studies assumed that the ultimate compressive strain of concrete limits the deformation. The reinforcement ratio of the tested slab panels ranged from 0.0025 to 0.005, while the ratio of the beams tested in Mattock's and Corley's studies ranged from 0.010 to 0.030. The floor slabs had significantly less reinforcement than their tested beams.

As shown in Fig. 4.13, the end rotations of BH2MN and BH3MN are 28 x 10\(^{-4}\) rad. and 43 x 10\(^{-4}\) rad. at the critical stages at which resistance decreased significantly due to the breaking of several reinforcing bars. The equivalent flexural stiffness, described in Section 5.4.2, are 54 kN x m\(^2\) for BH2MN and 49 kN x m\(^2\) for BH3MN at the critical stages. These
values are 4.0 and 3.6 percent of the respective initial flexural stiffnesses. An attempt was made to estimate the ductility of BH3MN and BH2MN by employing the simplified techniques and experimental findings noted in the previous sections of this chapter. Assumptions and procedures used are as follows:

1. Computation of the ultimate strength was based upon the flexural capacity of the critical section. (Section 5.4.1)

2. The sum of the flexural and shear deformations constituted the total deformation. The proportion of the flexural or shear deformation to the total deformation remained unchanged at all load levels. (Section 5.4.2)

3. The degradation factor $\alpha$ at the ultimate load was computed based upon the curvature at the critical section and an empirical hinge length of 250 mm. (Section 5.4.3)

4. The degradation factor $\alpha$ at the critical stage was taken as 0.038, the average of experimental values. Note that this value was purely empirical.

With the procedures described above, the ductility can be computed without using any of deflections in the tests. The displacement at the ultimate load is 2.8 mm (0.11 in.) for
BH2MN and 1.9 mm (0.075 in.) for BH3MN. The critical displacement is 8.6 mm (0.34 in.) for BH2MN and 5.8 mm (0.23 in.) for BH3MN. The ductility then is 3.1 for both slabs, which approximates the experimental ductility listed in Table 3.15. This coincidence, however, is rather obvious because the degradation factor 0.038 was chosen to match the experimental results. The critical displacement is easily affected by the value of degradation factor at the critical stage. A minor change in this factor at the critical stage can easily double or halve the ductility. The precise evaluation of the critical displacement which corresponds to the breaking of reinforcing bars, therefore, is crucial.

The instant when reinforcing bars would break is unpredictable since local conditions such as bond slip and secondary bending due to dowel action significantly affect the strain at critical regions. Although the experimental results show that the slab panels BH2MN and BH3MN can deflect up to about 7 mm without a significant loss in resistance, to expect this level of deflection in floor slabs having various loading conditions is controversial. Furthermore, to rely the ductility on the performance of reinforcing bars appears to be unwise. It is tentatively suggested not to expect ductile behavior of floor slabs unless they are reinforced more heavily than floor slabs with usual reinforcement.
5.6 Effect of Cyclic Loading

5.6.1 Effect on Strength

The experimental results have shown that reversed loading (negative loading in monotonic loading tests) or cyclic loading (represented by the spectrum shown in Fig. 3.17) reduces the in-plane strength of the tested slab panels by as much as 20 to 25 percent. The finite element analysis described in Chapter IV, on the other hand, did not show a reduction in the strength caused by either reversed or cyclic loading. The evaluation of the ultimate strength based on the flexural capacity at the critical sections, described in Section 5.3.3, did not indicate any such reduction, either.

As several reinforcing bars were broken in the monotonically loaded slab panels at the end of positive loading, these bars obviously did not function in resisting negative loading. The flexural capacity of the critical section was recomputed with these bars removed. The reduction of the ultimate load, however, was not more than 4 percent. All removed bars, located either in the compression region or the vicinity of the neutral axis, had very little effect on the ultimate strength. Clearly, some characteristics not represented by either the finite element analysis or flexural capacity calculation were responsible for the reduction of the ultimate
strength of the panels under reversed or cyclic loading.
It is suspected that precracked concrete lose some effectiveness in resisting compressive force even after cracks are closed.

5.6.2 Effect on Stiffness

5.6.2.1 Elastic Limit

In cyclic loading tests BH1CY and BH3CY, the initial stiffness listed in Table 3.13 remained unchanged during the cycles with the 0.25 mm amplitude. During the loading in the first cycle of the 0.76 mm amplitude, cracks developed along the slab-wall junctions, causing the stiffness to decrease significantly as shown in Fig. 3.27 and 3.28. The cracks occurred in BH1CY at 28 kN (6.3 kip) under positive loading and at -28 kN (-6.3 kip) under negative loading and in BH3CY at 18 kN (4.0 kip) under positive loading and -14 kN (-3.2 kip) under negative loading. These cracks loads were 20 and 25 percent lower than the cracking loads taken place in the monotonic loading tests. (Section 5.4.1) The three preceding cycles in a lower amplitude evidently accelerated the development of the cracks. Interestingly, the cracking loads and the ultimate loads decreased by the same percentage under cyclic loading. In BH2CY, the cracking load decreased to 78 percent,
while the ultimate load decreased to 79 percent. In BH3CY, the cracking load decreased to 74 percent, while the ultimate load decreased to 73 percent.

5.6.2.2 Stiffness in Post-Elastic Regions

The technique to evaluate the in-plane stiffness of floor slabs by treating them as deep beams was extended to the post-elastic regions for slabs subjected to cyclic loading. Figs. 5.16 and 5.17 show the proportions of flexural and shear deflections to total deflection of BH1CY and BH3CY, respectively. The definitions of the flexural and shear deflections and the procedures used to compute these deflections followed those described in Section 5.4.2. Rotation and deflection increments used in the calculations were taken in three different ways: 1) rotation and deflection increments from the point of the minimum deflection to the point of the maximum deflection of a hysteretic loop, which corresponds to the secant stiffness 1 defined in Section 3.6.3, 2) rotation and deflection increments from the point of zero deflection preceding the maximum deflection to the point of the maximum deflection of a hysteretic loop, which corresponds to the secant stiffness 2 defined in Section 3.6.3, and 3) rotation and deflection increments from the point of zero deflection preceding the minimum deflection to the point of the minimum
deflection of a hysteretic loop, which corresponds to the secant stiffness \(3\) defined in Section 3.6.3. The proportions of flexural and shear deflections to total deflection are computed for the first cycle of each amplitude based upon these three different types of rotation and deflection increments and plotted against the amplitude in Figs. 5.16 and 5.17.

As evidenced by these diagrams, the proportions are confined in a relatively narrow band and maintain the proportions found in the elastic region (indicated by dotted lines) regardless of amplitude or load (or displacement) level. This finding supports the conclusions derived in Section 5.4.2, which discusses slabs subjected to monotonic loading. First, both the equivalent flexural and shear stiffnesses degraded by the same degree regardless of loading condition, load level, or amplitude. Second, the proportions of flexural and shear deflections to total deflection in the elastic region remained unchanged in post-elastic regions. Consequently, the in-plane stiffness of floor slabs under cyclic loading can also be evaluated by means of the basic beam theory which includes both flexural and shear deformations once the degradation factor \(\alpha\) is specified. The degradation factor \(\alpha\) is a function of load level, deflection and previous loading history. The following section describes the derivation of the degradation factor \(\alpha\) for slabs subjected to cyclic loading.
5.6.2.3 Degradation Factor for Slabs in Cyclic Loading

Fig. 5.18 plots the secant stiffness $l$, defined in Section 3.6.3, of cyclic loading tests BH1CY and BH3CY against specified amplitudes. This diagram also plots the secant stiffness of the monotonic loading tests BH2MN and BH3MN, in which the stiffness is plotted against the deflection corresponding to the amplitudes. During the cycles of the two lowest amplitudes, 0.23 mm and 0.76 mm, the secant stiffnesses between the monotonic and cyclic loading tests are the same. According to Section 3.7.4.3, the initial stiffness of BH2MN is significantly lower than those of all other tests. During these cycles, damage caused by cracks occurring in the previous cycles had not yet reduced the stiffness. During the three cycles of the 1.3 mm amplitude, in which the load reached approximately half of the ultimate load for the first time, the secant stiffness $l$ equals approximately 75 percent of the secant stiffness of the corresponding monotonic loading tests. The secant stiffness $l$ constantly ranges from 75 to 80 percent of the secant stiffness of the corresponding monotonic loading tests in the succeeding cycles. Both the stiffness and the ultimate load are reduced due to cyclic loading by the same percentage.
Based upon this finding, a rule to evaluate the secant stiffness $1$ of slabs under cyclic loading from the secant stiffness of the slabs when they are loaded monotonically is proposed:

1. The secant stiffness $1$ of a slab subjected to cyclic loading with a given amplitude is $x$ times the secant stiffness of the slab subjected to monotonic loading at the deflection corresponding to the amplitude.

2. The fraction $x$ is the ratio of the ultimate load which can be reached if the slab is loaded monotonically to the ultimate load which can be reached if the slab is subjected to cyclic loading. The degradation factor $\alpha$ of the slab under cyclic loading, therefore, is $x$ times the degradation factor of the slab under monotonic loading.

3. During the cycles with amplitudes in which the load is less than half of the ultimate load, the secant stiffness $1$ may equal the corresponding secant stiffness derived from the monotonic loading.

The secant stiffness $1$ represents only a portion of the stiffness of the slab panel subjected to cyclic loading. Nevertheless, this stiffness is one of the most suitable representations of the stiffness of slabs under arbitrary
reversed loadings. As pointed out in Section 3.7.4.2, the five types of stiffness defined in Section 3.6.3 varied little regardless of loading condition or load level.

5.6.3 Effect on Ductility

Experimental results show that cyclic loading reduced critical displacements by about 10 percent and increased by about 40 percent the deflection at which 90 percent of the ultimate load was reached for the first time. The approximately 30 percent difference in ductility observed in monotonic and cyclic loading tests, as shown in Table 3.15, was attributed primarily to the larger deflection corresponding to 90 percent of the ultimate load taking place during the cyclic loading tests. However, it must be noted that the load-deflection relationship of cyclically loaded panels is heavily dependent on the loading spectrum. Consequently, significant variations may take place in the base deflection value for the calculation of ductility.

The ductility of the slab panels subjected to cyclic loading also depended upon the performance of reinforcing bars. The breaking of several reinforcing bars at their intersections with major cracks caused significant loss in resistance. Although the test results show that cyclic loading reduced the critical displacement by only 10 percent
and that resistance did not degrade in three cycles for most of the specified amplitudes of the test spectrum, these performances may not prevail under different loading systems. The suggestion offered in Section 5.5 can be pointed out again. With limited data available, we should not allow slabs to deflect under any loading conditions beyond the deflection level at which the ultimate load can be reached when the slabs are loaded monotonically.

