The Present Status of Application of the Plastic Concepts in Structural Design

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The different approaches of recognizing plastic behavior in the design of steel structures are reviewed and the design requirements that assure successful utilization of this behavior in practice are described. Partial recognition of plastic behavior is now an important part of the allowable-stress design provisions. Continuous beams and low-rise building frames are routinely designed by the plastic method, which represents a full recognition of plastic response. The multistory frames, because of the effect of overall instability, the design must be based on the stability limit load, not the plastic limit load. This complicates significantly the design process and may have affected wider adoption of the method. Inelastic behavior, however, is extensively utilized in the design of both low- and high-rise structures resisting seismic ground shaking. Some recent advances in this area are described.

Introduction

With the exception of very slender structures and pin-connected structures, the behavior of a structure in the plastic range is an important design consideration, whether from load-carrying or energy-absorption capacity point of view. Plastification of cross section and moment redistribution are the two important phenomena associated with inelastic response and are recognized either partially or completely in some of the current design specifications. Partial recognition of plastic behavior is included in some allowable-stress design provisions. When both phenomena are fully recognized, the design methodology is generally referred to as the plastic method and the basis of design is the plastic limit load. To achieve this limit load, two basic requirements must be met: (1) no premature failure due to member instability and/or overall instability can occur and (2) all loads are applied proportionally. The problem of member instability can be taken into account in the member selection phase of the design process. The effect of overall instability, however, tends to change the failure mode of the structure from plastic collapse to inelastic instability. The maximum strength of the structure is represented in the latter case by the stability limit load, which can be determined only by performing a second-order analysis. When the loads are applied non-proportionally, the problem of incremental collapse becomes a design concern and the shakedown load is sometimes used as the limit load. An example of this is the autostress design for continuous bridges (10, 11).

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Plastic behavior is recognized in four different ways in the current design practice:

(1) Partial recognition in allowable stress design

(2) Full recognition in plastic design using the plastic limit load as the basis of design

(3) Full recognition when design is based on the stability limit load

(4) For the case of variable repeated loading, limited moment redistribution is recognized when design is based on the shakedown load.

In designing structures subjected to seismic forces the major concern is energy absorption and the limit state is usually not defined in terms of load. The important concept is to utilize the plastic strength of the structural elements to achieve the good energy absorption capacity. In this connection the term "inelastic design", rather than plastic design is often used. At the present time, approaches (1) and (2) are widely used when the design requirements are met (2,6), and almost all the seismic structures are designed with consideration of inelastic behavior. The status of application of all these approaches is discussed in this paper, starting with a description of the general requirements common to all.

Requirements for Utilizing Plastic Strength in Design

As mentioned previously, premature failure of individual members must be avoided in order to allow plastic hinges to form at the critical sections and to achieve moment redistribution. The latter requires certain amount of rotate capacity available at the hinges. The most important considerations for members subjected to bending moment are local buckling and lateral buckling, and design requirements are necessary to insure not only the formation of plastic hinges but also sufficient rotation capacity. The local buckling requirement is usually given as the maximum permissible width-to-thickness ratios of the component plates and the lateral buckling requirement is defined in terms of the unbraced length. In setting the design requirements, the amount of rotation capacity necessary to achieve either partial or full moment redistribution must be carefully considered.