5.7 Effect of Vertical Load

5.7.1 Effect on Strength

The design service vertical load reduced the ultimate resistance of the tested slab by 15 percent. The vertical load also changed the crack pattern particularly in the middle portion of the bottom surface, where many cracks extended from the center of the slab in radial direction. Evidently, out-of-plane bending moment caused these cracks. The vertical load, however, did not change the general in-plane behavior of the test panel. The major crack still developed along the boundary between the column and middle strips, and its complete formation governed the ultimate resistance. The opening and closing of the major crack also controlled the deformation of the panel in post-ultimate load regions.
Under in-plane loading, the major crack developed along the boundary between the column and middle strips, where a significant decrease of reinforcement took place. The boundary was least reinforced because the out-of-plane bending moment caused by the vertical load was small along the boundary. In other words, the boundary was the most critical section under in-plane loading but relatively safe under out-of-plane (vertical) loading. Along the slab-wall junction, on the other hand, the bending moment caused by the vertical load was the greatest; therefore, the junction was heavily reinforced. The junction with substantial reinforcement, then, increased the flexural capacity of the slab in its own plane. The junction, therefore, was the most critical section under vertical loading but considerably stronger than the boundary against in-plane loading. The test result indicates that the boundary between the column and middle strips was still the most critical section under the combined vertical and in-plane loading.

Cyclic loading reduced the ultimate load further by 18 percent, which is about the same as the reduction on slab panels not subjected to vertical loading. The cyclic loading changed neither the crack pattern nor the development of major cracks.
5.7.2 Effect on Stiffness

5.7.2.1 Effect in Post-Elastic Region

In order to determine the suitability of treating floor slabs as deep beams and evaluating the in-plane stiffness of floor slabs subjected to the design service vertical load, the proportions of flexural and shear deflections to total deflection were computed for BV1MN and BV2CY. The procedures used to calculate the flexural and shear deflections were those described in Section 5.4.2. Three types of rotation and deflection increments (BV2CY), were employed to compute equations 5-6 through 5-8, as described in Section 5.6.2.2. According to Fig. 5.19, the proportions are confined in a relatively narrow band. They maintain the proportion in the elastic region regardless of amplitude or load level. This finding again supports the extension of the calculation procedure described in Section 5.4.2 for evaluating the in-plane stiffness of floor slabs under combined in-plane and out-of-plane loadings. The in-plane stiffness of floor slabs subjected to combined vertical and in-plane loads (either monotonic or cyclic) can be evaluated by means of the fundamental beam theory which includes both flexural and shear deformations once the degradation factor $\alpha$ is specified. The degradation factor can be a function of in-plane load level, vertical load level, deflection, and previous loading history.
5.7.2.2 Degradation Factor of Slabs with Vertical Load

The test results show that the design service vertical load reduced the initial in-plane stiffness by about 20 percent. (Section 3.7.4.7) The reduction was caused primarily because cracks developed under the vertical load. (Fig. 3.19) Unlike slab panels without vertical load, slab panels with vertical load immediately lost their initial in-plane stiffness. Afterward, the stiffness gradually degraded as the load increased or reversed.

For BH2MN, BV1MN, and BV2CY, the ratio of equivalent flexural stiffness \( \frac{EI}{E} \) to the elastic flexural stiffness is plotted against the displacement. (Fig. 5.20) This ratio matches the degradation factor. The equivalent flexural stiffness of BV1MN ranged from 60 to 85 percent of that of BH2MN. The difference between these equivalent flexural stiffnesses was appreciable particularly in the vicinity of the deflection at which BH2MN reached its ultimate load. (2.9 mm of deflection) The stiffness of BV1MN degraded rapidly in the deflection range in which BH2MN approached its ultimate load, while varying its tangent stiffness slightly. The secant stiffness of the slab panels with the design service vertical load and which were loaded monotonically in their own planes, therefore, comprised 60 to 85 percent of the secant stiffness of the panels with no vertical load.
Like the slab panels without vertical load, the slab panels with the design service vertical load also show a correlation between the secant stiffnesses under cyclic and monotonic loadings. For a deflection of less than 1.1 mm, the point at which BV2CY first reached about half of the ultimate load, each of the secant stiffnesses 1, 2, and 3 of BV2CY almost equalled the secant stiffness of BV1MN. For a larger deflection, the secant stiffnesses 1, 2, and 3 of BV2CY ranged from 80 to 90 percent of the secant stiffness of BV1MN. The relationship between the secant stiffnesses of BV1MN and BV2CY was identical to those between BH2MN and BH1CY, and BH3MN and BH3CY. The rule stipulated in Section 5.6.2.3, therefore, can be used to evaluate the secant stiffness 1 of slabs with vertical load.

The degradation factor of a slab to which vertical load is first applied and held constant during in-plane cyclic loading is \(x\) times \(y\) times the degradation factor of the slab to which only in-plane load is applied monotonically. The factor \(x\) is the ratio of the ultimate load which may be reached if the slab is loaded monotonically to the ultimate load which may be reached if the slab is subjected to cyclic loading. The factor \(y\) is the ratio of the ultimate load which may be reached if the slab is loaded monotonically with vertical load to the ultimate load which may be reached if the slab is loaded.
monotonically without vertical load. For the slab panels investigated in this study, x and y are 0.80 and 0.85 respectively. Data, however, are limited, and further research is indispensable to the proper evaluation of the combined effect of the intensity of vertical load and loading history on these factors, x and y.

As in panels without vertical load, the secant stiffness also is a reasonable representation of the stiffness in slabs with vertical load and subjected to cyclic loading. The five types of stiffness defined in Section 3.6.3 differed little regardless of load level, indicating that the secant stiffness reasonably matches the tangent stiffness of a hysteretic loop in most regions.

5.7.3 Effect on Ductility or Deformability

As indicated in Section 5.6.3, ductility in cyclic loading tests is questionable because it is difficult to pinpoint the deflection at which 90 percent of the ultimate resistance could first be reached. In addition, ductility was not computed for BV1MN since the test was terminated before the panel reached the critical displacement. The design service vertical load, on the other hand, reduced critical displacement by about 15 percent when the slab panel was subjected to cyclic loading, but increased critical displacement by about 25 percent when the panel was subjected to mono-
tonic loading. Like the panels without vertical load, the
panel with vertical load lost its resistance when several
reinforcing bars broke. The vertical load (at least up to the
design service load level) cannot be a major parameter reducing
the deformability of the slabs if the uncertainty of computing
ductility as well as the slab's allowable deflection are taken
into account.

5.8 Summary

In this Chapter, the critical in-plane characteristics
of floor slabs: strength, stiffness, and ductility, were examined
based on the experimental findings and data provided by the
proposed finite element analysis (Chapter IV). Practical proce­
dures to predict the ultimate in-plane strength and in-plane
stiffness of floor slabs have also been proposed. The important
findings can be summarized as follows:

Findings Concerning Strength

(1) Two failure modes were observed in the analyzed
slab panels depending on imposed loading and
boundary conditions. In the flexural failure mode,
the major crack developed parallel to the clamped
edge at a distance of about a quarter panel length
(350 mm) and controlled the ultimate in-plane
resistance. This location of major crack, labelled
the critical section, was very near the boundary between the column and middle strips of the panel. Many reinforcing bars were terminated here, according to out-of-plane bending considerations. The flexural capacity of the critical section governed the ultimate resistance.

(2) In the shear failure mode, slab panels reached their ultimate loads when a diagonal crack completely separated a triangular portion from the remainder of the panel. A strong beam attached to the loading edge of the slab panel, simulating the situation where slabs are supported by shearwalls would prevent the diagonal separation of the panel and promote the flexural failure mode.

(3) Increasing the amount of flexural reinforcement improved the in-plane strength of slab panels. On the other hand, increasing the size of concrete beams increased the elastic in-plane stiffness and the elastic limit load, but did not affect the ultimate resistance. When panels failed in the flexural mode, the effect of the magnitude of shear on the ultimate strength was minimal, indicating that the moment-to-shear ratio did not influence the flexural capacity of the critical sections.
Cyclic loading represented by the spectrum shown in Fig. 3.17 reduced the in-plane strength of slab panels by as much as 25 percent. Cumulative damage like cracking was responsible for the reduction. The effectiveness of concrete in resisting compression was reduced by cracking even after the cracks had been closed. The application of the design service vertical load reduced the ultimate in-plane resistance of slab panels by not more than 15 percent. Vertical load did not alter the general behavior of the panels; the major crack still developed along the boundary between column and middle strips.

The in-plane flexural strength of slab panels loaded monotonically without vertical load can be reasonably evaluated by using the theoretically calculated flexural capacity at the critical section by means of the Navier-Bernoulli hypothesis. (The procedure detail is described in Section 5.3.3) The standard ACI Code provision (11.5.6.2) for estimating the contribution of shear reinforcement to the shear capacity is applicable for the slab panels. The ACI provision (11.3.1.1) on the contribution of the cracked concrete overestimates the shear capacity.
Findings Concerning Stiffness

(1) Slab panels not subjected to vertical loading started losing their initial in-plane stiffness upon appearance of a crack at the edge of the slab-wall junction; this point, therefore, was defined as the elastic limit. The load corresponding to the elastic limit was approximately 30 percent of the ultimate load, whereas the secant stiffness at this stage was about 30 percent of the initial value. The secant stiffness continuously decreased as load increased, reaching approximately 15 percent of the initial value at the ultimate load stage.

(2) In cyclically loaded slab panels without vertical load application, the secant stiffness degraded in a similar manner as in monotonically loaded slab panels. Hysteretic load-deflection loops of these panels were narrow in width, indicating that energy absorption in a complete cycle was small. The secant stiffness, therefore, appears to be a relatively suitable representation of the stiffness of slabs subjected to arbitrary reversed loading.

(3) Upon the application of the design service vertical load, two cracks developed on the slab panel's top surface, one along the slab-wall junction and the other along the loading line. These cracks apparently changed the
pattern of stiffness degradation under monotonic loading. Tangent stiffness of the panel degraded gradually, measuring near zero at the ultimate load. The vertical load promoted the development of cracks, and cracking in low load levels made the transfer of forces from concrete to reinforcing bars more gradual. When slab panels with the vertical load were loaded cyclically, however, the effect of the vertical load on stiffness degradation was small. The slab panel with the design service vertical load constantly showed an about 20 percent lower secant stiffness than the panel without such load.

(4) In the elastic range, the stiffness characteristics of the slab panels may be reasonably estimated by treating them as deep beams considering both flexural and shear deformations (equivalent beams). The use of nominal moment of inertia and shear area, those computed from the gross cross section, is sufficient for evaluating the in-plane stiffness of floor slabs regardless of aspect ratio or relative beam size.

(5) The equivalent beam simulation also can be used in post-elastic ranges if a hypothetical reduction factor is introduced. This factor represents a stiffness reduction of the equivalent beam caused by various inelastic
behavior such as cracking or yielding of reinforcing bars. Once a reduction factor is known at a given load level, the in-plane stiffness of a floor slab in the post-elastic region is computed by using the same procedure as for elastic deep beams. A simple procedure to compute the reduction factor is proposed in Section 5.4.3. This equivalent beam simulation is proven to be applicable not only for floor slabs subjected to monotonic in-plane loadings but also for floor slabs subjected to in-plane cyclic loadings or combined in-plane and out-of-plane loadings.

Findings Concerning Ductility (Deformability)

(1) Once the ultimate load was reached, the deflection of the tested slab panels was controlled primarily by the opening and closing of the major crack. Few new cracks started in this post-ultimate load region, while the level of resistance remained relatively constant. Finally, the breaking of reinforcing bars at the major crack reduced the resistance drastically. The effect of either cyclic loadings or combined in-plane and out-of-plane loadings on ductility was minimal. The slab panels under these loadings de-
formed without a significant change in resistance to approximately 80 percent of the limit deflection of monotonically loaded slab panels.