Utilization of Plastic Strength in Allowable-Stress Design

Plastic behavior is recognized partially in specifications with allowable-stress design provisions. Section 1.5.1.4.1 of the AISC Specifications is an example in which both cross-sectional plastification and moment redistribution are utilized. A 10 percent increase of the allowable bending stress from 0.6Fy to 0.66Fy reflects the increase in bending moment capacity from the yield moment My to the plastic moment Mp. Partial moment redistribution is recognized by permitting a 10 percent reduction in the maximum negative moment in the design of continuous beams and frames. It is, however, required to increase the maximum positive moment by 10 percent of the average negative
moments. The limiting width-to-thickness ratios of the flange and the web of WF members are $b_f/2t_f = 65\sqrt{F_y}$ and $d/t_w = 640/\sqrt{F_y}$, respectively (where $b_f$ is the flange width, $t_f$ the flange thickness, $d$ the overall depth of section and $t_w$ the web thickness). For A36 steel, the ratios are 10.8 and 10.7. Two limiting unbraced lengths are specified: $76b_f/y$ and $20,000/(d/bft_f)F_y$. Both the limiting width-to-thickness ratios and unbraced lengths are more liberal than those specified for plastic design, because the design permits only a partial moment redistribution. Although the basic concepts behind these provisions appear to be quite reasonable, the adequacy of these limiting values for structures other than continuous beams has not been fully established. The provisions must be viewed as empirical and the real margin of safety of structures is difficult to assess.

It is interesting to note that continuous beams designed by the allowable-stress method with the increased allowable stress and the 10 percent moment redistribution may be considerably lighter than those designed by the plastic method (22).

Design Based on Plastic Limit Load

In conventional plastic design, it is assumed that sufficient number of plastic hinges develop so that, at the ultimate state, the structure would deform as a kinematic mechanism. The various limiting values for the width-to-thickness ratios and bracing spacing were selected to insure that all hinges can rotate sufficiently to reach strain hardening. This is a very severe design requirement. The limiting values for $b_f/2t_f$ and $d/t_w$ are 8.5 and 69 for A36 steel, which are considerably smaller than the values discussed previously. The required bracing spacing depends on the bending moment distribution near the hinge. The requirements are also more restrictive than those specified in the allowable-stress design.

Despite these severe design requirements, experience has shown that the plastic method can provide economical designs for a wide range of structures with a consistent margin of safety. The first structure designed plastically in the U.S. was in 1957. Although the plastic concept is now widely accepted by the engineering profession, it has not been so extensively applied. Two possible reasons are: (1) the high cost of fabricating and erecting moment connections and (2) the more economic design that can sometimes be achieved by allowable-stress design, as mentioned previously.

Design Based on Stability Limit Load

As indicated before, the strength of multistory structure may be significantly reduced because of the effect of overall instability. Figure 1 shows the lateral load vs. drift relationships of a frame subjected to proportionally increasing gravity and lateral loads, the proportionality constant being equal to $\alpha$. Two types of frame analysis can be performed. The first-order analysis in which the effect of instability or $P$-$\Delta$ moment is ignored, and the second-order analysis in which this effect is included. If the frame is perfectly elastic, the first-order analysis gives a linear relationship, shown as curve (a).
The second-order analysis leads to the non-linear load vs. drift curve (b) which approaches the elastic stability limit load of the frame when the drift becomes very large (equal to infinity in theory). When the effect of yielding is included in the analysis, the load vs. drift relationships shown as curves (c) and (d) in Fig. 1 are obtained. The first-order curve becomes horizontal at the plastic limit load. This is the load at which the frame will deform continuously as a kinematic mechanism if the effect of deformation and the associated P-Δ moment is not present. The "real" behavior of the structure is closely represented by curve (d) obtained from the second-order elastic-plastic analysis in which the combined effects of yielding and P-Δ moment are taken into account. The peak of the curve corresponds to the inelastic stability limit load (or simply the stability limit load), and it usually does not coincide with the formation of plastic mechanism.

The stability limit load represents the best estimate of the ultimate strength of a frame, and can serve as a rational basis of design. A number of empirical formulas and procedures have been proposed to account for the affect of strength reduction due to the P-Δ.

For tall buildings, it appears that the best approach in design would be to use a second-order computer program to calculate directly the stability limit load (14, 17). Such programs have been available for sometime (13, 20). One of these, referred to as SOFRAN-LIN (Second-Order FFrame ANalysis - Load INcrement), is described briefly here. The program is based on the slope-deflection equations which govern the member behavior under combined axial force and bending moment. The member stiffness are modified to take into account the effects of axial force and yielding. Concentrated plastic hinges are assumed to form at the critical sections of the members. The plastic moment values are determined as recommended in Ref. 1. The program repeatedly solves the frame until the structural response converges. If the user specifies a series of proportionally increasing gravity and wind loads, the load-drift curve of the frame under proportional loads can be traced up to the maximum values of the two loads.