(2) In-plane ductilities of tested slab panels were compared with theoretical ductilities computed by means of a conventional procedure. The computed ductilities were approximately twice as large as the experimental ductilities. The discrepancy was attributed to the difference in material behavior controlling the critical deformation. The breaking of reinforcing bars controlled deformation in post-ultimate load regions for tested slab panels. In contrast, the computational procedure assumes that the reinforcing bars can yield indefinitely and the ultimate compressive strain of the concrete limits the deformation.

(3) A technique is proposed to compute the ductility of floor slabs by using empirically defined ultimate stiffness reduction factors. This technique functions satisfactorily for the tested panels, but additional confirmation is still needed before its usefulness may be expanded. With insufficient data available about the deflection corresponding to the breaking of reinforcing bars, it is tentatively suggested that in-plane ductile behavior of floor slabs should not be expected unless they are reinforced more heavily than usual.
VI DYNAMIC RESPONSE OF BUILDING INCLUDING IN-PLANE FLOOR SLAB DEFORMATION

6.1 General

This chapter examines influence of in-plane deformation of floor slabs on the total seismic lateral load applied to a structure as well as on the distribution of this lateral load to the vertical load resisting elements. Shear forces and bending moments applied to floor slabs are also investigated.

A relatively simple building model representing a reinforced concrete building with usual dimensions was selected, and the dynamic response of the model was investigated. The direct integration method was used to obtain the time-history relationship of this structural model. The ground motion selected for this study was the N-S component of the El Centro earthquake recorded in Imperial Valley, May 18, 1940. The various in-plane characteristics of the floor slabs defined in Chapter V as well as the stiffness and strength characteristics of the vertical elements (shearwalls and columns) were incorporated into the dynamic analysis.

At the beginning of this investigation, elastic dynamic analysis was executed based on the elastic characteristics of the floor slabs and vertical elements. The results obtained from this elastic analysis were used as the basis for comparison
with the results derived from the succeeding analyses. In these analyses, the stiffness characteristics of the floor slabs as well as vertical elements altered. First, the propriety of the rigid floor slab assumption was examined. Then, the change in the model structure's response due to the non-linear actions of vertical elements and floor slabs was investigated.

6.2 Building Model Selected

A reinforced concrete building model with seven stories, six bays in width, and one bay in depth was selected for this study. Fig. 6-1 illustrates the plan and elevation views of this building and its critical dimensions. The story height is 3.0 m from second through seventh stories, while the first story is 3.75 m in height. The center-to-center span length is 6.0 m in two horizontal directions. A shearwall is located at each end, whereas the interior spans are supported by flexible frames. The thickness of the shearwalls is 320 mm, and the cross section of the columns is 600 mm x 600 mm with no capital. The floor slabs are 200 mm in thickness. Beams (300 mm x 500 mm) are attached to the lower sides of these slabs and along column lines in both directions. The ground motion was applied in the transverse direction of the structure.
This model has a relatively slender cross section, whose aspect ratio is 1 : 6. This arrangement was chosen intentionally so that the floor slabs would play a significant role on the distribution of lateral force to vertical elements. As discussed in Chapter II, a slender cross section tends to amplify the bending action of the floor slabs, referred to as bowing. In addition, a great difference in lateral stiffness between adjoined vertical elements increases the force to be transmitted by the floor slab connecting these components. This increase in force may cause severe distortion of the floor slab.

6.3 Analysis of Building Model

6.3.1 Simplification of Model

To investigate the response of the building model, the structure was simplified to a discrete model. In this model, all structural components: floor slabs, frames, and walls, were treated as beams, named equivalent beams. These equivalent beams were assembled and jointed at each connection, forming a seventy-story six-bay frame. Since the ground motion was applied in the transverse direction, equivalent beam properties for the deformation in this direction were needed. The beam properties were determined so that these equivalent beams would appropriately simulate the original properties. Walls were treated as deep beams which have both flexural and shear de-
formations. Floor slabs also were treated as deep beams; Chapter V has discussed this treatment in detail. Frames, composed of columns and beams, were represented as beams that have particular combinations of flexural and shear rigidities. The selection of both geometrical and material properties of the equivalent beams is described in further detail in Section 6.4. Since both geometry of the model structure and input ground motion were symmetrical about the vertical axis (Fig. 6.1), only half of the structure must be analyzed.

6.3.2 Formulation of Equations of Motion

In dynamic response analysis, the equations of motion can be written as:

\[ [M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{R(t)\} \] (6-1)

where:

- \([M]\) = mass matrix
- \([C]\) = damping matrix
- \([K]\) = stiffness matrix
- \{\{u\}\} = displacement vector
- \{\{\dot{u}\}\} = \frac{du}{dt}
- \{\{\ddot{u}\}\} = \frac{d^2u}{dt^2}
- \{R(t)\} = vector of arbitrary time varying loads
If the structure is uniformly subjected to the ground motion $u_g$, the equation can be expressed as:

$$[M]\{\ddot{u}_r\} + [C]\{\dot{u}_r\} + [K]\{u_r\} = -[M] \times u_g \quad (6-2)$$

where $\{u_r\}$ is the relative displacement vector of the structure with respect to the ground, i.e., $\{u_r\} = \{u\} - \{1\} \times u_g$.

6.3.3 Formulation of Mass Matrix

The mass of the structure was represented by lumped masses at junctions. Each lumped mass equalled to the mass within a tributary region of a junction. The mass of the service design vertical load, 3.8 kPa, acting on the floor slabs was added to the mass calculated from the structure's dead weight. Since rotational inertia was neglected, (Section 6.3.4), the lumped masses caused only translational inertia at the junctions. The mass matrix $[M]$, then, is a diagonal matrix.

6.3.4 Formulation of Stiffness Matrix

The element stiffness matrix of equivalent beams representing floor slabs and walls can be expressed in terms of the deflections and rotations at end points:
\[
\left\{ \begin{array}{c}
P_i \\ M_i \\ P_j \\ M_j \\
\end{array} \right\} = \frac{1}{A\ell^2 + 2B} \left[ \begin{array}{cccc}
2 & \ell & -2 & \ell \\
\ell & \frac{2\ell^2}{3} + \frac{B}{3A} & -\ell & \frac{\ell^2}{3} - \frac{B}{3A} \\
-2 & -\ell & 2 & -\ell \\
\ell & \frac{\ell^2}{3} - \frac{B}{3A} & -\ell & \frac{2\ell^2}{3} + \frac{B}{3A} \\
\end{array} \right] \left\{ \begin{array}{c}
\delta_i \\ \Theta_i \\ \delta_j \\ \Theta_j \\
\end{array} \right\}
\]

where:
\[
A = \frac{\ell}{6EI_x}
\]
\[
B = \frac{\ell}{AG}
\]

Element stiffness matrices were first assembled for each floor or vertical span (wall or frame), making a set of substructure stiffness matrices.

The torsional rigidity of walls, frame and floor slabs was taken to be zero. Elements associated with rotation (moment) were then condensed for each substructure stiffness matrix. This static condensation was permissible since the rotational inertia has been eliminated. After the static condensation, the substructure stiffness matrix, therefore, related trans-
latory displacements to lateral forces. A global stiffness matrix was composed by assembling substructure stiffness matrixes and related the out-of-plane forces at each joint of the planar frame to the corresponding displacements. The procedure for making the stiffness matrix is schematically illustrated in Fig. 6.2.

6.3.5 Formulation of Damping Matrix

The Rayleigh damping was chosen to represent the viscous damping characteristics of the structure. The damping matrix is:

\[ [C] = \alpha[M] + \beta[K] \]

\( \alpha \) and \( \beta \) are constants to be determined by given damping ratios that correspond to two unequal frequencies of vibrations. \( \alpha \) and \( \beta \) were calculated by using the two lowest circular frequencies \( \omega_1 \) and \( \omega_2 \) of the structure and then by employing 5 and 10 percent of the critical damping in these two modes. After applying the mode superposition technique to the original equations of motion, the relationship between \( \alpha \) and \( \beta \), and \( \omega_1 \) and \( \omega_2 \) can be expressed as:

\[ \alpha + \beta \times (0.05)^2 = \omega_1 \times 0.10 \]
\[ \alpha + \beta \times (0.10)^2 = \omega_2 \times 0.20 \]

\( 6-4 \)
6.3.6 Method of Integrating Equations of Motion

Various integration techniques have been developed to solve the equations of motion for linear and nonlinear structures. The techniques are divided primarily into two groups. The first group, referred to as the implicit solution technique, includes the techniques such as the Houbolt method, the Newmark method, the Wilson θ method, and the Park stiffly-stable method. The second group, referred to as the explicit solution technique, includes the central difference method, the two-cycle iteration with the trapezoidal rule, and the Runge-Kutta method. Algorithms for these solution techniques as well as their stability and accuracy have been investigated by many researchers. While the stability and accuracy of these techniques for linear structures have been evaluated extensively, those for nonlinear structures can be studied only through numerical experimentation. In the present study, the Newmark method \(^{(6.6)}\) was chosen to solve the equations of motion. This selection was based upon the investigation completed by Nickell, \(^{(6.7)}\) Mondkar and Powel, \(^{(6.5)}\) and Adeli et al. \(^{(6.1)}\)

Basically, the Newmark integration technique is an extension of the linear acceleration method and makes the following assumptions:
\[ \ddot{u}_{t+\Delta t} = \ddot{u}_t + [(1 - \delta) \ddot{u}_t + \delta \dddot{u}_{t+\Delta t}] \Delta t \]  

\[ u_{t+\Delta t} = u_t + \dot{u}_{t+\Delta t} + [(\frac{1}{2} - \alpha) \ddot{u}_t + \alpha \dddot{u}_{t+\Delta t}] \Delta t^2 \]

\(\alpha\) and \(\delta\) are parameters to be chosen by considering integration accuracy and stability. The present study used \(\alpha = 1/4\) and \(\delta = 1/2\), which Newmark originally proposed as an unconditionally stable constant-average-acceleration method.

The equilibrium equations at time \((t+\Delta t)\) are:

\[ [M][\dddot{u}_{t+\Delta t}] + [C][\ddot{u}_{t+\Delta t}] + [K][u_{t+\Delta t}] = -[M][\dddot{u}_t] \]  

Solving from equation 6-6 for \(\dddot{u}_{t+\Delta t}\) in terms of \(u_{t+\Delta t}\) and substituting it into equation 6-5 result in two equations for \(\dddot{u}_{t+\Delta t}\) and \(\ddot{u}_{t+\Delta t}\), each in terms of the unknown displacement \(u_{t+\Delta t}\) only. These equations for \(\dddot{u}_{t+\Delta t}\) and \(\ddot{u}_{t+\Delta t}\) are then substituted into equation 6-7 to solve for \(u_{t+\Delta t}\), after which, using equations 6-5 and 6-6, \(\dddot{u}_{t+\Delta t}\) and \(\ddot{u}_{t+\Delta t}\) can also be calculated.

The complete algorithm of the Newmark method is presented in Bathe and Wilson (6.3) and Bathe (6.2).
6.4 Selected Force-Deflection Relationship

6.4.1 Force-Deflection Model for Floor Slabs

The stiffness of the floor slabs in post-elastic range was determined by using the concept of reduction factor defined in Chapter V. Each equivalent beam representing a floor slab had a particular reduction factor according to the magnitude of the forces being applied to the beam as well as its previous history. As discussed in Sections 5.2.2, 5.4.1, and 5.4.2, the reduction factors for flexural and shear stiffnesses were assumed to be identical at any stage during loading and dependent upon the magnitude of the maximum bending moment applied to the equivalent beam. In the present study, the reduction factor vs. moment relationship was approximated in the following form: the reduction factor is 1.0, indicating that the floor slab is elastic when $M/M_{\text{max}}$ is less than 0.3. At 0.3 of $M/M_{\text{max}}$ the reduction factor is decreased to 0.3, assuming that cracking occurs. As the moment increases, the reduction factor decreases linearly until the moment reaches its maximum value, at which the reduction factor is 0.15. This reduction factor vs. moment relationship is illustrated by the bold line in Fig. 5.17, which shows that this line reasonably approximates the actual degradation pattern of the reduction factor. After reaching its maximum value, the moment is assumed to remain unchanged under continued deformation. On the other hand, the secant stiffness measured when unloading...
begins is used for succeeding loading until the moment reaches the maximum value a second time. When this takes place, the stiffness is once again set at zero. This assumed moment-stiffness relationship was based on the experimental finding that the width of the hysteretic loops of slab panels was relatively narrow and that the tangent stiffness measured from a hysteretic loop of the slab panel varied only slightly regardless of the moment (or deformation) level. Detailed discussion of this characteristics can be found in Sections 3.7.4.2. and 5.6.2.3. Fig. 6.3 illustrates the hysteretic law adopted for the floor slabs.

6.4.2 Force-Deflection Model for Walls

The behavior of shearwalls under seismic loadings has been investigated extensively as already discussed in Chapter II. Here in this study, the experimental work performed by Oesterle et al. (2.32) was referred to for determining the characteristics of the model structure's shearwalls. One of their tested shearwalls (R-2 specimen in their report), which did not have any boundary frame, was utilized to represent the shearwalls.

A number of analytical force-deflection models have been proposed for simulating hysteretic behavior of structural members and systems. These proposed models include for example:
1) the ordinary elasto-plastic model, and 2) the degrading stiffness models, (6.4, 6.9) which consider such nonlinear behavior as the Bauschinger effect and the pinching effect. A comprehensive review and critique of those models has been made by Riddell and Newmark. (6.8)

It was decided to represent the hysteretic behavior of the shearwalls by the force-deflection model used for floor slabs. (Section 6.4.2) This decision was made based upon the following reasons: (1) Oesterle et al. reported that flexure dominates the behavior of the shearwall R-2 (2) The load-deflection relationship of the shearwall R-2 is similar to those of the tested floor slabs. (However, the pinching effect is more appreciable in the shearwall.) (3) The algorithm needed for this model is much simpler than those for other models. (6.9)

6.4.3 Model for Frames

Frames were also assumed to be beams which include both flexural and shear deformations (equivalent beams). Moment of inertia of the equivalent beam was taken as that of the frame's cross section. Shear area of the beam, on the other hand, was evaluated by equating the shear displacement of the equivalent beam to the lateral displacement of the frame.
Fig. 6.4 schematically shows the procedure to compute the shear area of equivalent beams. In this study, the frames were assumed to behave linear-elastically.

6.5 Input Material and Geometrical Properties

Table 6.1 lists the moment of inertia and shear area of equivalent beams representing floor slabs, walls, and frames. This table also shows the maximum moment levels which the critical sections of floor slabs or walls can reach. The elastic constants for all elements are: 24.1 GPa for the modulus of elasticity and 0.15 of poisson's ratio. The lumped masses at junctions are listed in Table 6.2. According to an eigenvalue analysis, the frequencies of the model structure are 0.355 Hz for the first mode and 0.0852 Hz for the second mode. From these frequencies and equation 6.4, the damping coefficients $\alpha$ and $\beta$ are computed to be 0.561 and 0.00362 respectively.

6.6 Results and Discussion

The model structure was analyzed for the following four cases, referred to as cases 1 through 4: (1) linear elastic analysis, (2) linear elastic analysis with rigid floor slab assumption, (3) non-linear analysis, in which floor slabs are allowed to behave non-linearly as defined in the previous section, (4) non-linear analysis, in which both floor slabs
and shearwalls are allowed to behave non-linearly. The numerical results including maximum displacement, maximum acceleration, total base shear, base shear distributed in each shearwall or frame, and maximum shear and moment applied to floor slabs are tabulated for the four analyses in Tables 6.3 through 6.5. Fig. 6.5 shows lateral force distribution in shearwalls and frames, while shear forces in floor slabs are plotted in Fig. 6.6. Fig. 6.7 shows several displacement, base shear, bending moment vs. time relationships. For convenience of the following discussion, all comparisons of results are referred to that of the elastic analysis (case 1).

When floor slabs are assumed to be infinitely rigid, the total shear is reduced to 88 percent, while it is increased to 102 percent when slabs behave non-linearly. (Table 6.3) The change in the total base shear, however, is not great despite that basically three different structures (case 1 to 3) are analyzed. As anticipated, the rigid slab assumption causes significant underestimate of the base shear resisted by the frames. (Table 6.4) The base shear in the middle frame (frame 3) is only 23 percent if the rigid slab assumption is adopted. Non-linearity of the floor slabs (case 3) also changes the lateral load distribution significantly. Decreased stiffness of floor slabs in the inelastic range causes frame base shear to increase by nearly 100 percent. Here, the
interaction among the vertical elements is lessened, resulting that each element responds more independently from one another.

In the elastic analyses, shearwalls take a major portion of the shear: 75 and 92 percent in cases 1 and 2 respectively. When slabs and shearwalls behave non-linearly, on the other hand, the base shear transmitted by the shearwalls is decreased significantly (to 65 percent). This reduction apparently is caused by the degraded stiffness in the shearwalls. Frames, in turn, carry approximately 2.5 times larger base shear.

Floor slabs behave like simply supported beams with distributed lateral force, causing maximum shear force at the junctions with shearwalls (exterior bents) and maximum bending moment at the mid-span (at the junction with the middle frame: frame 3). (Fig. 6.7) The shear force transmitted in the floor slabs is much smaller than the shear force which would be considered to be critical. The ACI Code formula, (11.3.1.1) which specifies the lower limit for shear effect consideration, gives 1.05 MN of shear force, while the maximum shear force carried by the floor slabs is not more than 0.428 MN. Bending moment, on the other hand, is significant particularly at the mid spans, and exceeds the specified maximum moment in many occasions. As tabulated in Table 6-5, the bending moments of the floor slabs in
the lowest five stories exceed the maximum moment level at the mid spans in case 1 (elastic analysis), while the top five stories reach this level when the floor slabs are allowed to behave non-linearly. A comparison between cases 3 and 4 reveals that the level of bending moment decreases as the variation in stiffness among the vertical elements decreases.

As shown in Fig. 6.6, the distribution of lateral force applied to each story is approximately triangular except for the roof level. Lateral force is reduced at this level because masses at this roof level are significantly smaller than those in other stories. No live load was assigned to this level. (Table 6.2) The distribution of lateral force in shearwalls, however, is far from triangular, rather, shear force is nearly constant for all floors. The lateral force distribution in frames is close to triangular although the variation among story shears decreases considerably when non-linear behavior of floor slabs or shearwalls is considered (cases 3 and 4). This near rectangular distribution is appreciable particularly in the middle frame (frame 3). The complexity of the lateral force distribution in frames and shearwalls would be caused by the change in the relative story stiffness of the vertical elements (shearwalls and frames) at each story level. The story stiffness of frames is relatively constant, while the story stiffness in shearwalls decreases significantly in upper stories.
6.7 Summary

The findings in this chapter may be itemized as follows:

1. The effect of the in-plane stiffness of the floor slabs on the base shear applied to vertical elements is appreciable in the analyzed model structure. The rigid floor assumption significantly underestimates the base shear of the frames.

2. The stiffness of the shearwalls (major lateral force resisting elements in the model structure) also affects the base shear. Degraded stiffness of the shearwalls shifts some base shear to the frames.

3. Shear force applied to the floor slabs is very small and practically negligible. Bending moment, however, is significant, particularly at the mid-span of the structure, reaching the maximum moment level in many stories. The degraded stiffness of the floor slabs alters the base shear distribution, placing more load to the frames.

4. The distribution of lateral force applied to each story is approximately triangular. The distribution of lateral force in shearwalls or frames, on the other hand, does not follow the triangular distribution. The change in the relative story stiffness of the vertical elements at each story level most probably causes this complexity.
VII SUMMARY, CONCLUSIONS, AND RECOMMENDED RESEARCH

This report presents a comprehensive study of the in-plane characteristics of reinforced concrete beam-supported floor slabs with various loading and support conditions and studies the effect of those characteristics on seismic building response. The discussion of experimental and analytical results and the application of the results and findings to the evaluation of the in-plane characteristics have been described in Chapters III through V. In Chapter VI, the effect of the in-plane deformation of floor slabs in building response has been examined. Important findings, suggestions and proposals for practical designs, and recommendations for future work are summarized as follows:

Experimental Findings

(1) The development of a crack, labelled a major crack, which developed along the boundary between the column and middle strips, controlled the ultimate in-plane strength of the test slab panels.

(2) The cyclic loading represented by the spectrum adopted in the tests reduced the in-plane strength of the test slab panels by as much as 25 percent. Cumulative damage like cracking was responsible for the reduction.

(3) The application of the design service vertical (gravity) load reduced the in-plane ultimate strength of the test slab panels by about 15 percent.
(4) The test slab panels with in-plane loading lost their initial stiffness after the formation of a crack at the clamped edge. The load at which this crack formed was approximately 30 percent of the ultimate load.

(5) The application of the design service vertical load caused two cracks on the top surface of the test slab panels, one along the slab-wall junction, and the other along the loading line. The vertical load promoted the development of cracks, and cracking in low load levels made the transfer of forces from concrete to reinforcing bars more gradual.

(6) The deflection in the post ultimate load range was controlled by the opening and closing of the major crack, while the breaking of reinforcing bars at the major crack caused the test slab panels to fail.

(7) The effect of cyclic loading or combined in-plane and out-of-plane loadings on ductility of the test slab panels was minimal.

(8) Because of insufficient available data regarding the performance of reinforcing bars which control the ductility of floor slabs, it is tentatively suggested that a high degree of ductility should not be expected unless the slabs are reinforced more heavily than the test slab panels.
Analytical Findings and Proposals for Practical Designs

(1) The in-plane ultimate strength of a floor slab is reached when the slab fails either by flexure or by shear. In the flexural mode of failure, a major crack develops along the boundary between the column and middle strips of the slab.

(2) In the shear mode, on the other hand, the slab reaches the ultimate load when a diagonal crack separates a triangular portion from the remainder of the panel.

(3) The in-plane flexural strength of the slab can be predicted by the theoretical flexural capacity of the critical section of the slab. The detailed procedure is described in Section 5.3.3.

(4) The standard ACI Code Provision (11.5.6.2) gives an accurate estimate of the in-plane shear strength of the slab.