With the aid of the various available computer programs, ultimate strength (based on the stability limit load) design may be carried out in three steps:

1. Preliminary analysis and selection of member sizes Computer program to be used: PDUF (Preliminary Design of Unbraced Frames)

2. One-story subassemblage analysis to check the member sizes Computer program to be used: SMOA (Subassemblage Method of Unbraced Frames)

3. Overall frame analysis Computer program to be used: SOFRAN-LIN

Within each step, certain iterative calculations may be performed and the member sizes modified. The details of the PDUF and SMOA program can be found in Ref. 9, and the theoretical basis of the subassemblage method of analysis is given in Ref. 7.
Very few unbraced multistory frames have been designed plastically on the basis of the stability limit load (8). Second-order analysis, however, has been used in recent years to assess the ultimate strength of some complex studies designed by the allowable-stress method (15).

Design Based on Shakedown Load

For steel structures subjected to non-proportional static loading, the theoretical limit load is the shakedown load in order to avoid incremental collapse failure. This type of failure, however, is believed to be highly unlikely in practice for a number of reasons. The following is quoted from Ref. 1.

"Practically every recent investigator of the subject has concluded that the problem of variable repeated loading may be disregarded for building frames designed for the usual conditions of static loading. The probability of failure by a single overload appears to be much greater than the probability of failure by alternating plasticity or by loss of deflection stability."

"Of particular significance is the fact that the ratio of live load to dead load must be very large before the load-carrying capacity is reduced because of load repetitions. In nearly all the tests performed, extreme examples were chosen in which all of the load was considered to be live load. It is unusual to find such extreme load variations in building structures. The live load is seldom more than two-thirds of the total load and usually it is of the order of one-third of the total."

"It must be remembered that the load factor does not provide for possible overloads alone. It also accounts for such additional factors as variation in material properties, dimensions, workmanship, fabrication, methods of analysis, etc. Therefore, variation in live load alone could not properly be assumed to account for the full value of the factor of safety."

"The results of the most recent tests using rolled shapes have shown that the observed shakedown load was always greater than the theoretically predicted value. Since the theoretical values are seldom more than 20% below the plastic limit load, the practicality of this problem loses much of its significance."

To the author's knowledge, only a few building structures have been designed explicitly to satisfy the shakedown requirements (3). For structures designed to resist high live load or severe deflection constraints, a careful assessment of the effect of load repetitions may be necessary.
Inelastic Design of Seismic Structures

Because of the increased activities in seismic hazard mitigation research in the recent years, new and more rational concepts of designing steel structures have been developed and applied in practice. Three areas of the new development are selected here for a brief review: (1) explicit inelastic design concept, (2) eccentrically braced seismic-resistant frames, and (3) analytical modeling of complex structures for inelastic analysis and design.

Most of the design procedures for seismic-resistant building structures utilize an elastic analysis in determining the bending moment and axial force distributions. All members and joints in the structure are then proportioned to satisfy certain strength and ductility requirements, regardless whether or not they would participate in the overall inelastic action during a strong earthquake. The effect of inelastic deformation is therefore taken into account only in an implicit manner. The ductile or non-ductile behavior of a particular type of structural system is recognized empirically in determining the design base shear.

Although such design procedures have produced buildings which have performed well in the past earthquakes, the inconsistency involved in the structural analysis phase and the member-design phase has along been recognized. The requirements that all members and joints must be designed to achieve a "sufficient" ductility sometimes result in uneconomical structures which are difficult to construct. In the recent years, several design procedures which consider explicitly the overall inelastic behavior of the structure in determining its design base shear or in proportioning the critical members or joints. For steel buildings Kato and his colleagues in Japan have developed an inelastic design method which allows the base shear to be adjusted in accordance with the story ductility of the structure (12).