(5) The in-plane stiffness of the floor slab can be evaluated by representing the slab as a deep beam considering both flexural and shear deformations. The use of nominal moment of inertia and shear area is appropriate when computing the initial in-plane stiffness regardless of geometry of the slab. (Section 5.4.1)

(6) The deep beam analogy is also valid when the in-plane stiffness of the floor slab in the post-elastic range is determined. A reduction factor is incorporated into the
computation in order to reflect the stiffness degradation in this range. (Section 5.4.2)

(7) A procedure to compute the reduction factor is proposed. (Section 5.4.3) A reduction factor at a given load level can be estimated on the basis of the curvature at the critical section and an empirical hinge length.

Effects of Floor Slab In-Plane Deformation on Seismic Building Response

(1) The stiffness of floor slabs and vertical elements, walls and frames, can significantly affect the base shear distribution to the vertical elements. The rigid slab assumption underestimated the base shear distributed to flexible frames by as much as 75 percent.

(2) Nonlinear action of the floor slabs also altered the lateral load distribution, causing the frames to take base shear approximately twice as large as that computed in the elastic analysis.

(3) Shear force applied to the floor slabs was very small and practically negligible. Bending moment, however, was significant particularly at the mid-spans of the building, reaching the specified maximum moment level in many stories.

(4) The distribution of lateral force applied to each story was considerably triangular. The distribution
of lateral force in each vertical element, however, did not follow the triangular distribution. The change in the relative story stiffness of the vertical elements at each story level caused this complexity.

Future Research Needed

(1) The in-plane behavior of floor slabs under cyclic loadings must be investigated further. It is known that the load-deflection relationship of slab panels depends heavily on the loading history. A different loading spectrum very possibly would affect the ultimate strength as well as the stiffness degradation.

(2) The in-plane behavior of floor slabs under combined in-plane and out-of-plane loadings is also a subject for further study. The design service vertical load reduced the ultimate resistance but did not change the general pattern of stiffness degradation or behavior in post-ultimate load regions. The intensity of the vertical load can yet be a major parameter to control the in-plane behavior of floor slabs. Although the critical section was located at the boundary between the column and middle strips in the test slab panels, a larger vertical load
or, alternatively, more reinforcement at this boundary may shift the critical section to the center line of the span where out-of-plane bending moment is the greatest.

(3) Future study should be extended to investigate the interaction of floor slabs with supporting elements, that is, the combined effect of frame and diaphragm actions. When floor slabs are supported by columns, lateral load applied to the floor system induces both in-plane and out-of-plane deformations in the slabs. The in-plane characteristics of these slabs may be greatly affected by the out-of-plane bending. If a floor slab is supported by a wall on one edge and by columns on the other, warping can take place in the slab because of the difference between the rotational stiffnesses of the two vertical members. The combined effect of warping and in-plane force on the in-plane characteristics of the floor slab may also be significant. To examine this combined effect, experiment must be performed by using subassemblies of floor systems rather than isolated slab panels.
APPENDIX

The proportionality constant $\lambda$ used in the flow rule can be derived as follows:

From equations 4-17 and 4-18:

$$
\begin{align*}
\text{e} \frac{d E}{d} &= \text{a} \frac{d E}{d}, \\
\text{s} \frac{d E}{s} &= \text{a} \frac{d E}{s} - \lambda \frac{\partial f}{\partial \sigma_d} \\
\text{e} \frac{d E}{e} &= \text{a} \frac{d E}{a} - \lambda \frac{\partial f}{\partial \tau_s} \\
\end{align*}
$$

(A-1) (A-2)

The elastic strain increments, $\text{e} \frac{d E}{d}$ and $\text{e} \frac{d E}{s}$ can be related to the stress increments, $\text{d} \sigma_d$ and $\text{d} \tau_s$ as:

$$
\begin{align*}
\text{d} \sigma_d &= \text{D} \frac{e}{d} \left( \text{d} \varepsilon - \lambda \frac{\partial f}{\partial \sigma_d} \right) \\
\text{d} \tau_s &= \text{D} \frac{e}{s} \left( \text{d} \varepsilon - \lambda \frac{\partial f}{\partial \tau_s} \right) \\
\end{align*}
$$

(A-3) (A-4)

Substituting equations A-3 and A-4 to equation 4-20:

$$
\text{d} f = \frac{\partial f}{\partial \sigma_d} \text{D} \frac{e}{d} \left( \text{d} \varepsilon - \lambda \frac{\partial f}{\partial \sigma_d} \right) + \frac{\partial f}{\partial \tau_s} \text{D} \frac{e}{s} \left( \text{d} \varepsilon - \lambda \frac{\partial f}{\partial \tau_s} \right) = 0
$$

(A-5)

and

$$
\lambda \left\{ \left( \frac{\partial f}{\partial \sigma_d} \right)^2 + \left( \frac{\partial f}{\partial \tau_s} \right)^2 \right\} = \text{D} \frac{e}{d} \left( \frac{\partial f}{\partial \sigma_d} \right) \text{d} \varepsilon + \text{D} \frac{e}{s} \left( \frac{\partial f}{\partial \tau_s} \right) \text{d} \varepsilon
$$

Then:

$$
\lambda = \frac{\text{D} \frac{e}{d} \left( \frac{\partial f}{\partial \sigma_d} \right) \text{d} \varepsilon + \text{D} \frac{e}{s} \left( \frac{\partial f}{\partial \tau_s} \right) \text{d} \varepsilon}{\left\{ \left( \frac{\partial f}{\partial \sigma_d} \right)^2 + \left( \frac{\partial f}{\partial \tau_s} \right)^2 \right\}^{1/2}}
$$

(4-21)
Symbols used in the text are defined where they first appear. A summary of frequently used symbols is presented below for convenience. Symbols with dual meanings are listed twice.

\[ A \] shear area of equivalent beam
\[ A_e \] effective shear area
\[ A_s \] area of steel
\[ A_v \] area of reinforcement within a distance \( s \) (ACI Code)
\[ A_1, A_2 \] areas of steel placed in 1 and 2 directions and intersecting interface
\[ (AG)_e \] equivalent shear stiffness
\[ [B] \] matrix relating centroidal displacements to concrete strains
\[ [B_b] \] matrix relating centroidal displacements to truss element's strain
\[ b_w \] web width (ACI Code)
\[ [C] \] viscous damping matrix
\[ [D] \] constitutive matrix
\[ D \] dead load
\[ [D_e] \] elastic constitutive matrix
\[ [D_{ep}] \] elasto-plastic constitutive matrix
\[ D_{11}^e, D_{22}^e \] coefficients of elastic constitutive matrix
d \quad \text{distance from extreme compression fiber to centroid of tension reinforcement (ACI Code)}

E \quad \text{modulus of elasticity of concrete}

E_b \quad \text{modulus of elasticity of truss element}

E_{s1}, E_{s2} \quad \text{modulus of elasticity of steel in 1 and 2 directions}

(EI)_e \quad \text{equivalent flexural stiffness}

e \quad \text{strain in concrete}

f \quad \text{yield function}

f_c \quad \text{concrete stress}

f_c' \quad \text{compressive strength of concrete (ACI Code)}

f_y \quad \text{yield strength of nonprestressed reinforcement (ACI Code)}

h \quad \text{centroidal distance of two triangular elements}

h_1, h_2 \quad \text{legs from centroid of element 1 and 2 to interface}

I \quad \text{moment of inertia of equivalent beam}

I_e \quad \text{effective moment of inertia}

[K] \quad \text{element stiffness matrix (Chap. IV)}

[K] \quad \text{stiffness matrix (Chap. VI)}

k_d \quad \text{normal spring constant}

k_s \quad \text{shear spring constant}

L \quad \text{live load}

\ell \quad \text{length of truss element (Chap. IV)}
\( \ell \)  
span length (Chap. V)

\( \ell_1 \)  
length of projecting part in equivalent beam  
(Chap. V)

\( \ell_1, \ell_2 \)  
distances between two centroids projected on  
local 1 and 2 axes (Chap. IV)

\( M \)  
external moment applied at edge

\( [M] \)  
matrix relating element displacements to  
relative displacements (Chap. IV)

\( [M] \)  
mass matrix (Chap. VI)

\( m_1, m_2, n_1, n_2 \)  
coefficients of transformation matrix

\( m_3 \)  
direction cosine of orthogonal steel

\( n_3 \)  
direction sine of orthogonal steel

\( P \)  
applied in-plane load

\( \{ P \} \)  
external force vector

\( [Q] \)  
matrix relating centroidal displacements to  
element displacements

\( [R] \)  
transformation matrix from global coordinates  
to local coordinates

\( s \)  
spacing of shear reinforcement (ACI Code)

\( t \)  
slab thickness

\( \{ U \} \)  
vector of element displacements in global  
coordinates

\( \{ \bar{U} \} \)  
vector of element displacements in local coordinates

\( \{ U_1 \} \)  
vector of centroidal displacements

\( \{ u \} \)  
displacement vector
\{u\} \quad \text{velocity vector}

\{\ddot{u}\} \quad \text{acceleration vector}

u_g \quad \text{ground displacement}

V_b \quad \text{total potential energy functional of truss element}

V_c \quad \text{total potential energy functional of triangular element (Chap. IV)}

V_c \quad \text{nominal shear strength provided by concrete (ACI code, Chap. V)}

V_s \quad \text{total potential energy functional of smeared triangular element (Chap. IV)}

V_s \quad \text{nominal shear strength provided by shear reinforcement (ACI Code, Chap. V)}

W \quad \text{design load of floor slab}

W, W_1, W_2 \quad \text{arbitrary point at interface}

x_g, y_g \quad \text{coordinates of center of gravity of triangular element}

\alpha \quad \text{degradation factor (Chap. V)}

\alpha, \beta \quad \text{Rayleigh damping coefficients}

\delta \quad \text{deflection along loading line in strength test}

\{\delta\} \quad \text{vector of relative displacements}

\delta_a, \delta_s \quad \text{edge deflections in stiffness test under anti-symmetrical and symmetrical loadings (Chap. III)}

\delta_f, \delta_s \quad \text{flexural and shear deflections in equivalent beam calculation (Chap. V)}

\delta_1, \delta_2 \quad \text{relative displacements measured with respect to local 1 and 2 axes}

\{\varepsilon\} \quad \text{strain vector}
\{d\varepsilon\} \quad \text{vector of strain increments}
\{d\varepsilon_e\} \quad \text{vector of elastic strain increments}
\{d\varepsilon_p\} \quad \text{vector of plastic strain increments}
\varepsilon_b \quad \text{strain in truss element}
\varepsilon_d, \varepsilon_s \quad \text{normal and shear strain coefficients in strain vector}
\varepsilon_1, \varepsilon_2 \quad \delta_1/l_1 \text{ and } \delta_2/l_2
\varepsilon_t \quad \text{cracking strain}
\Theta \quad \text{end rotation in strength test}
\Theta_a, \Theta_s \quad \text{wall rotations in stiffness test under anti-symmetrical and symmetrical loadings}
\Theta_f \quad \text{end rotation due to flexure in equivalent beam calculation}
\lambda \quad \text{proportionality constant used in flow rule}
\nu \quad \text{poisson's ratio}
\xi \quad \text{vertical deflection in strength test}
\xi_1, \xi_2 \quad \text{viscous damping coefficients}
\{\sigma\} \quad \text{stress vector}
\sigma_b \quad \text{stress in truss element}
\sigma_p \quad \text{yield stress in uniaxial loading}
\sigma_t \quad \text{tensile strength of concrete}
\sigma_y \quad \text{compressive strength of concrete}
\{d\sigma\} \quad \text{vector of stress increments}
\( \tau_p \)  
yield stress in pure shear

\( \phi \)  
curvature at critical section

\( \omega \)  
circular frequency
Table 3.1 Dimensions of Test Specimen

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<thead>
<tr>
<th>Items</th>
<th>Dimensions</th>
</tr>
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<tr>
<td></td>
<td>(mm)</td>
</tr>
<tr>
<td>Center-to-Center Span Length</td>
<td>1630</td>
</tr>
<tr>
<td>Exterior Panels</td>
<td>2030 x 2440</td>
</tr>
<tr>
<td>Interior Panel</td>
<td>1030 x 2440</td>
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<tr>
<td>Slab Thickness</td>
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<tr>
<td>Beam Width</td>
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<tr>
<td>Beam Depth</td>
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<tr>
<td>(projecting depth)</td>
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<tr>
<td>Column Length</td>
<td>280</td>
</tr>
<tr>
<td>(from the mid-plane of the slab)</td>
<td></td>
</tr>
<tr>
<td>Column Cross Section</td>
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</tr>
<tr>
<td>Wall Thickness</td>
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<tr>
<td>Wall Length</td>
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</tr>
<tr>
<td>Total Wall Height</td>
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Table 3.2 Selected Reinforcing Bar Sizes

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<tr>
<td>Column Longitudinal Bars</td>
<td>#3</td>
</tr>
<tr>
<td>Column Stirrups</td>
<td>D2.0</td>
</tr>
<tr>
<td>Beam Longitudinal Bars</td>
<td>D2.0, D3.0</td>
</tr>
<tr>
<td>Beam Stirrups</td>
<td>W14</td>
</tr>
<tr>
<td>Wall Reinforcing Bars</td>
<td>D5.0, #3, #4</td>
</tr>
</tbody>
</table>
Table 3.3 Design Detail of Concrete Slab

<table>
<thead>
<tr>
<th>Strip</th>
<th>Sign</th>
<th>( A_s ) Required ( (\text{mm}^2) )</th>
<th>( A_s ) Provided</th>
<th>Required Steel Ratio</th>
<th>Over Reinforcement Ratio</th>
<th>Over-Strength Ratio</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>Column</td>
<td>Negative 58* D2.0 x 6</td>
<td></td>
<td>0.0018</td>
<td>1.3</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Positive 58* D2.5 x 2 + D2.0 x 4</td>
<td></td>
<td>0.0018</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>Negative 58* D2.0 x 6</td>
<td></td>
<td>0.0018</td>
<td>1.3</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>Interior</td>
<td>Negative 61 D2.0 x 6</td>
<td></td>
<td>0.0019</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>Negative 58* D2.5 x 2 + D2.0 x 4</td>
<td></td>
<td>0.0018</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>120 D3.0 x 7</td>
<td></td>
<td>0.0038</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>Column</td>
<td>Negative 58* D2.0 x 6</td>
<td></td>
<td>0.0018</td>
<td>1.3</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>Positve</td>
<td>58* D2.0 x 6</td>
<td></td>
<td>0.0018</td>
<td>1.3</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>Negative 29* D2.0 x 2</td>
<td></td>
<td>0.0018</td>
<td>1.1 ( \sigma )</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>Interior</td>
<td>Positive 29* D2.0 x 2</td>
<td></td>
<td>0.0018</td>
<td>1.1 ( \sigma )</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>Interior</td>
<td>Negative 67 D2.0 x 6</td>
<td></td>
<td>0.0021</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>Positive 58* D2.0 x 5</td>
<td></td>
<td>0.0018</td>
<td>1.1</td>
<td>1.9</td>
</tr>
</tbody>
</table>

* Controlled by temperature requirement

** Based on flexural resistance
Table 3.4 Design Detail in Concrete Beam

<table>
<thead>
<tr>
<th>Region</th>
<th>Required Area</th>
<th>Actual Area</th>
<th>Ratio (Actual/Required)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Moment</td>
<td>54.3 mm² (0.0842 in.²)</td>
<td>D3.0 x 3</td>
<td>1.07</td>
</tr>
<tr>
<td>(Around Columns)</td>
<td></td>
<td>(58 mm²)</td>
<td></td>
</tr>
<tr>
<td>Negative Moment</td>
<td>33.2 (0.0515)</td>
<td>D2.0 x 3</td>
<td>1.17</td>
</tr>
<tr>
<td>(Around Walls)</td>
<td></td>
<td>(39 mm²)</td>
<td></td>
</tr>
<tr>
<td>Positive Moment</td>
<td>27.9 (0.0432)</td>
<td>D2.0 x 3</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(39 mm²)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.5 Concrete Mix Proportions

<table>
<thead>
<tr>
<th></th>
<th>For 1.0 m$^3$ (1.3 cu.yd)</th>
<th>Concrete</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>27.6 MPa</td>
<td>34.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4,000 psi)</td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5,000 psi)</td>
</tr>
<tr>
<td>Type 1 Portland Cement</td>
<td>330 kg (730 lbs.)</td>
<td>400 kg (880 lbs.)</td>
<td></td>
</tr>
<tr>
<td>6.4 mm Crushed Limestone</td>
<td>770 kg (1700 lbs.)</td>
<td>770 kg (1700 lbs.)</td>
<td></td>
</tr>
<tr>
<td>Concrete Sand</td>
<td>1000 kg (2300 lbs.)</td>
<td>1000 kg (2300 lbs.)</td>
<td></td>
</tr>
<tr>
<td>Total Water</td>
<td>0.23 m$^2$ (60 gals.)</td>
<td>0.23 m$^3$ (60 gals.)</td>
<td></td>
</tr>
<tr>
<td>WRDA-19 Plastisizer</td>
<td>0.012 m$^3$ (110 oz.)</td>
<td>0.015 m$^3$ (130 oz.)</td>
<td></td>
</tr>
<tr>
<td>Water Cement Ratio</td>
<td>0.68</td>
<td>0.57</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.6 Concrete Compressive Strength (7 Day and 28 Day Tests)

<table>
<thead>
<tr>
<th>Type</th>
<th>Slump (mm)</th>
<th>7-Day Strength (MPa)</th>
<th>28-Day Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.6 MPa (4,000 psi)</td>
<td>Batch 1 110</td>
<td>23.2</td>
<td>27.7</td>
</tr>
<tr>
<td></td>
<td>Batch 2 120</td>
<td>24.0</td>
<td>28.8</td>
</tr>
<tr>
<td>34.5 MPa (5,000 psi)</td>
<td>Batch 1 110</td>
<td>29.9</td>
<td>36.1</td>
</tr>
<tr>
<td></td>
<td>Batch 2 130</td>
<td>26.8</td>
<td>33.1</td>
</tr>
</tbody>
</table>
Table 3.7 Mechanical Properties of Reinforcing Bars

<table>
<thead>
<tr>
<th>Size</th>
<th>Area (mm$^2$)</th>
<th>Yield Stress (MPa)</th>
<th>Yield Strain (m/m)</th>
<th>Ultimate Stress (MPa)</th>
<th>Ultimate Strain (m/m)</th>
<th>Modulus of Elasticity (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2.0</td>
<td>13.4</td>
<td>368</td>
<td>$1.93 \times 10^{-3}$</td>
<td>411</td>
<td>$78.3 \times 10^{-3}$</td>
<td>191</td>
</tr>
<tr>
<td>D2.5</td>
<td>17.2</td>
<td>609</td>
<td>$3.11 \times 10^{-3}$</td>
<td>668</td>
<td>$49.2 \times 10^{-3}$</td>
<td>196</td>
</tr>
<tr>
<td>D3.0</td>
<td>21.5</td>
<td>590</td>
<td>$2.72 \times 10^{-3}$</td>
<td>590</td>
<td>$62.5 \times 10^{-3}$</td>
<td>190</td>
</tr>
<tr>
<td>Type</td>
<td>Compressive Strength on Testing (MPa)</td>
<td>Tensile Strength Split Test (MPa)</td>
<td>Modulus of Elasticity (GPa)</td>
<td>Initial Poisson's Ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------------------------------</td>
<td>---------------------------------</td>
<td>---------------------------</td>
<td>------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.6 MPa (4,000 psi)</td>
<td>Batch 1 28.0</td>
<td>2.13</td>
<td>21</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Batch 2 29.0</td>
<td>2.40</td>
<td>22</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.5 MPa (5,000 psi)</td>
<td>Batch 1 37.3</td>
<td>3.57</td>
<td>30</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Batch 2 35.0</td>
<td>4.90</td>
<td>30</td>
<td>0.20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.9(a) Designation of Test Program
(Five character alphanumeric code)

<table>
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<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>slab-on-beam floor system</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>Horizontal (in-plane) loading only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>Vertical (out-of-plane) load included</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>panel 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>panel 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>panel 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>a whole specimen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-5</td>
<td>SS</td>
<td>Stiffness test, Symmetrical loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SA</td>
<td>Stiffness test, Anti-symmetrical loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MN</td>
<td>Monotonic loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CY</td>
<td>Cyclic loading</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE**

**BHIMN**

slab-on-beam floor system
horizontal loading only
panel 1 is tested
under monotonic loading
Table 3.9(b) Sequence of Test Program

<table>
<thead>
<tr>
<th>B-1</th>
<th>Test Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH 6SS</td>
<td>Stiffness test, symmetrical loading</td>
</tr>
<tr>
<td>BH 6SA</td>
<td>Stiffness test, anti-symmetrical loading</td>
</tr>
<tr>
<td>BH 2MN</td>
<td>Strength test, monotonic loading without vertical load, Moment-to-shear ratio of 1.63m</td>
</tr>
<tr>
<td>BH 3MN</td>
<td>Strength test, monotonic loading without vertical load, Moment-to-shear ratio of 3.25m</td>
</tr>
<tr>
<td>BH 1CY</td>
<td>Strength test, cyclic loading without vertical load, Moment-to-shear ratio of 1.63m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B-2</th>
<th>Test Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH 6SS</td>
<td>Stiffness test, symmetrical loading</td>
</tr>
<tr>
<td>BH 6SA</td>
<td>Stiffness test, anti-symmetrical loading</td>
</tr>
<tr>
<td>BV 1MN</td>
<td>Strength test, monotonic loading with vertical load, Moment-to-shear ratio of 1.63m</td>
</tr>
<tr>
<td>BH 3CY</td>
<td>Strength test, cyclic loading without vertical load, Moment-to-shear ratio of 3.25m</td>
</tr>
<tr>
<td>BV 2CY</td>
<td>Strength test, cyclic loading with vertical load, Moment-to-shear ratio of 1.63m</td>
</tr>
</tbody>
</table>
Table 3.10 Stiffness Test Results