It is generally recognized that moment-resisting frames have the desired ductile characteristics for use in an aseismic structure. However, if the building is also required to resist lateral load due to wind, the moment-resisting frames tend to be too flexible. The sizes of the girders are often increased to satisfy drift requirements. On the other hand, diagonally braced frames usually have good stiffness characteristics, but their behavior in the inelastic range is not considered to be desirable for aseismic design. The braces tend to buckle under axial compression. The "eccentrically braced frame", which utilizes strong braces to force yielding in the girders, seems to have both the advantages of high stiffness for resisting wind and good ductility for resisting earthquake. The recent work of Popov and his colleagues have resulted in design criteria for eccentrically braced frames (18, 19, 21), and they have already been applied to several high-rise buildings in California.

The behavior of frames with eccentric K braces has recently been studied experimentally and analytically as part of the on-going US-Japan Cooperative Research Program in Earthquake Engineering. A full-scale, six-story steel building structure, consisting of two moment frames and a braced frame, has been tested at the Japanese
Building Research Institute. The dimensions of the frames are shown in Fig. 2. In one phase of the program, eccentric K braces were installed in one of the bays and portions of the girders, which acted as shear links, were strengthened with web stiffeners. The structure was tested to large lateral drift levels using the pseudo-dynamic testing method. Very ductile behavior was observed throughout the test.

To be able to predict analytically the seismic response of structures is an important part of the design process. Modeling techniques and computer programs have been developed to perform inelastic analysis on three-dimensional structures subjected to multi-component earthquake ground motions (4, 5). Complex steel and mixed steel and concrete structures can now be analyzed to determine the critical locations, where inelastic action may occur and where special design details may be required. The computer programs can also be used to analyze earthquake damaged buildings using recorded ground motions. Some success in this area has been reported (17). Figure 3 shows the predicted inelastic seismic response of a four-story moment frame damaged during the Miyagi-Ken-Oki earthquake of June 1978 in Japan. Also indicated is the permanent deformation of the structure observed during a post earthquake inspection of the building.

Summary

Many different types of structures are now designed by utilizing their reserve strength in the plastic range. A discussion of the various methods that are being employed to utilize this strength in design has been presented. Plastic strength is recognized partially in the allowable-stress design, but the real factor of safety of structures designed by such an approach is difficult to assess. For continuous girders and low-rise building frames, designs are routinely made using the plastic limit load as the basis of design. The plastic limit load, however, overestimates the strength of a high-rise structure because of the P-Δ effect. To recognize this problem a proposal has been made to base the design directly on the stability limit load, which can be determined from a second-order elastic-plastic analysis. Very few actual designs, however, have been carried out on this basis. For structures subjected to variable repeated loading, the appropriate limit load is the shakedown load and the limit state is defined by deflection instability. The use of shakedown load in bridge design has recently been proposed.

Inelastic behavior is recognized extensively in design of aseismic structures and other dynamically loaded structures. A description has been given of an explicit inelastic design approach using the energy concept. The eccentrically braced frame with ductile shear links represents an ideal system to meet both the stiffness and energy absorption requirements. The advances in analytical modeling have made it possible to analyze complex structures for their response under multi-directional earthquake ground shaking.

Inelastic behavior is recognized in different ways in the current design practice. More extensive application of the plastic concepts in structural design is expected in the future through continuing research and updating of design specifications.
Appendix--References


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Fig. 1 Load-Deflection Curves of Frame under Proportional Loads
Fig. 2 Dimensions of Frames in Test Building

- **Exterior Frames A and C**
- **Interior Frame B**

**Legend:**
- $B_c =$ Concentric Brace (Phase I)
- $B_e =$ Eccentric Brace (Phase II)
Fig. 3 Seismic Response of a Damaged Building
Key Words

Structures, steel, building, bridge, plastic behavior, instability, buckling, ductility, earthquake, dynamic loading