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>SAP IV</th>
<th>Beam Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B-1</td>
<td>B-2</td>
<td></td>
</tr>
<tr>
<td>Symmetrical loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$ (m/GN)</td>
<td>6.24</td>
<td>6.50</td>
<td>5.42</td>
</tr>
<tr>
<td>$\Theta_s$ (rad./GN)</td>
<td>1.17</td>
<td>1.25</td>
<td>1.08</td>
</tr>
<tr>
<td>Anti-symmetrical loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$ (m/GN)</td>
<td>11.60</td>
<td>12.50</td>
<td>11.90* (8.30)</td>
</tr>
<tr>
<td>$\Theta_a$ (rad./GN)</td>
<td>4.15</td>
<td>4.32</td>
<td>3.81* (2.42)</td>
</tr>
</tbody>
</table>

* With artificially stiff beams along the Boundaries
### Table 3.11  Vertical Reflection under Design Service Vertical Load

<table>
<thead>
<tr>
<th>Test</th>
<th>Scale #1 (mm)</th>
<th>Scale #2 (mm)</th>
<th>Scale #3 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BV 1MN</td>
<td>0.76</td>
<td>1.30</td>
<td>1.02</td>
</tr>
<tr>
<td>BV 2CY</td>
<td>0.76</td>
<td>0.89</td>
<td>1.52</td>
</tr>
<tr>
<td>Elastic Theory</td>
<td>0.42</td>
<td>1.17</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Table 3.12 Strength Test Results (Ultimate Resistance and Deflection)

<table>
<thead>
<tr>
<th>Test</th>
<th>Ultimate Load (kN)</th>
<th>Maximum Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>BH 2MN</td>
<td>120.0</td>
<td>- 88.5</td>
</tr>
<tr>
<td>BH 1CY</td>
<td>94.7</td>
<td>- 96.5</td>
</tr>
<tr>
<td>BH 3MN</td>
<td>56.9</td>
<td>- 38.7</td>
</tr>
<tr>
<td>BH 3CY</td>
<td>41.8</td>
<td>- 40.5</td>
</tr>
<tr>
<td>BV 1MN</td>
<td>102.0</td>
<td>- 89.8</td>
</tr>
<tr>
<td>BV 2CY</td>
<td>85.0</td>
<td>- 83.2</td>
</tr>
</tbody>
</table>
Table 3.13 Initial Stiffness in Strength Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Initial Stiffness (MN/m)</th>
<th>Experiment/Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH 2MN</td>
<td>218</td>
<td>0.663</td>
</tr>
<tr>
<td>BH 1CY</td>
<td>272</td>
<td>0.796</td>
</tr>
<tr>
<td>BH 3MN</td>
<td>166</td>
<td>0.790</td>
</tr>
<tr>
<td>BH 3CY</td>
<td>175</td>
<td>0.830</td>
</tr>
<tr>
<td>BV 1MN</td>
<td>222</td>
<td>0.675</td>
</tr>
<tr>
<td>BV 2CY</td>
<td>201</td>
<td>0.589</td>
</tr>
</tbody>
</table>
Table 3.14 Stiffness Change in Monotonic Loading Tests

<table>
<thead>
<tr>
<th></th>
<th>BH 2MN (MN/m)</th>
<th>BH 3MN (MN/m)</th>
<th>BV 1MN (MN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stiffness</td>
<td>218</td>
<td>166</td>
<td>222</td>
</tr>
<tr>
<td>Tangent stiffness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at ultimate resistance</td>
<td>33.8 (0.16)</td>
<td>22.0 (0.13)</td>
<td>0.0 (0.00)</td>
</tr>
<tr>
<td>Secant stiffness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at ultimate resistance</td>
<td>42.9 (0.20)</td>
<td>28.5 (0.17)</td>
<td>-</td>
</tr>
<tr>
<td>Tangent stiffness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>upon unloading</td>
<td>67.2 (0.31)</td>
<td>33.7 (0.20)</td>
<td>83.3 (0.38)</td>
</tr>
<tr>
<td>Tangent stiffness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in negative</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stable range</td>
<td>4.2 (0.019)</td>
<td>2.8 (0.017)</td>
<td>3.2 (0.015)</td>
</tr>
</tbody>
</table>
Table 3.15 Critical Displacement and Ductility in Strength Tests

<table>
<thead>
<tr>
<th></th>
<th>Critical Displacement Positive (mm)</th>
<th>Critical Displacement Negative (mm)</th>
<th>Ductility Positive</th>
<th>Ductility Negative</th>
<th>Total Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH 2MN</td>
<td>7.62 - 7.62</td>
<td>3.2</td>
<td>--</td>
<td>--</td>
<td>14.86</td>
</tr>
<tr>
<td>BH 1CY</td>
<td>6.75 - 6.30</td>
<td>2.1</td>
<td>2.0</td>
<td></td>
<td>13.05</td>
</tr>
<tr>
<td>BH 3MN</td>
<td>7.11 - 5.72</td>
<td>3.6</td>
<td>--</td>
<td>--</td>
<td>12.83</td>
</tr>
<tr>
<td>BH 3CY</td>
<td>4.60 - 5.42</td>
<td>2.6</td>
<td>2.5</td>
<td></td>
<td>10.02</td>
</tr>
<tr>
<td>BV 1MN</td>
<td>9.22* - 9.22*</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>18.44</td>
</tr>
<tr>
<td>BV 2CY</td>
<td>6.27 - 5.08</td>
<td>3.1</td>
<td>2.8</td>
<td></td>
<td>11.35</td>
</tr>
</tbody>
</table>

* No breaking of bars, "Critical Displacement" not reached
Table 4.1 Comparison of Proposed Model With SAP IV (Elastic Analysis)

<table>
<thead>
<tr>
<th>Load</th>
<th>Selected point (Fig.4.8)</th>
<th>Direction of Displacement</th>
<th>Proposed Model/SAP IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>x</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>x</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>1.03</td>
</tr>
<tr>
<td>Case 2</td>
<td>1</td>
<td>x</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>x</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>x</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>y</td>
<td>1.07</td>
</tr>
</tbody>
</table>
Table 5.1(a) Dimensions and Material Properties of Slab Panel in Effective Moment of Inertia and Shear Area Calculations

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>d</td>
<td>1630 mm</td>
</tr>
<tr>
<td>Length</td>
<td>( \ell )</td>
<td>1630 x mm (ASPECT RATIO)</td>
</tr>
<tr>
<td>Thickness</td>
<td>t</td>
<td>39.6 mm</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>E</td>
<td>22 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>( \nu )</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 5.1(b) Relative Beam Size of Slab Panel in Effective Moment of Inertia and Shear Area Calculations

<table>
<thead>
<tr>
<th>Relative Beam Size</th>
<th>Width x Depth (mm x mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0 0</td>
</tr>
<tr>
<td>1.43</td>
<td>68 96</td>
</tr>
<tr>
<td>1.61</td>
<td>81 114</td>
</tr>
<tr>
<td>2.29</td>
<td>118 166</td>
</tr>
<tr>
<td>2.71</td>
<td>136 192</td>
</tr>
</tbody>
</table>

* The beam size of the tested slab panels
Table 5.2 Effective Moment of Inertia and Shear Area

<table>
<thead>
<tr>
<th>ASPECT RATIO</th>
<th>RELATIVE BEAM SIZE</th>
<th>MOMENT OF INERTIA ($I_e$)</th>
<th>SHEAR AREA ($A_e$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>NOMINAL VALUES</td>
<td>(m$^4$)</td>
<td>(m$^2$)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.0141</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.01)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0142</td>
<td>0.0203</td>
<td>0.0232</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.01)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0142</td>
<td>0.0203</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.01)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0142</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.01)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>1.25</td>
<td>0.0142</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.01)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>1.50</td>
<td>0.0141</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0141</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>2.50</td>
<td>0.0141</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0141</td>
<td>0.0202</td>
<td>0.0231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
</tbody>
</table>
Table 5.3 Maximum Load and Failure Mode of Analyzed Slab Panels

<table>
<thead>
<tr>
<th>Case</th>
<th>Moment-to-Shear Ratio</th>
<th>Experiment Maximum Load</th>
<th>FEM Model Maximum Load</th>
<th>Failure Mode</th>
<th>Maximum Load Based On Flexural Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.63m (64 in.)</td>
<td>120 kN</td>
<td>116 kN</td>
<td>Flexural</td>
<td>125 kN</td>
</tr>
<tr>
<td>2</td>
<td>3.25m (128 in.)</td>
<td>56 kN</td>
<td>53 kN</td>
<td>Flexural</td>
<td>54 kN</td>
</tr>
<tr>
<td>3</td>
<td>∞</td>
<td></td>
<td>M = 153 kN x m</td>
<td>Flexural</td>
<td>M = 155 kN x m</td>
</tr>
<tr>
<td>4</td>
<td>0.812m (32 in.)</td>
<td></td>
<td>131 kN</td>
<td>Shear Failure</td>
<td>340 kN</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
<td>124 kN</td>
<td>Shear Failure</td>
<td>125 kN</td>
</tr>
<tr>
<td>6</td>
<td>1.63m (64 in.)</td>
<td></td>
<td>153 kN</td>
<td>Shear Failure</td>
<td>239 kN</td>
</tr>
<tr>
<td>7</td>
<td>1.63m (64 in.)</td>
<td></td>
<td>144 kN</td>
<td>Shear Failure</td>
<td>181 kN</td>
</tr>
<tr>
<td>8</td>
<td>1.63m (64 in.)</td>
<td></td>
<td>117 kN</td>
<td>Flexural</td>
<td>125 kN</td>
</tr>
<tr>
<td>4'</td>
<td>0.812m (32 in.)</td>
<td></td>
<td>398 kN</td>
<td>Flexural</td>
<td>332 kN</td>
</tr>
<tr>
<td>6'</td>
<td>1.63m (64 in.)</td>
<td></td>
<td>224 kN</td>
<td>Flexural</td>
<td>239 kN</td>
</tr>
<tr>
<td>7'</td>
<td>1.63m (64 in.)</td>
<td></td>
<td>175 kN</td>
<td>Flexural</td>
<td>181 kN</td>
</tr>
</tbody>
</table>
### Table 6.1 Sectional Properties and Maximum Moment Levels of Model Structure

<table>
<thead>
<tr>
<th></th>
<th>(I^4) (Moment of inertia)</th>
<th>(A^2) (Shear area)</th>
<th>Maximum moment level (MN (\times) m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab</td>
<td>3.63</td>
<td>1.08</td>
<td>2.94</td>
</tr>
<tr>
<td>Wall</td>
<td>5.79</td>
<td>1.60</td>
<td>24.5</td>
</tr>
<tr>
<td>Frame (1st story)</td>
<td>3.24</td>
<td>0.0265</td>
<td>-</td>
</tr>
<tr>
<td>Frame (2nd-7th story)</td>
<td>3.24</td>
<td>0.0414</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 6.2 Lumped Masses in Model Structure

<table>
<thead>
<tr>
<th>Span Story</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4 (half)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15.9</td>
<td>20.3</td>
<td>20.3</td>
<td>10.2</td>
</tr>
<tr>
<td>6</td>
<td>30.1</td>
<td>37.3</td>
<td>37.3</td>
<td>18.6</td>
</tr>
<tr>
<td>5</td>
<td>30.1</td>
<td>37.3</td>
<td>37.3</td>
<td>18.6</td>
</tr>
<tr>
<td>4</td>
<td>30.1</td>
<td>37.3</td>
<td>37.3</td>
<td>18.6</td>
</tr>
<tr>
<td>3</td>
<td>30.1</td>
<td>37.3</td>
<td>37.3</td>
<td>18.6</td>
</tr>
<tr>
<td>2</td>
<td>30.1</td>
<td>37.3</td>
<td>37.3</td>
<td>18.6</td>
</tr>
<tr>
<td>1</td>
<td>31.9</td>
<td>38.0</td>
<td>38.0</td>
<td>19.0</td>
</tr>
</tbody>
</table>

Unit: kg
(1) Linear elastic analysis
(2) Linear elastic analysis (rigid floor assumption)
(3) Non-linear analysis (floor slabs can behave non-linearly)
(4) Non-linear analysis (floor slabs and walls can behave non-linearly)

Table 6.3 Maximum Displacement, Acceleration, and Total Base Shear

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum Displacement</th>
<th>Maximum Acceleration</th>
<th>Total Base Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(m/sec$^2$)</td>
<td>(MN)</td>
</tr>
<tr>
<td>1</td>
<td>34.2 (5.00) 33.3 (4.78)</td>
<td>9.74 (4.98) 9.12 (5.00)</td>
<td>8.24 (4.78)</td>
</tr>
<tr>
<td>2</td>
<td>29.6 (4.72) 29.6 (4.72)</td>
<td>10.0 (4.72) 10.0 (4.72)</td>
<td>7.29 (4.72)</td>
</tr>
<tr>
<td>3</td>
<td>54.2 (5.04) 30.0 (5.04)</td>
<td>13.8 (2.34) 6.52 (4.86)</td>
<td>8.38 (5.04)</td>
</tr>
<tr>
<td>4</td>
<td>63.7 (5.08) 53.9 (5.10)</td>
<td>13.2 (2.34) 8.25 (5.04)</td>
<td>8.87 (5.04)</td>
</tr>
</tbody>
</table>

Number in parenthesis indicates the time when the maximum value is reached.

1 7th story middle
2 7th story end

Table 6.4 Base Shear in Walls and Frames

<table>
<thead>
<tr>
<th>Case</th>
<th>Wall (MN)</th>
<th>Frame 1 (MN)</th>
<th>Frame 2 (MN)</th>
<th>Frame 3 (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.16</td>
<td>0.322</td>
<td>0.462</td>
<td>0.515</td>
</tr>
<tr>
<td>2</td>
<td>3.34</td>
<td>0.119</td>
<td>0.119</td>
<td>0.119</td>
</tr>
<tr>
<td>3</td>
<td>2.43</td>
<td>0.555</td>
<td>0.931</td>
<td>1.09</td>
</tr>
<tr>
<td>4</td>
<td>2.05</td>
<td>0.795</td>
<td>1.18</td>
<td>1.33</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Story</th>
<th>Maximum shear in floor slab (MN)</th>
<th>Maximum moment in floor slab (MN x m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>7</td>
<td>0.0265</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.194</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.327</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.391</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.420</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.428</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0.422</td>
<td>-</td>
</tr>
</tbody>
</table>

* Reaching the specified maximum moment level
** Exceeding the specified maximum moment level
Fig. 3.2 Dimension and Supporting Condition of Test Specimen
Fig. 3.3 Reinforcement Detail in Concrete Slab and Beam
Fig. 3.4 Pedestal-Wall Connection
Fig. 3.5 Overall View of Test Setup
Fig. 3.6  Lateral Load Distribution Frame Placed for Strength Test
Fig. 3.7 Lateral Load Distribution Frame

UNIT: mm
Fig. 3.8 Lateral Loading Jack
Fig. 3.9 Inserts embedded in slab
Fig. 3.10 Vertical Loading Frame
Fig. 3.11 Loading Conditions and Measurement in Stiffness Test

Walls Allowed to Rotate, Columns Free to Slide
Fig. 3.12 Loading Condition and Measurement in Strength Test

UNIT: mm
Fig. 3.13 Testing Procedure in Strength Test
Fig. 3.14 Vertical Deflection Measurement
Fig. 3.15 Strain Gages in Slab
Fig. 3.16 Rosette Gages on Slab
Fig. 3.17 Loading Spectrum for Cyclic Loading Tests
Fig. 3.18 Strain Distribution in Slab Under Design Service Vertical Load

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Fig. 3.18(Continued) Strain Distribution in Slab
Under Design Service Vertical Load

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Fig. 3.19 Crack Pattern in Slab Under Design Service Vertical Load
Fig. 3.20 Load-Deflection Curve of BH2MN, Monotonic Loading without Vertical Load
Fig. 3.21 Load-Deflection Curve of BHICY, Cyclic Loading without Vertical Load
Fig. 3.21(continued)
Load Deflection Curve of BHICY, Cyclic Loading Without Vertical Load
Fig. 3.22 Load-Deflection Curve of BH3MN, Monotonic Loading without Vertical Load
Fig. 3.23 Load-Deflection Curve of BH3CY, Cyclic Loading without Vertical Load
Fig. 3.23(continued) Load-Deflection Curve of BH3CY, Cyclic Loading without Vertical Load
Fig. 3.24 Load-Deflection Curve of BVIMN, Monotonic Loading with Vertical Load
Fig. 3.25 Load-Deflection Curve of BV2CY, Cyclic Loading with Vertical Load
Fig. 3.25 (continued) Load-Deflection Curve of BV2CY, Cyclic Loading with Vertical Load
Fig. 3.26 Measured Stiffnesses in Cyclic Loading Strength Tests
Fig. 3.27 Stiffness Degradation in BHICY
Fig. 3.28 Stiffness Degradation in BH3CY
Fig. 3.29  Stiffness Degradation in BV2CY
Fig. 3.30 Crack Pattern of BH2MN

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Fig. 3.31 Crack Pattern of BHICY

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Fig. 3.32 Crack Pattern of BH3MN
Fig. 3.33 Crack Pattern of BH3CY
Fig. 3.34 Crack Pattern of BVIMN
Fig. 3.35 Crack Pattern of BV2CY
Fig. 3.36 Load-Vertical Deflection Curve of BV1MN
Fig. 3.37(a) Load-Vertical Deflection Curve of BV2CY (Scale #1)
Fig. 3.37(b) Load-Vertical Deflection Curve of BV2CY (Scale #2)
Fig. 3.37(c) Load-Vertical Deflection Curve of BV2CY (Scale #3)
Fig. 3.38 Finite Element Model of Test Slab
Fig. 4.1 Two Rigid Triangular Elements

Fig. 4.2 Displacement, Rotation, Force, and Moment of Triangular Elements
Fig. 4.3 Relative Displacement of Two Triangular Elements

Fig. 4.4 Normal and Shear Springs Along Interface
Fig. 4.5 Orthogonal Reinforcement
(Smeared Triangular Elements)

Fig. 4.6 Truss Element
Fig. 4.7 Iteration Procedure (Initial Stress Method)
Fig. 4.8 Example Problem, Plate With a Clamped Edge

Fig. 4.9 Discretization of Plate (Example Problem)
Fig. 4.10 Analyzed Slab Panel (BH2MN, BH3MN, BHICY)
Fig. 4.11 Finite Element Model of Slab Panel
Fig. 4.12 Analytical Load-Deflection Curves (BH2MN, BH3MN)
Fig. 4.13 Analytical Load-End Rotation Curves (BH2MN, BH3MN)
Fig. 4.14 Analytical Crack Patterns (BH2MN)
Fig. 4.14(continued) Analytical Crack Patterns (BH2MN, BH3MN)

3. $P = \text{ULTIMATE \ BH2MN}$

4. $P = 36\, \text{kN}$ \ BH3MN
Fig. 4.14 (continued) Analytical Crack Pattern (BH3MN)

P = 51kN

BH3MN

P = ULTIMATE BH3MN

Fig. 4.14 (continued) Analytical Crack Pattern (BH3MN)
Fig. 4.15 Analytical Load-Deflection Curve of BH2MN Under Load Reversal
EXPERIMENT

POSITIVE LOADING  

NEGATIVE LOADING

MODEL

BH2MN

Fig. 4.16 Analytical Crack Pattern of BH2MN Under Load Reversal
Fig. 4.17 Analytical Load-Deflection Curve of BHICY
Fig. 5.1 Equivalent Beam Calculation in Stiffness Tests
Fig. 5.2 Procedure to Compute Effective Moment of Inertia and Shear Area
Fig. 5.3 Loading and Boundary Conditions of Analyzed Slab Panels
FLEXURAL FAILURE

SHEAR FAILURE

$V_S = \frac{A_V \times f_y \times d}{s}$

Fig. 5.4 Failure Mode of Analyzed Slab Panels
Neutral axis by $T_t + C + T_c = 0$

Fig. 5.5 Procedure to Compute Flexural Capacity of Critical Section
Fig. 5.6 Strain Distribution of Slab Panels in Elastic Range
Fig. 5.7 Equivalent Beam Calculation in Strength Tests
Fig. 5.8 Moment - Equivalent Moment of Inertia Relationship
Fig. 5.9 Moment - Equivalent Shear Area Relationship
Fig. 5.10 Proportion of Flexural and Shear Deformations in Slab Panels
Fig. 5.11 Proportion of Flexural and Shear Deformations in BH2MN and BH3MN (plotted against deflection level)
Fig. 5.12. Elastic and Hinge Rotations in Equivalent Beam
(Used for the calculation of degraded stiffness)
Fig. 5.13 Moment-Curvature Relationship at Critical Sections
Fig. 5.14 Hinge Length in Slab Panels
Fig. 5.15 Analytical Moment – Equivalent Moment of Inertia Relationships
Fig. 5.16 Proportion of Flexural and Shear Deformations in BHICY
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Fig. 5.18 Comparison of StiffnessDegradation
Between Monotonic and Cyclic Loading Tests
Fig. 5.19 Proportion of Flexural and Shear Deformations in Slab Panels with Vertical Load
Fig. 5.20 Degradation of Equivalent Moment of Inertia in Slab Panels with Vertical Load
Fig. 6.1 Dimensions of Model Structure
Fig. 6.2 Formulation of Stiffness Matrix
Fig. 6.3 Hysteretic Moment-Rotation Model for Floor Slabs and Walls
Fig. 6.4 Computation of Shear Area of Equivalent Beam
Fig. 6.5 Lateral Force Distribution in Vertical Elements
Fig. 6.6 Lateral Force Distribution in Floor Slabs

(1) Elastic

(3) Slab nonlinear

(4) Slab, wall nonlinear
Fig. 6.7(a) Time-Displacement (7th floor middle) Relationship in Elastic Analysis

Fig. 6.7(b) Time-Base Shear (wall) Relationship in Elastic Analysis

Fig. 6.7(c) Time-Base Shear (frame 3) Relationship in Elastic Analysis
5.0MN \times m

7th floor Moment

0

4

8 time(sec)

Fig. 6.7(d) Time-Maximum Moment (7th floor) Relationship in Elastic Analysis

5.0MN \times m

4th floor Moment

0

4

8 time(sec)

Fig. 6.7(e) Time-Maximum Moment (4th floor) Relationship in Elastic Analysis

5.0MN \times m

1st floor Moment

0

4

8 time(sec)

Fig. 6.7(f) Time-Maximum Moment (1st floor) Relationship in Elastic Analysis
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